12th Edition

Complete Solutions Manual

A Problem Solving Approach to

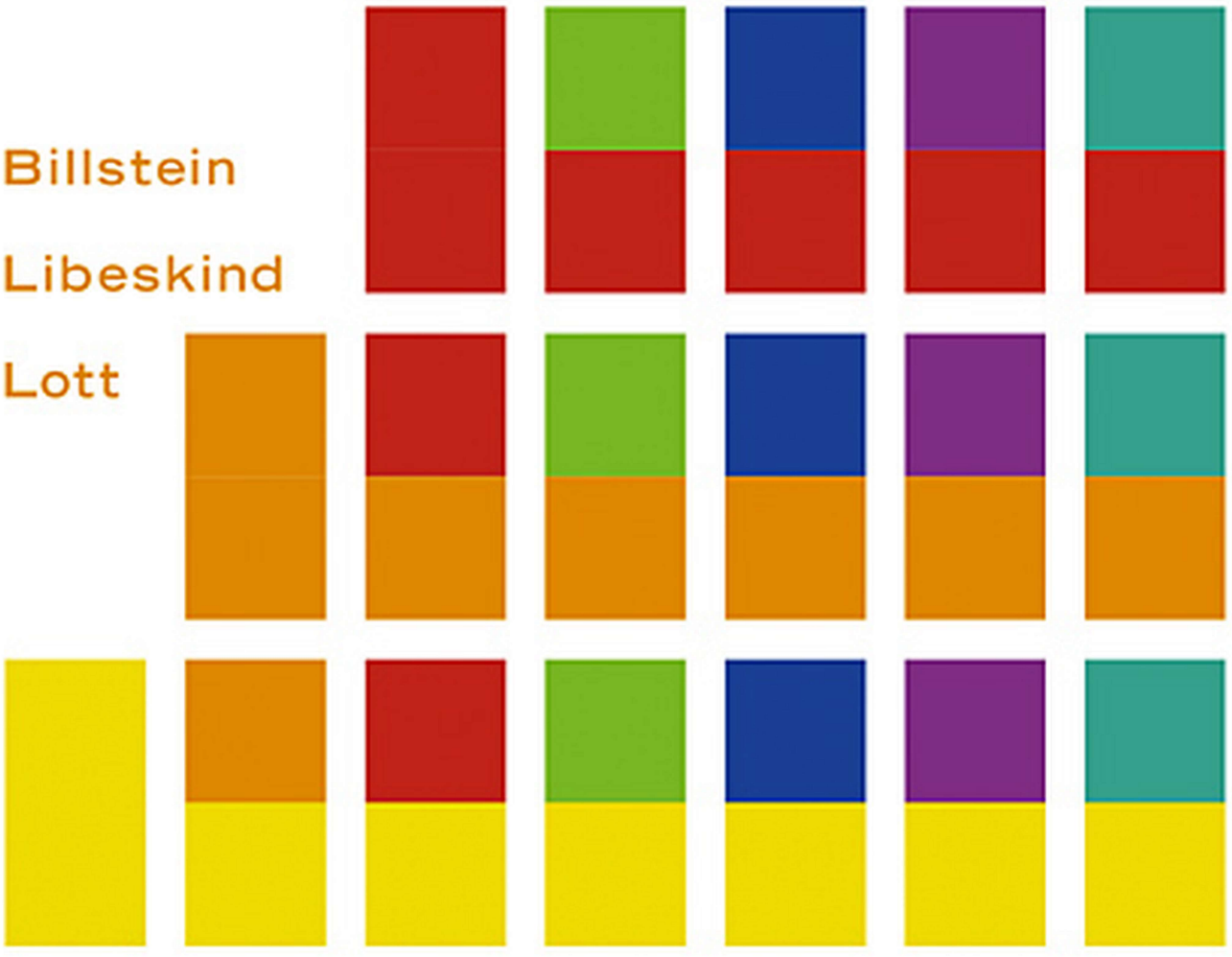
Mathematics

for Elementary School Teachers I

Billstein

Libeskind

Lott



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CHAPTER 1

AN INTRODUCTION TO PROBLEM SOLVING

Assessment 1-1A: Mathematics and Problem Solving

1. (a) List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 2 & + & \cdots & + & 98 & + & 99 \\ 99 & + & 98 & + & \cdots & + & 2 & + & 1 \\ \hline 100 & + & 100 & + & \cdots & + & 100 & + & 100 \end{array}$$

There are 99 sums of 100. Thus the total can be found by computing $\frac{99 \cdot 100}{2} = \mathbf{4950}$. (Another way of looking at this problem is to realize there are $\frac{99}{2} = 49.5$ pairs of sums, each of 100; thus $49.5 \cdot 100 = 4950$.)

(b) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{1001-1}{2} + 1 = 501$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 3 & + & \cdots & + & 999 & + & 1001 \\ 1001 & + & 999 & + & \cdots & + & 3 & + & 1 \\ \hline 1002 & + & 1002 & + & \cdots & + & 1002 & + & 1002 \end{array}$$

There are 501 sums of 1002. Thus the total can be found by computing $\frac{501 \cdot 1002}{2} = \mathbf{251{,}001}$.

(c) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{300-3}{3} + 1 = 100$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 3 & + & 6 & + & \cdots & + & 297 & + & 300 \\ 300 & + & 297 & + & \cdots & + & 6 & + & 3 \\ \hline 303 & + & 303 & + & \cdots & + & 303 & + & 303 \end{array}$$

There are 100 sums of 303. Thus the total can be found by computing $\frac{100 \cdot 303}{2} = 15{,}150$.

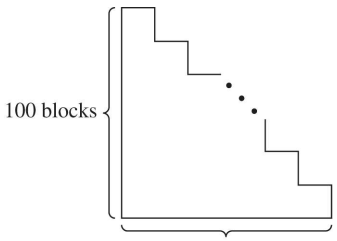
(d) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{400-4}{4} + 1 = 100$ terms.

List the numbers:

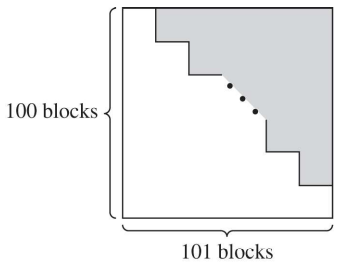
$$\begin{array}{rcrcccrcr} 4 & + & 8 & + & \cdots & + & 396 & + & 400 \\ 400 & + & 396 & + & \cdots & + & 8 & + & 4 \\ \hline 404 & + & 404 & + & \cdots & + & 404 & + & 404 \end{array}$$

There are 100 sums of 404. Thus the total can be found by computing $\frac{100 \cdot 404}{2} = 20{,}200$.

2. (a)



(b)



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When the stack in (a) and a stack of the same size is placed differently next to the original stack in (a), a rectangle containing 100 (101) blocks is created. Since each block is represented twice, the desired sum is $100(101)/2 = \mathbf{5050}$.

While the above represents a specific example, the same thinking can be used for any natural number n to arrive at a formula $n(n+1)/2$.

3. There are $\frac{147-36}{1} + 1 = 112$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 36 & + & 37 & + & \cdots & + & 146 & + & 147 \\ 147 & + & 146 & + & \cdots & + & 37 & + & 36 \\ \hline 183 & + & 183 & + & \cdots & + & 183 & + & 183 \end{array}$$

There are 112 sums of 183. Thus the total can be found by computing $\frac{112 \cdot 183}{2} = \mathbf{10,248}$.

4. **(a)** Make a table as follows; there are 9 rows so there are 9 different ways.

6-cookie packages	2-cookie packages	single-cookie packages
1	2	0
1	1	2
1	0	4
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

(b) Make a table as follows; there are 12 rows so there are **12 different ways**.

6-cookie packages	2-cookie packages	single-cookie packages
2	0	0
1	3	0
1	2	2
1	1	4
1	0	6
0	6	0
0	5	2
0	4	4
0	3	6
0	2	8
0	1	10
0	0	12

5. If each layer of boxes has 7 more than the previous layer we can add powers of 7:

$7^0 = 1$ (red box)

$7^1 = 7$ (blue boxes)

$7^2 = 49$ (black boxes)

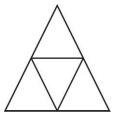
$7^3 = 343$ (yellow boxes)

$7^4 = 2401$ (gold boxes)

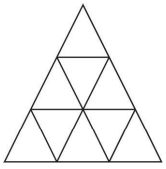
$1 + 7 + 49 + 343 + 2401 = 2801$ boxes altogether.

6. Using strategies from Poyla's problem solving list identify subgoals (solve simpler problems) and make diagrams to solve the original problem.

1 triangle; name this the "unit" triangle.



This triangle is made of 4 unit triangles. Counting the large triangle there are 5 triangles



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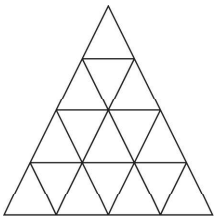
Unit triangles	4 unit triangles	9 unit triangles
9	3	1

13 total triangles



Unit triangles	4 unit triangles	9 unit triangles	16 unit triangles
16	7	3	1

There are 27 triangles in the original figure.

7. Observe that $E = (1 + 1) + (3 + 1) + \cdots + (97 + 1) = O + 49$. Thus, E is 49 more than O..

Alternative strategy:

$$O + E = 1 + 2 + 3 + 4 + 5 + 6 + \cdots + 97 + 98$$

$$= \frac{98(99)}{2} = 49(99)$$

$$E = 2(1 + 2 + 3 + 4 + \cdots + 49) = 2\left(\frac{49(50)}{2}\right) = 49(50)$$

$$O = O + E - E = 49(99) - 49(50) = 49(49).$$

So O is 49 less than E.

8. Bubba is last; Cory must be between Alababa and Dandy; Dandy is faster than Cory. Listing from fastest to slowest, the finishing order is then Dandy, Cory, Alababa, and Bubba.

9. Make a table.

$20 bills	$10 bills	$5 bills
2	1	0
2	0	2
1	3	0
1	2	2
1	1	4
1	0	6
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

There are twelve rows so there are twelve different ways. 12

10. The diagonal from the left, top corner to the right, bottom corner sums to $17 + 22 + 27 = 66$.

The first row sums to $17 + a + 7 = 24 + a$. So $a = 66 - 24 = \mathbf{42}$. The last column sums to $7 + b + 27 = 34 + b$. So $b = 66 - 34 = \mathbf{32}$. The first column sums to $17 + 12 + c = 29 + c$. So $c = 66 - 29 = \mathbf{37}$. The second column sums to $42 + 22 + d = 64 + d$. So $d = 66 - 64 = \mathbf{2}$.

11. Debbie and Amy began reading on the same day, since 72 pages for Debbie ÷ 9 pages per day = 8 days. Thus Amy is on 6 pages per day × 8 days = **page 4**8.

12. The last three digits must sum to 20, so the second to last digit must be $20 - (7 + 4) = 9$. Since the sum of the 11th , 12th, and 13th digits is also 20, the 11th digit is $20 - (7 + 9) = 4$.

A		7								4	7	9	4

We can continue in this fashion until we find that A is 9, or we can observe the repeating pattern from back to front, 4, 9, 7, 4, 9, 7, … and discover that A is 9.

13. Choose the box labeled Oranges and Apples (Box B). Retrieve a fruit from Box B. Since Box B is mislabeled, Box B should be labeled as having the fruit you retrieved. For example, if you retrieved an apple, then Box B should be labeled Apples. Since Box A is mislabeled, the Oranges and Apples label should be placed on Box A. These leave only one possibility for Box C; it should be labeled Oranges. If an orange was retrieved from Box B, then Box C would be labeled Oranges and Apples and Box A should be labeled Apples.

14. The electrician made $1315 for 4 days at $50 per hour. She spent $15 per day on gasoline so 4 • $15 = $60 on gasoline. The total is then $1315 + $60 = $ 1375. At $50 per hour, she worked $\frac{1375}{50} = 27.5$ hours.

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15. Working backward: Top – 6 rungs – 7 rungs + 5 rungs – 3 rungs = top – 11 rungs, which is located at the middle.

From the middle rung travel up 11 to the top or down 11 to the bottom. Along with the starting rung, then, there are 11 + 11 + 1 = **23 rungs.**

Assessment 1-1B

1. (a) List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 2 & + & \cdots & + & 48 & + & 49 \\ 49 & + & 48 & + & \cdots & + & 2 & + & 1 \\ \hline 50 & + & 50 & + & \cdots & + & 50 & + & 50 \end{array}$$

There are 49 sums of 50. Thus the total can be found by computing $\frac{49 \cdot 50}{2} = \mathbf{1225}$. (Another way of looking at this problem is to realize there are $\frac{49}{2} = 24.5$ pairs of sums, each of 50; thus $24.5 \cdot 50 = 1225$.)

(b) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{2009-1}{2} + 1 = 1005$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 1 & + & 3 & + & \cdots & + & 2007 & + & 2009 \\ 2009 & + & 2007 & + & \cdots & + & 3 & + & 1 \\ \hline 2010 & + & 2010 & + & \cdots & + & 2010 & + & 2010 \end{array}$$

There are 1005 sums of 2010. Thus the total can be found by computing $\frac{1005 \cdot 2010}{2} = \mathbf{1,010,025}$.

(c) The number of terms in any sequence of numbers may be found by subtracting the first term from the last, dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{600-6}{6} + 1 = 100$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 6 & + & 12 & + & \cdots & + & 594 & + & 600 \\ 600 & + & 594 & + & \cdots & + & 12 & + & 6 \\ \hline 606 & + & 606 & + & \cdots & + & 606 & + & 606 \end{array}$$

There are 100 sums of 606. Thus the total can be found by computing $\frac{606 \cdot 100}{2} = 30,300$.

(d) The number of terms in any sequence of numbers may be found by subtracting the first term from the last (or the last from the first if the first is greater than the last), dividing the result by the common difference between terms, and then adding 1 (because both ends must be accounted for). Thus $\frac{1000-5}{5} + 1 = 200$ terms.

List the numbers:

$$\begin{array}{rcrcccrcr} 1000 & + & 995 & + & \cdots & + & 10 & + & 5 \\ 5 & + & 10 & + & \cdots & + & 995 & + & 1000 \\ \hline 1005 & + & 1005 & + & \cdots & + & 1005 & + & 1005 \end{array}$$

There are 200 sums of 1005. Thus the total can be found by computing $\frac{1005 \cdot 200}{2} = 100,500$.

2. (a) The diagram illustrates how the numbers can be paired to form 50 sums of 101. The sum of the first 100 natural numbers is $50(101) = \mathbf{5050}$.

(b) A diagram similar to the one in 2a would illustrate how the numbers can be paired to form 100 sums of 202. Since the last term is odd, the middle term, 101, is left unpaired. So, the sum of the first 201 natural numbers is 100 • 202 + 101 = 20,301.

3. There are $\frac{203-58}{1} + 1 = 146$ terms. terms

List the numbers:

$$\begin{array}{rcrcccrcr} 58 & + & 59 & + & \cdots & + & 202 & + & 203 \\ 203 & + & 202 & + & \cdots & + & 59 & + & 58 \\ \hline 261 & + & 261 & + & \cdots & + & 261 & + & 261 \end{array}$$

There are 146 sums of 261. Thus the total can be found by computing $\frac{146 \cdot 261}{2} = \mathbf{19,053}$.

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4. There are many answers to this problem. A systematic list is a good approach. Using only two numbers and addition, 5 rows give 5 different ways.

One number	Eleven minus the number
1	10
2	9
3	8
4	7
5	6

We can view 6 + 5 as a different way than 5 + 6 and continue in this manner to find 10 different ways. Or, we can find 7 more ways using three numbers as follows.

1	1	9
2	1	8
3	1	7
4	1	6
5	1	5
1	2	8

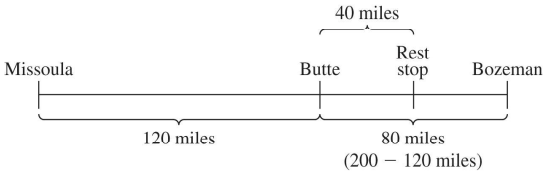
5. After two socks are drawn, the two socks match or they don't. If they match, we are done. If they don't match, the next sock drawn will match one of the two socks already drawn.

6. There are 13 squares of one unit each; 4 squares of four units each; and one square 9 units; for a total of 18 squares.

7.
$$\begin{aligned} P &= 1 + 3 + 5 + 7 + \ldots + 99 \\ Q &= \quad 5 + 7 + \ldots + 99 + 101 \\ Q - P &= (5 + \ldots + 99 + 101) - \\ &\quad (1 + 3 + 5 + \ldots + 99) \\ &= (101) - (1 + 3) \\ &= 97 \end{aligned}$$

Q is larger than P by 97.

8. A diagram will help.



The next step is 120 miles + 40 miles = **160 miles** from Missoula.

9. **(a)** Marc must have five pennies to make an even \$1.00. The minimum number of coins would have as many quarters as possible, or three quarters. The remaining $20¢$ must consist of at least one dime and one nickel; the only possibility is one dime and two nickels. The minimum is 5 pennies, 2 nickels, 1 dime, and 3 quarters, or 11 coins.

(b) The maximum number of coins is achieved by having as many pennies as possible. It is a requirement to have one quarter, one dime, and one nickel $= 40¢$, so there may then be 60 pennies for a total of 63 coins.

10. Adding the numbers gives 99. This means that each row, diagonal, and column must add to $99 \div 3 = 33$. Write 33 as a sum of the numbers in all possible ways:

$19 + 11 + 3$
$19 + 9 + 5$
$17 + 13 + 3$
$17 + 11 + 5$
$17 + 9 + 7$
$15 + 13 + 5$
$15 + 11 + 7$
$13 + 11 + 9$

Summarizing the pattern:

Number	Nr. sums with number
3	2
5	3
7	2
9	3
11	4
13	3
15	2
17	3
19	2

Thus 11 must be in the center of the square and 5, 9, 13, and 17 must be in the corners. One solution would be:

17	7	9
3	11	19
13	15	5

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11. Answers may vary; two solutions might be to:

(a) Put four marbles on each tray of the balance scale. Take the heavier four and weigh two on each tray. Take the heavier two and weigh one on each tray; the heavier marble will be evident on this third weighing.

(b) This alternative shows the heavier marble can be found more efficiently, two steps rather than three. Put three marbles in each tray of the balance scale.

(i) If the two trays are the same weight, the heavier marble is one of the remaining two. Weigh them to find the heavier.

(ii) If one side is heavier, take two of the three marbles and weigh them. If they are the same weight, the remaining marble is the heavier. If not, the heavier will be evident on this second weighing.

12. **(a)** There are:

$1 \text{ partridge} \times 12 \text{ days} = 12 \text{ gifts};$
$2 \text{ doves} \times 11 \text{ days} = 22 \text{ gifts};$
$3 \text{ hens} \times 10 \text{ days} = 30 \text{ gifts};$
$4 \text{ birds} \times 9 \text{ days} = 36 \text{ gifts};$
$5 \text{ rings} \times 8 \text{ days} = 40 \text{ gifts};$
$6 \text{ geese} \times 7 \text{ days} = 42 \text{ gifts};$
$7 \text{ swans} \times 6 \text{ days} = 42 \text{ gifts};$
$8 \text{ maids} \times 5 \text{ days} = 40 \text{ gifts};$
$9 \text{ ladies} \times 4 \text{ days} = 36 \text{ gifts};$
$10 \text{ lords} \times 3 \text{ days} = 30 \text{ gifts};$
$11 \text{ pipers} \times 2 \text{ days} = 22 \text{ gifts; and}$
$12 \text{ drummers} \times 1 \text{ day} = 12 \text{ gifts.}$

So the gifts given the most by your true love was 42 geese and 42 swans.

(b) $12 + 22 + \cdots + 22 + 12 =$ **364** gifts total.

13. **(a)** There must be 1 or 3 quarters for an amount ending in 5. Then dimes can add to \$1.15 plus 4 pennies to realize \$1.19. Thus:

Quarters	Dimes	Pennies	Total
3	4	4	\$1.19
1	9	4	\$1.19

and in neither case can change for \$1.00 be made.

(b) Two or zero quarters would allow an amount ending in 0. Then more combinations of dimes or pennies could add to \$1.00.

14. If the price of 15 sandwiches equals the price of 20 salads, each sandwich will buy $\frac{20}{15} = \frac{4}{3}$ salads. Thus 3 sandwiches $= 3\left(\frac{4}{3}\right) =$ **4 salads**.

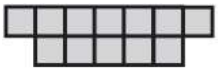
15. Use a variable and a table

12 AM	5 AM	9 AM	12 PM
T	$T - 15$	$2(T - 15)$	$2(T - 15) + 10$

So, $2(T - 15) + 10 = 32$
$2T - 30 + 10 = 32$
$2T - 20 = 32$
$2T = 52$
$T =$ **26 degrees**.

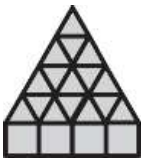
Assessment 1-2A: Explorations with Patterns

1. **(a)** Each figure in the sequence adds one box each to the top and bottom rows. The next would be:



(b) Each figure in the sequence adds one upright and one inverted triangle. The next would be:

(c) Each figure in the sequence adds one box to the base and one row to the overall triangle. The next would be:



2. **(a)** Terms that continue a pattern are 17, 21, 25, , … . This is an arithmetic sequence because each successive term is obtained from the previous term by addition of 4.

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(b) Terms that continue a pattern are 220, 270, 320, … . This is arithmetic because each successive term is obtained from the previous term by addition of 50.

(c) Terms that continue a pattern are 27, 81, 243, … . This is geometric because each successive term is obtained from the previous term by multiplying by 3.

(d) Terms that continue a pattern are $10^9, 10^{11}, 10^{13}, \ldots$. This is geometric because each successive term is obtained from the previous term by multiplying by 10^2.

(e) Terms that continue a pattern are $193+10\cdot 2^{30}, 193+11\cdot 2^{30}, 193+12\cdot 2^{30}, \ldots$. This is arithmetic because each successive term is obtained from the previous term by addition of 230.

3. In these problems, let a_n represent the nth term in a sequence, a_1 represent the first term, d represent the common difference between terms in an arithmetic sequence, and r represent the common ratio between terms in a geometric sequence. In an arithmetic sequence, $a_n = a_1 + (n-1)d$; in a geometric sequence $a_n = a_1 r^{n-1}$. Thus:

(a) Arithmetic sequence: $a_1 = 1$ and $d = 4$:

(i) $a_{100} = 1 + (100-1)\cdot 4$
$= 1 + 99\cdot 4 = 397.$

(ii) $a_n = 1 + (n-1)\cdot 4$
$= 1 + 4n - 4 = 4n - 3.$

(b) Arithmetic sequence: $a_1 = 70$ and $d = 50$:

(i) $a_{100} = 70 + (100-1)\cdot 50$
$= 70 + 99\cdot 50 = 5020.$

(ii) $a_n = 70 + (n-1)\cdot 50$
$= 70 + 50n - 50$ or $50n + 20.$

(c) Geometric sequence: $a_1 = 1$ and $r = 3$:

(i) $a_{100} = 1\cdot 3^{100-1} = 3^{99}.$

(ii) $a_n = 1\cdot 3^{n-1} = 3^{n-1}.$

(d) Geometric sequence: $a_1 = 10$ and $r = 10^2$:

(i) $a_{100} = 10\cdot (10^2)^{(100-1)} = 10\cdot (10^2)^{99}$
$= 10\cdot 10^{198} = 10^{199}.$

(ii) $a_n = 10\cdot (10^2)^{(n-1)}$
$= 10\cdot 10^{(2n-2)} = 10^{2n-1}.$

(e) Arithmetic sequence:
$a_1 = 193 + 7\cdot 2^{30}$ and $d = 2^{30}$:

(i) $a_{100} = 193 + 7\cdot 2^{30} + (100-1)\cdot 2^{30}$
$= 193 + 7\cdot 2^{30} + 99\cdot 2^{30}$
$= 193 + 106\cdot 2^{30}.$

(ii) $a_n = 193 + 7\cdot 2^{30} + (n-1)\cdot 2^{30}$
$= 193 + (n+6)\cdot 2^{30}.$

4. 2, 7, 12, … . Each term is the 5th number on a clock face (clockwise) from, the preceding term.

5. (a) Make a table.

Number of term	Term
1	$1\cdot 1\cdot 1 = 1$
2	$2\cdot 2\cdot 2 = 8$
3	$3\cdot 3\cdot 3 = 27$
4	$4\cdot 4\cdot 4 = 64$
5	$5\cdot 5\cdot 5 = 125$
6	$6\cdot 6\cdot 6 = 216$
7	$7\cdot 7\cdot 7 = 343$
8	$8\cdot 8\cdot 8 = 512$
9	$9\cdot 9\cdot 9 = 729$
10	$10\cdot 10\cdot 10 = 1000$
11	$11\cdot 11\cdot 11 = 1331$

The 11^{th} term 1331 is the least 4-digit number greater than 1000.

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(b) The 9^{th} term 729 is the greatest 3-digit number in this pattern.

(c) The greatest number less than 10^4 is $21 \cdot 21 \cdot 21 = 9261$.

(d) The cell A14 corresponds to the 14th term, which is $14 \cdot 14 \cdot 14 = 2744$.

6. (a) The number of matchstick squares in each windmill form an arithmetic sequence with $a_1 = 5$ and $d = 4$. The number of matchstick squares required to build the 10th windmill is thus $5 + (10 - 1) \cdot 4 = 5 + 9 \cdot 4 =$ **41 squares**.

(b) The ***n***th windmill would require $5 + (n - 1) \cdot 4 = 5 + 4n - 4 = \mathbf{4n + 1}$ squares.

(c) There are 16 matchsticks in the original windmill. Each additional windmill adds 12 matchsticks.

This is an arithmetic sequence with $a_1 = 16$ and $d = 12$, so $a_n = 16 + (n - 1) \cdot 12 = \mathbf{12n + 4}$ matchsticks.

7. (a) Each cube adds four squares to the preceding figure; or 6, 10, 14, … . This is an arithmetic sequence with $a_1 = 6$ and $d = 4$. Thus $a_{15} = 6 + (15 - 1) \cdot 4 = 62$ *squares* to be painted in the 10th figure.

(b) This is an arithmetic sequence with $a_1 = 6$ and $d = 6$. The ***n***th term is thus: $a_n = 6 + (n - 1) \cdot 4 = \mathbf{4n + 2}$.

8. Since the first year begins with 700 students, after the first year there would be 760, after the second there would be 820, … , and after the twelfth year the number of students would be the 13^{th} term in the sequence.

This then is an arithmetic sequence with $a_1 = 700$ and $d = 60$, so the 13^{th} term (current enrollment + twelve more years) is: $700 + (13 - 1) \cdot 60 = 1420$ *students*.

9. Using the general expression for the *n*th term of an arithmetic sequence with $a_1 = 24,000$ and $a_9 = 31,680$ yields:

$$31680 = 24000 + (9 - 1)d$$
$$31680 = 24000 + 8d \Rightarrow d = 960,$$

the amount by which Joe's income increased each year.

To find the year in which his income was \$45120:

$$45120 = 24000 + (n - 1) \cdot 960$$
$$45120 = 23040 + 960n$$
$$\Rightarrow n = 23.$$

Joe's income was \$45,120 in his **23rd year**.

10. (a) If the first difference of the sequence increases by 2 for each term, then the five first differences between the first six terms of the original sequence are 2, 4, 6, 8, 10. If the first term of the original sequence is 3, then the first six terms are 3, 5, 9, 15, 23, 33.

(b) If the first term is a, then $a + (a + 2) = 10 \Rightarrow a = 4$. Thus the first six terms of the original sequence are 4, 6, 10, 16, 24, 34.

(c) If the fifth term is 35, then:

Sixth term $35 + 10 = 45$

Fourth term $35 - 8 = 27$

Third term $27 - 6 = 21$

Second term $21 - 4 = 17$

First term $17 - 2 = 15$.

Thus the sequence is 15, 17, 21, 27, 35, 45.

11. (a) Look for the differences:

5 6 14 32 64 115 191

1 8 18 32 51 76

7 10 14 19 25

3 4 5 6

1 1 1

The third difference row is an arithmetic sequence with fixed difference of 1. Thus the 6th term in the second difference row $25 + 7 = 32$; the 7^{th} term in the first difference row is $76 + 32 = 108$; and the 8th term in the original sequence is $191 + 108 = 299$. Using the same reasoning, the next three terms in the original sequence will be 299, 447, 644.

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(b) Look for the differences:

0	2	6	12	20	30	42
	2	4	6	8	10	12
		2	2	2	2	2

The first difference row is an arithmetic sequence with fixed difference 2. Thus the 7th term in the first difference row is $12 + 2 = 14$; the 8th term in the original sequence is $42 + 14 = 56$. Using the same reasoning, the next three terms in the original sequence are 56, 72, 90.

12. (a) Using the general expression for the nth term of an arithmetic sequence with $a_1 = 51, a_n = 251$, and $d = 1$ yields:

$251 = 51 + (n - 1) \cdot 1 \Rightarrow$

$251 = 50 + n \Rightarrow n = 201.$

There are 201 terms in the sequence.

(b) Using the general expression for the ***n***th term of a geometric sequence with $a_1 = 1$, $a_n = 2^{60}$, and $r = 2$ yields:

$2^{60} = 1(2)^{n-1} = 2^{n-1}$

$\Rightarrow n - 1 = 60 \Rightarrow n = 61.$

There are 61 terms in the sequence.

(c) Using the general expression for the ***n***th term of an arithmetic sequence with $a_1 = 10$, $a_n = 2000$, and $d = 10$ yields:

$2000 = 10 + (n - 1) \cdot 10 \Rightarrow$

$2000 = 10n \Rightarrow n = 200.$

There are 200 terms in the sequence.

(d) Using the general expression for the n^{th} term of a geometric sequence with $a_1 = 1$, $a_n = 1024$, and $r = 2$ yields:

$1024 = 1(2)^{n-1} \Rightarrow$

$2^{10} = 2^{n-1} \Rightarrow n - 1 = 10$

$\Rightarrow n = 11.$

There are 11 terms in the sequence.

13. (a)

First term:	$(1)^2 + 2 = \mathbf{3}$;
Second term:	$(2)^2 + 2 = \mathbf{6}$;
Third term:	$(3)^2 + 2 = \mathbf{11}$;
Fourth term:	$(4)^2 + 2 = \mathbf{18}$; and
Fifth term:	$(5)^2 + 2 = \mathbf{27}$.

(b)

First term:	$5(1) + 1 = 6$;
Second term:	$5(2) + 1 = 11$;
Third term:	$5(3) + 1 = 16$;
Fourth term:	$5(4) + 1 = 21$; and
Fifth term:	$5(5) + 1 = 26$.

(c)

First term:	$10^{(1)} - 1 = \mathbf{9}$;
Second term:	$10^{(2)} - 1 = \mathbf{99}$;
Third term:	$10^{(3)} - 1 = \mathbf{999}$;
Fourth term:	$10^{(4)} - 1 = \mathbf{9999}$; and
Fifth term:	$10^{(5)} - 1 = \mathbf{99999}$.

(d)

First term:	$3(1) - 2 = 1$**;**
Second term:	$3(2) - 2 = 4$**;**
Third term:	$3(3) - 2 = 7$**;**
Fourth term:	$3(4) - 2 = 10$**;** and
Fifth term:	$3(5) - 2 = 13$**.**

14. Answers may vary; examples are:

(a) If $n = 5$, then $\frac{5+5}{5} = 2 \neq 5 + 1 = 6$.

(b) If $n = 2$, then $(2 + 4)^2 = 36 \neq 2^2 + 16 = 20$.

15. (a) There are 1, 5, 11, 19, 29 tiles in the five figures. Each figure adds 2n tiles to the preceding figure, thus a_6 the 6th term has $29 + 12 =$ **41 tiles**.

(b) $n^2 = 1, 4, 9, 16, 25, \ldots$. Adding $(n - 1)$ to n^2 yields $1, 5, 11, 19, 29, \ldots$, which is the proper sequence. Thus the n*th* term has $n^2 + (n - 1)$.

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(c) If $n^2 + (n-1) = 1259$;

Then $n^2 + n - 1260 = 0$. This implies $(n-35)(n+36) = 0$, so $n = 35$.

There are 1259 tiles in the 35th figure.

16. The nth term of the arithmetic sequence is $200 + n(200)$. The sequence can also be generated by adding 200 to the previous term. The n^{th} term of the geometric sequence is 2^n. The sequence can also be generated by multiplying the previous term by 2. Make a table.

Number of the term	Arithmetic term	Geometric term
7	1600	128
8	1800	256
9	2000	512
10	2200	1024
11	2400	2048
12	2600	4096

With the 12th term, the geometric sequence is greater.

17. **(a)** Start with one piece of paper. Cutting it into five pieces gives us 5. Taking each of the pieces and cutting it into five pieces again gives $5 \cdot 5 = 25$ pieces. Continuing this process gives a geometric sequence: 1, 5, 25, 125, … . After the 5^{th} cut there are $5^5 = 3125$ pieces of paper.

(b) The number of pieces after the nth cut would be 5^n.

18. **(a)** For an arithmetic sequence there is a common difference between the terms. Between 39 and 69 there are three differences so we can find the common difference by subtracting 39 from 69 and dividing the answer by three: $69 - 39 = 30$ and $30 \div 3 = 10$. The common difference is 10 and we can find the missing terms: 39 – 10 = 29 and 39 + 10 = 49 and 49 + 10 = 59.

(b) For an arithmetic sequence there is a common difference between the terms. Between 200 and 800 there are three differences so we can find the common difference by subtracting 200 from 800 and dividing the answer by three: $800 - 200 = 600$ and $600 \div 3 = 200$. The common difference is 200 and we can find the missing terms: 200 – 200 = 0 and 200 + 200 = 400 and 400 + 200 = 600.

(c) For a geometric sequence there is a common ration between the terms. Between 5^4 and 5^{10} there are three common ratios used so we can find the common ratio by dividing 5^{10} by 5^4 and then taking the cube root:

$5^{10} \div 5^4 = 5^6$ and $(5^6)^{(\frac{1}{3})} = 5^2$. The common ratio is 52 and we can find the missing terms: $5^4 \div 5^2 = 5^2, 5^4 \cdot 5^2 = 5^6, 5^6 \cdot 5^2 = 5^8$.

19. **(a)** Let's call the missing terms a, b, c, d, and f, then the sequence becomes:

$a, b, 1, 1, c, d, e, f$

$b + 1 = 1 \rightarrow b = 0$

$a + b = 1 \rightarrow a + 0 = 1 \rightarrow a = 1$

$1 + 1 = c \rightarrow c = 2$

$1 + c = d \rightarrow 1 + 2 = d \rightarrow d = 3$

$c + d = e \rightarrow 2 + 3 = e \rightarrow e = 5$

$d + e = f \rightarrow 3 + 5 = f \rightarrow = 8.$

The missing terms are 1, 0, 2, 3, 5, and 8.

(b) Let's call the missing terms a, b, c, and d, then the sequence becomes:

$a, b, c, 10, 13, d, 36, 59$

$c + 10 = 13 \rightarrow c = 3$

$b + c = 10 \rightarrow b + 3 = 10 \rightarrow b = 7$

$a + b = c \rightarrow a + 7 = 3 \rightarrow = {}^{-}4$

$10 + 13 = d \rightarrow d = 23$

The missing terms are -4, 7, 3, and 23.

(c) If a Fibonacci-type sequence is a sequence in which the first two terms are arbitrary and in which every term starting from the third is the sum of the previous two terms, then we can add 0 and 2 to get the third term and continue the pattern:

$0 + 2 = 2$

$2 + 2 = 4$

$2 + 4 = 6$

$4 + 6 = 10$

$6 + 10 = 16$

$10 + 16 = 26$

The missing terms are 2, 4, 6, 10, 16, and 26.

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20. Starting with 1 and 1 the Fibonacci sequence would be 1,1,2,3,5,8,13,21,34,55, … .

(a) 1 + 1 + 2 = 4 which is one less than the fifth term.

(b) 1 + 1 + 2 + 3 = 7 which is one less than the sixth term.

(c) 1 + 1 + 2 + 3 + 5 = 12 which is one less than the seventh term.

(d) The sum of the first n terms is one less than the $(n + 2)$ term. OR $F1 + F2 + F3 + \ldots + F_n = F_n + 2 - 1$.

(e) The sum of the first n-2 terms of the Fibonacci sequence is one less than the nth term.

21. (a)

Year 1 $80 + .05(80) = 84$

Year 2 $84 + .05(84) = 88.2$

Year 3 $88.2 + .05(88.2) = 92.61$

Year 4 $92.61 + .05(92.61) = 97.2405$

Year 5 $97.2405 + .05(97.2405) = 102.102525$
$\approx \$102.10.$

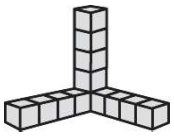
(b) This is a geometric sequence with $a1 = 80$ and $r = 1.05$, so the price after n years is $80 \cdot 1.05^n$.

Assessment 1-2B

1. (a) In a clockwise direction, the shaded area moves to a new position separated from the original by one open space, then two open spaces, then by three, etc. The separation in each successive step increases by one unit; next would be:

(b) Each figure in the sequence adds one row of boxes to the base. Next would be:

(c) Each figure in the sequence adds one box to the top and each leg of the figure. Next would be:



2. (a) Terms that continue a pattern are 18, 22, 26, … . This is an arithmetic sequence because each successive term is obtained from the previous term by addition of 4.

(b) Terms that continue a pattern are 39, 52, 65, … . This is an arithmetic sequence because each successive term is obtained from the previous term by addition of 13.

(c) Terms that continue a pattern are 256, 1024, 4096, … . This is a geometric sequence because each successive term is obtained from the previous term by multiplying by 4.

(d) Terms that continue a pattern are $2^{14}, 2^{18}, 2^{22}, \ldots$. This is a geometric sequence because each successive term is obtained from the previous term by multiplying by 24.

(e) Terms that continue a pattern are $100 + 4 \cdot 2^{50}, 100 + 6 \cdot 2^{50}, 100 + 8 \cdot 2^{50}, \ldots$.
This is a arithmetic sequence because each successive term is obtained from the previous term by adding by $2 \cdot 2^{50}$.

3. In these problems, a_n represents the nth term in a sequence, a_1 represents the first term, d represent the common difference between terms in an arithmetic sequence, and r represents the common ratio between terms in a geometric sequence.

In an arithmetic sequence, $a_n = a_1 + (n - 1)d$; in a geometric sequence, $a_n = a_1 r^{n-1}$. Thus:

(a) Arithmetic sequence: $a_1 = 2$ and $d = 4$.

(i) $a_{100} = 2 + (100 - 1) \cdot 4 = 398.$

(ii) $a_n = 2 + (n - 1) \cdot 4$
$= 2 + 4n - 4 = 4n - 2.$

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(b) Arithmetic sequence: $a_1 = 0$ **and** $d = 13$.

(i) $a_{100} = 0 + (100 - 1) \cdot 13 = 1287.$

(ii) $a_n = 0 + (n - 1) \cdot 13$
$= 13n - 13.$

(c) Geometric sequence: $a_1 = 4$ and $r = 4$.

(i) $a_{100} = 4 \cdot 4^{99} = 4^{100}.$

(ii) $a_n = 4 \cdot 4^{n-1} = 4^n.$

(d) Geometric sequence: $a_1 = 2^2$ and $r = 2^4$.

(i) $a_{100} = 2^2 \cdot (2^4)^{99} = 2^2 \cdot 2^{396} = 2^{398}.$

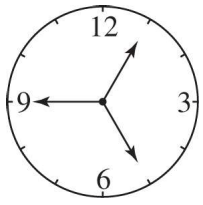
(ii) $a_n = 2^2 \cdot (2^4)^{(n-1)}$
$= 2^2 \cdot 2^{4n-4} = 2^{4n-2}.$

(e) Arithmetic sequence:
$a_1 = 100 + 4 \cdot 2^{50}$ and $d = 2^{51}$:

(i) $a_{100} = 100 + 4 \cdot 2^{50} + (100 - 1) \cdot 2^{51}$
$= 100 + 2 \cdot 2^{51} + 99 \cdot 2^{51}$
$= 100 + 101 \cdot 2^{51}.$

(ii) $a_n = 100 + 4 \cdot 2^{50} + (n - 1) \cdot 2^{51}$
$= 100 + 2 \cdot 2^{51} + (n - 1) \cdot 2^{51}$
$= 100 + (n + 1) \cdot 2^{51}.$

4. The hands must move 8 hours to move from 1 to 9 on the clock face. To move from 9 to 5, the hand must move 8 hours also. To move from 5 to 1, the hand must move another 8 hours. If we add 8 hours to 1 o'clock, we will land on the 9. This pattern will continue, so the next three terms are 9, 5, 1.



5. (a) Answers may vary:

(i) **The sum of the first n odd numbers is n^2;** e.g., $1 + 3 + 5 + 7 = 4^2$.

(ii) **Square the average of the first and last terms;** e.g.,
$1 + 3 + 5 + 7 = \left(\frac{1+7}{2}\right)^2 = 4^2.$

(b) There are $\frac{35-1}{2} + 1 = 18$ terms in this sequence.

(i) $1 + 3 + 5 + 7 + \cdots + 35 = 18^2$
$= \mathbf{324}.$

(ii) $\left(\frac{1+35}{2}\right)^2 = 18^2 = \mathbf{324}.$

6. (a) Note that 5 toothpicks are added to form each succeeding hexagon. This is an arithmetic sequence $a_1 = 6$ and $d = 5$, so $a_{10} = 6 + (10 - 1) \cdot 5 = 6 + 9 \cdot 5 = \mathbf{51}$ toothpicks.

(b) n hexagons would require $6 + (n - 1) \cdot 5 = 6 + 5n - 5 = \mathbf{5n + 1}$ toothpicks.

7. (a) Looking at the third figure, there are $5 + 3 + 1 = 9$ triangles. The fourth figure would than have $7 + 5 + 3 + 1 = 16$ triangles. An alternative to simply adding $7, 5, 3,$ and 1 together is to note that $7 + 1 = 8$ and $5 + 3 = 8$. There are $\frac{4}{2} = 2$ of these sums, and $2 \cdot 8 = 16$. Then the 100th figure would have $100 + 99 = 199$ triangles in the base, $99 + 98 = 197$ triangles in the second row, and so on until the 100th row where there would be 1 triangle. $199 + 1 = 200$; $197 + 3 = 200$; etc. and so the sum of each pair is 200 and there are $\frac{100}{2} = 50$ of these pairs. $50 \cdot 200 = 10,000$, or 10,000 triangles in the 100th figure.

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(b) The number of triangles in the ***n***th figure is $\frac{n}{2}$(number of triangles in base + 1). The number of triangles in the base is $n + (n - 1)$, or $2n - 1$. $(2n - 1) + 1 = 2n$. Then $\frac{n}{2}(2n) = n^2$, or ***n***2 triangles in the ***n***th figure.

8. This is a geometric sequence with $a_1 = \dfrac{15{,}360}{2}$ and $r = \frac{1}{2}$. The *n*th term of a geometric sequence is $a_n = a_1 r^{n-1}$; thus the 10th term would be $15360\left(\frac{1}{2}\right)^{10} =$ **15 liters**.

Note the progression of terms in the following table:

After Day	Amount of Water Remaining
1	$15{,}360 \cdot \frac{1}{2} = 7680$ liters
2	$7680 \cdot \frac{1}{2} = 3840$ liters
⋮	⋮
9	$60 \cdot \frac{1}{2} = 30$ liters
10	$30 \cdot \frac{1}{2} = 15$ liters

9. This is an arithmetic sequence with $a_1 = 8\frac{1}{6}$ (i.e., 8 a.m. plus 10 minutes, or $\frac{10}{60}$ of an hour) and $d = \frac{5}{6}$ (or $\frac{50}{60}$ of an hour). Thus

$a_8 = 8\frac{1}{6} + (8 - 1) \cdot \frac{5}{6} = 14$, or **2:00 p.m.**

(14 is 2:00 p.m. on a 24-hour clock.)

10. **(a)** If the first difference of the sequence increases by 3 for each term, then five first differences between the first six terms of the original sequence are 3, 6, 9, 12, 15. If the first term of the original sequence is 3, then the first six terms are 3, 6, 12, 21, 33, 48.

(b) If the first term is a, then $a + (a + 3) = 7 \Rightarrow a = 2$. Thus the first six terms of the original sequence are 2, 5, 11, 20, 32, 47.

(c) If the fifth term (a_5) is 34, then:

$a_6 = 34 + 15 = 49$

$a_4 = 34 - 12 = 22$

$a_3 = 22 - 9 = 13$

$a_2 = 13 - 6 = 7$

$a_1 = 7 - 3 = 4$.

Thus the sequence is 4, 7, 13, 22, 34, 49.

11. **(a)** Look for the differences:

3 8 15 24 35 48

5 7 9 11 13

The first difference row is an arithmetic sequence with fixed difference of 2. Thus the 6th term in the first difference row is $13 + 2 = 15$; the 7th term in the original sequence is $48 + 15 = 63$. Using the same reasoning, the next three terms in the original sequence are 63, 80, 99.

(b) Look for the differences:

1 7 18 37 67 111

6 11 19 30 44

5 8 11 14

The second difference row is an arithmetic sequence with fixed difference of 3. Thus the 5th term in the second difference row is $14 + 3 = 17$; the 6th term in the original sequence is $111 + 61 = 172$. Using the same reasoning, the next three terms in the original sequence are 172, 253, 357.

12. **(a)** The ***n*th** term for this geometric sequence is 3^{n-1}. Thus $3^{99} = 3^{n-1}$.

So $99 = n - 1$, and $n = 100$.

There are 100 terms in the sequence.

(b) The ***n*th** term for this arithmetic sequence is $9 + (n - 1) \cdot 4$. Thus $353 = 9 + (n - 1) \cdot 4$. Solving for ***n***, $n = 87$. There are 87 terms in the sequence.

(c) The ***n*th** term for this arithmetic sequence is $38 + (n - 1) \cdot 1$. Thus $238 = 38 + (n - 1) \cdot 1$. Solving for ***n***, $n = 201$. There are 201 terms in the sequence.

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13. **(a)**
First term: $5(1) - 1 = 4$
Second term: $5(2) - 1 = 9$
Third term: $5(3) - 1 = 14$
Fourth term: $5(4) - 1 = 19$
Fifth term: $5(5) - 1 = 24$

(b)
First term: $6(1) - 2 = \mathbf{4}$
Second term: $6(2) - 2 = \mathbf{10}$
Third term: $6(3) - 2 = \mathbf{16}$
Fourth term: $6(4) - 2 = \mathbf{22}$
Fifth term: $6(5) - 2 = \mathbf{28}$

(c)
First term: $5 \cdot 1 + 1 = \mathbf{6}$
Second term: $5 \cdot 2 + 1 = \mathbf{11}$
Third term: $5 \cdot 3 + 1 = \mathbf{16}$
Fourth term: $5 \cdot 4 + 1 = \mathbf{21}$
Fifth term: $5 \cdot 5 + 1 = \mathbf{26}$

(d)
First term: $1^2 - 1 = \mathbf{0}$
Second term: $2^2 - 1 = \mathbf{3}$
Third term: $3^2 - 1 = \mathbf{8}$
Fourth term: $4^2 - 1 = \mathbf{15}$
Fifth term: $5^2 - 1 = \mathbf{24}$

14. Answers may vary; examples are:

(a) If $n = 6$, then $\frac{3+6}{3} = 3 \neq 6$.

(b) If $n = 4$, then $(4 - 2)^2 = 4 \neq 4^2 - 2^2 = 12$.

15. **(a)** The first figure has 2 tiles, the second has 5 tiles, the third has 8 tiles, … . This is an arithmetic sequence where the $\boldsymbol{n^{th}}$ term is $2 + (n - 1) \cdot 3$.

Thus the 7^{th} term has $2 + (7 - 1) \cdot 3 = 20$ tiles.

(b) The nth term is $2 + (n - 1) \cdot 3 = 2 + 3n - 3 = \mathbf{3n - 1}$.

(c) The question can be written as: Is there an n such that $3n - 1 = 449$. Since $3n - 1 = 449 \Rightarrow 3n = 450 \Rightarrow n = 150$, the answer is yes, the 150th figure.

16. The n^{th} term of the arithmetic sequence is $-100 + n(300)$. The sequence can also be generated by adding 300 to the previous term. The n^{th} term of the geometric sequence is 3^{n-1}. The sequence can also be generated by multiplying the previous term by 3. Make a table

Number of the term	Arithmetic term	Geometric term
7	2000	729
8	2300	2187
9	2600	6561

With the **9th term**, the geometric sequence is greater.

17. Use a table of Fibonacci numbers to find the pattern, F_n is the nth Fibonacci number:

Generation	Male	Female	Number in Generation	Total
1	1	0	1	1
2	0	1	1	2
3	1	1	2	4
4	1	2	3	7
5	2	3	5	12
6	3	5	8	20
⋮	⋮	⋮	⋮	⋮
n	F_{n-2}	F_{n-1}	F_n	$F_{n+2}-1$

The sum of the first n Fibonacci numbers is $F_{n+2} - 1$. $F_{12} = 144$, so there are **143 bees** in all 10 generations.

18. **(a)** For an arithmetic sequence there is a common difference between the terms. Between 49 and 64 there are three differences so we can find the common difference by subtracting 49 from 64 and dividing the answer by three: $64 - 49 = 15$ and $15 \div 3 = 5$. The common difference is 5 and we can find the missing terms: 49 – 5 = 44 and 49 + 5 = 54 and 54 + 5 = 59.

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(b) For a geometric sequence there is a common ratio between the terms. Between 1 and 625 there are four common ratios used so we can find the common ratio by dividing 625 by 1 and then taking the fourth root:

$625 \div 1 = 625$ and $625^{(\frac{1}{4})} = 5$. The common ratio is 5 and we can find the missing terms: $1 \cdot 5 = 5, 5 \cdot 5 = 25, 25 \cdot 5 = 125$.

(c) For a geometric sequence there is a common ratio between the terms. Between 310 and 319 there are three common ratios used so we can find the common ratio by dividing 319 by 310 and then taking the cube root:

$3^{19} \div 3^{10} = 3^9$ and $(3^9)^{(\frac{1}{3})} = 3^3$. The common ratio is 33 and we can find the missing terms: $3^{10} \div 3^3 = 3^7, 3^{10} \cdot 3^3 = 3^{13}, 3^{13} \cdot 3^3 = 3^{16}$.

(d) For an arithmetic sequence there is a common difference between the terms. Between $\boldsymbol{a}$ and $5\boldsymbol{a}$ there are four differences so we can find the common difference by subtracting $\boldsymbol{a}$ from $5\boldsymbol{a}$ and dividing the answer by four: $5a - a = 4a$ and $4a \div 4 = a$. The common difference is $\boldsymbol{a}$ and we can find the missing terms: $\boldsymbol{a} + \boldsymbol{a} = 2\boldsymbol{a}$, : $2\boldsymbol{a} + \boldsymbol{a} = 3\boldsymbol{a}$, : $3\boldsymbol{a} + \boldsymbol{a} = 4\boldsymbol{a}$.

19. (a) Let's call the missing terms x and y, then the sequence becomes 1, x, y, 7, 11 and if it is a Fibonacci-type sequence then:

$1 + x = y$

$x + y = 7$

$y + 7 = 11 \rightarrow y = 11 - 7 = 4$

and $x + y = 7 \rightarrow x + 4 = 7 \rightarrow x = 3$.

The missing terms are 3 and 4.

(b) Let's call the missing terms x, y. and z, then the sequence becomes x, 2, y, 4, z and if it is a Fibonacci-type sequence then:

$x + 2 = y$

$2 + y = 4 \rightarrow y = 2$

$y + 4 = z \rightarrow 2 + 4 = z \rightarrow z = 6$

and $x + 2 = 2 \rightarrow x = 0$.

The missing terms are 0, 2, and 6.

(c) Let's call the missing terms x, y. and z, then the sequence becomes x, y, 3, 4, z and if it is a Fibonacci-type sequence then:

$x + y = 3$

$y + 3 = 4 \rightarrow y = 4 - 3 = 1$

$3 + 4 = 7$

and $x + y = 3 \rightarrow x + 1 = 3 \rightarrow x = 2$.

The missing terms are 2, 1, and 7.

20. Starting with 1 and 1 the Fibonacci sequence would be 1,1,2,3,5,8,13,21,34,55, … .

(a) $F_1 + F_3 = F_4 \rightarrow 1 + 2 = 3$

(b) $F_1 + F_3 + F_5 = F_6 \rightarrow 1 + 2 + 5 = 8$.

(c) $F_1 + F_3 + F_5 + F_7 = F_8 \rightarrow 1 + 2 + 5 + 13 = 21$.

(d) $F_1 + F_3 + F_5 + \ldots + F_{2k-1} = F_{2k}$.

Mathematical Connections 1-2: Review Problems

17. Order the teams from 1 to 10, and consider a simpler problem of counting how many games are played if each team plays each other once. The first team plays nine teams. The second team also plays nine teams, but one of these games has already been counted. The third team also plays 9 teams, but two of these games were counted in the previous two summands. Continuing in this manner, the total is 10 + 9 + 8 + … + 3 + 2 + 1 = 9(10) / 2=45 games. Double this amount to obtain 90 games must be played for each team to play each other twice.

18. 7 ways. Make a table:

Quarters	Dimes	Nickels
1	1	1
1	0	3
0	4	0
0	3	2
0	2	4
0	1	6
0	0	8

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19. If the problem is interpreted to stated that at least one 12-person tent is used, then there are 10 ways. This can be seen by the table below, which illustrates the ways 2,3,5, and -6 person tents can be combined accommodate 14 people.

6-Person	5-Person	3-Person	2-Person
2	0	0	1
1	1	1	0
1	0	2	1
1	0	0	4
0	2	0	2
0	1	3	0
0	1	1	3
0	0	4	1
0	0	2	4
0	0	0	7

Chapter 1 Review

1. Make a plan. Every 7 days (every week) the day will change from Sunday to Sunday. 365 days per year $\div 7$ days per week ≈ 52 weeks per year $+\frac{1}{7}$ weeks per year. Thus the day of the week will change from Sunday to Sunday 52 times and then change from Sunday to Monday. July 4 will be a Monday.

2. $\$5.90 \div 2 = \$2.95$ more on one of the items. That is $\$20 + \$2.95 = \$22.95$ for the more expensive item and $\$20 - \$2.95 = \$17.05$ for the less expensive item. Check that both items add up to \$40: $\$22.95 + \$17.05 = \$40$.

3. The information in the rhyme is that the scholar is a "ten o'clock scholar" who "used to come at ten o'clock." The question is "what makes you come sooner." If we assume that ten o'clock means 10 AM, then the rhyme makes no sense because the scholar came later than usual. If we assume that ten o'clock means 10 PM, the question makes sense but is not answered.

4. **(a)** 15, 21, 28 Neither. The successive differences of terms increases by one; e.g., 10 + 5, 15 + 6, … .

(b) 32, 27, 22. Arithmetic Subtract 5 from each term to obtain the subsequent term.

(c) 400, 200, 100. Geometric Each term is half the previous term.

(d) 21, 34, 55. Neither Each term is the sum of the previous two terms—this is the Fibonacci sequence.

(e) 17, 20, 23. Arithmetic Add 3 to each term to obtain the subsequent term.

(f) 256, 1024, 4096. Geometric Multiply each term by 4 to obtain the subsequent term.

(g) 16, 20, 24. Arithmetic Add 4 to each term to obtain the subsequent term.

(h) 125, 216, 343. Neither Each term is the 3rd power of the counting numbers $= 13, 23, 33, \ldots$.

5. **(a)** The successive differences are 3. Each term is 3 more than the previous term. This suggests that it is an arithmetic sequence of the form $3n + ?$. Since the first term is 5, $3(1) + ? = 5$. The n^{th} term would be $\mathbf{3n + 2}$.

(b) Each term given is 3 times the previous term. This suggests that the sequence is geometric. $\boldsymbol{n^{th}}$ term will be $3^{\boldsymbol{n}}$.

(c) We are reminded of the sequence 1; 8; 27; 64; … , which is given by n^3. Since the terms in the original sequence are one less than the terms given by n^3, $\boldsymbol{n^3 - 1}$ is a possible n^{th} term.

6. **(a)** $3(1) - 2 = \mathbf{1}$;
$3(2) - 2 = \mathbf{4}$;
$3(3) - 2 = \mathbf{7}$;
$3(4) - 2 = \mathbf{10}$; and
$3(5) - 2 = \mathbf{13}$.

(b) $1^2 + 1 = \mathbf{2}$;
$2^2 + 2 = \mathbf{6}$;
$3^2 + 3 = \mathbf{12}$;
$4^2 + 4 = \mathbf{20}$; and
$5^2 + 5 = \mathbf{30}$.

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(c) $4(1) - 1 = \mathbf{3}$;
$4(2) - 1 = \mathbf{7}$;
$4(3) - 1 = \mathbf{11}$;
$4(4) - 1 = \mathbf{15}$; and
$4(5) - 1 = \mathbf{19}$.

7. (a) $a_1 = 2, d = 2, a_n = 200$.
So $200 = 2 + (n-1) \cdot 2 \Rightarrow n = 100$.
Sum is $\frac{100(2+200)}{2} = \mathbf{10,100}$.

(b) $a_1 = 51,\ d = 1, a_n = 151$.
So $151 = 51 + (n-1) \cdot 1 \Rightarrow n = 101$.
Sum is $\frac{101 \cdot (51+151)}{2} = \mathbf{10,201}$.

8. (a) $5 + 3 = 8$, which is not odd.

(b) 15 is odd; and it does not end in a 1 or a 3.

(c) The sum of any two even numbers is always even. An even number is one divisible by 2, so any even number can be represented by $2 + 2 + 2 + \cdots$. Regardless of how many twos are added, the result is always a multiple of 2, or an even number.

9. All rows, columns, and diagonals must add to 34; i.e., the sum of the digits in row 1. Complete rows or columns with one number missing, then two, etc. to work through the square:

16	3	2	13
5	10	**11**	**8**
9	**6**	7	12
4	**15**	14	**1**

10. The ten middle tables will hold two each and the two end tables will hold three each, totaling 26 people.

11. (a) $\square + 2^{60} = 2^{61}$
$\square = 2^{61} - 2^{60}$
$\square = 2(2^{60}) - 2^{60}$
$\square = 2^{60}(2-1)$
$\square = 2^{60}$

(b) $\square^2 = 625$
$\sqrt{\square^2} = \sqrt{625}$
$\square = 25$

12. $100 \div 5 = 20$ plus $1 = \mathbf{21}$ posts. 1 must be added because both end posts must be counted.

13. 1 mile $=$ 5280 feet.
$5280 \div 6$ feet $=$ 880 turns per mile.
880×50000 miles $= \mathbf{44,000,000\ turns}$.

14. There are 9 students between 7 and 17 (8 through 16). There must be 9 between them in both directions, since they are direct opposites.
$9 + 9 + 2 = \mathbf{20\ students}$.

15. Let l be a large box, m be a medium box, and s be a small box:
$3l + (3l \times 2m \text{ each}) + [(3 \times 2)m \times 5s \text{ each}]$
$3l + 6m + 30s = \mathbf{39\ total\ boxes}$.

16. Look for the differences:

5 15 37 77 141
10 22 40 64
12 18 24

The second difference row is an arithmetic sequence with $d = 6$. Thus the 4th term in the second difference row is $24 + 6 = 30$; the5th term in the first difference row is $64 + 30 = 94$; and the 6th term in the original sequence is $141 + 94 = \mathbf{235}$.

17. Extend the pattern of doubling the number of ants each day. This is a geometric sequence with $a_1 = 1500$, $a_n = 100{,}000$, and $r = 2$.

$$100{,}000 = 1500 \cdot 2^{n-1} \Rightarrow$$
$$66\tfrac{2}{3} = 2^{n-1}.$$

Since $2^{7-1} < 66\frac{2}{3}$ and $2^{8-1} > 66\frac{2}{3}$, the ant farm will fill sometime between **the 7th and 8th day**.

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18. The best strategy would be one of guessing and checking:

(***i***) Ten 3's + two 5's = 40… close but too low.

(***ii***) Nine 3's + three 5's = 42… still too low.

(***iii***) Eight 3's + Four 5's = 44.

They must have answered four 5-point questions.

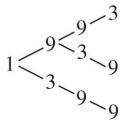
19. **Yes**. Let ℓ = length of the longest piece,
m = length of the middle-sized piece, and
s = lenth of the shortest piece.

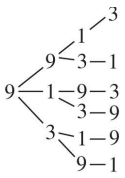
Then $\ell = 3m$ and $s = m - 10$.

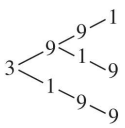
So $\ell + m + s = 90 \Rightarrow$
$3m + m + (m - 10) = 90 \Rightarrow$
$5m = 100$.

Thus $m = \mathbf{20\,cm}$;
$\ell = 3m = \mathbf{60\,cm}$; and
$s = m - 10 = 10$ **cm**.

20. Make a diagram that demonstrates all the ways four-digit numbers can be formed from left (thousands place) to right (ones place). **12 four-digit** numbers can be formed.







21. If n is 1 larger than m (i.e., n-m = 1, or equivalently, $n = m$+1), there would be 2 terms: m, and n (since $n = m + 1$). The number of terms can be found by subtracting m from n and adding 1: $n - m + 1 = 2$.

If n is 2 larger than m (i.e., n-m = 2, or equivalently, $n = m$+2) there would be 3 terms: m, $m + 1$, and n, since $n = m + 2$). The number of terms can be found by subtracting m from n and adding 1: $n - m + 1 = 3$.

If n is 3 larger than m there would be 4 terms: m, $m + 1$, $m + 2$, and n, since $n = m + 3$. The number of terms can be found by subtracting m from n and adding 1: $n - m + 1 = 4$.

Therefore, given that n is larger than m there would be $n - m + 1$ terms.

22. Answer may vary. Fill the 4-cup container with water and pour the water into the 7-cup container. Fill the 4-cup container again and pour water into the 7-cup container until it is full. Four minus three (1) cups of water will remain in the 4-cup container. Empty the 7-cup container and pour the contents of the 4-cup container into the 7- cup container. The 7-cup container now holds 1 cup of water. Refill the 4-cup container and pour it into the 7-cup container. The 7-cup container now contains exactly 5 cups of water.

23. A possible pattern is to increase each rectangle by one row of dots and one column of dots to obtain the next term in the sequence. Make a table.

Number of the term	Row of dots	Column of dots	Term (row × column)
1	1	2	2
2	2	3	6
3	3	4	12
4	4	5	20
5	5	6	30
6	6	7	42
7	7	8	56
⋮			
100	100	101	10100
⋮			
n	n	$n + 1$	$n(n + 1)$

We also observe that the number of the term corresponds to the number of rows in the arrays and that the number of columns in the array is the number of the term plus one. Thus, the next three terms are **30, 42, and 56**. The 100th term is 10100 and the n^{th} term is $n(n + 1)$.

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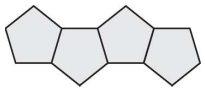
24. A possible pattern is that each successive figure is constructed by adjoining another pentagon to the previous figure

(a)



(b) Observe that the perimeter of the first figure is 5 and that when a new pentagon is adjoined 4 new sides are added and one side (where the new pentagon is adjoined) is lost. Make a table.

Number of terms	1-unit sides (perimeter)
1	5
2	$5 - 1 + 4 = 8$
3	$8 - 1 + 4 = 11$
4	$11 - 1 + 4 = 14$

(c) and **(d)** Looking at the terms in the sequence and noting that the difference of terms is three, we suspect that the sequence is arithmetic and conjecture that the n^{th} term is $3n+2$. However, we need to be sure. Looking at the 4th term (part a) we observe that the pentagons on the end Contribute 4 sides to the perimeter and the "middle" pentagons contribute 3 sides. Thus, in the n^{th} figure there will be 2 "end" pentagons that contribute 4 1-unit sides and $n - 2$ "middle" pentagons that contribute $3(n - 2)$ 1 unit sides. The total will be $\mathbf{3(n - 2) + 2(4)} = 3n + 2$ units. Thus the 100th term is $3(100) + 2 = \mathbf{30}2$.

25. **(a)** The circled terms will constitute an arithmetic sequence because the common difference will be twice the difference in the original series.

(b) The new sequence will be a geometric sequence because the ratio will be the square of the ratio of the original series.

26. If the sequence is $a_1, a_2, a_3, a_4, \ldots$ to find the first term we substitute $n = 1$ and get $1^2 - 1$ or 0, so a_1=0. For $n = 2$, we get 2^2-2 or 2. Thus $a_1 + a_2$=2; hence $a_2 = 2$. For $n = 3$, $a_1 + a_2 + a_3 = 3^2 - 3 = 6$. Substituting for a_1 and a_2, we get $a_3 = 6 - 2 = 4$. For $n = 4$, $a_1 + a_2 + a_3 + a_4 = 4^2 - 4 = 12$. Substituting for a_1, a_2, and a_3, we get $0 + 2 + 4 + a_4 = 12$. Hence $a_4 = 6$.

27. **(a)** Let's call the missing terms a, and b then the sequence becomes:

$13, a, b, 27$

$13 + a = b \rightarrow a = b - 13$

$a + b = 27 \rightarrow b - 13 + b = 27$

$\rightarrow 2b = 40$

$\rightarrow b = 20$

$a = b - 13 \rightarrow a = 7$

So 7 and 20 are the missing terms.

(b) Let's call the missing terms a, and b then the sequence becomes:

$137, a, b, 163$

$137 + a = b \rightarrow a = b - 137$

$a + b = 163 \rightarrow b - 137 + b = 163$

$\rightarrow 2b = 300$

$\rightarrow b = 150$

$a = b - 137 \rightarrow a = 13$

So 13 and 150 are the missing terms.

(c) Let's call the missing terms x, and y, then the sequence becomes:

b, x, y, a

$b + x = y \rightarrow x = y - b$

$x + y = a \rightarrow y - b + y = a$

$\rightarrow 2y = a + b$

$\rightarrow y = \dfrac{a+b}{2}$

$x = \dfrac{a+b}{2} - b \rightarrow x = \dfrac{a+b}{2} - \dfrac{2b}{2}$

$\rightarrow x = \dfrac{a-b}{2}$

So the missing terms are $\dfrac{a-b}{2}$ and $\dfrac{a+b}{2}$.

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CHAPTER 2

INNTRODUCTION TO LOGIC AND SETS

Assessment 2-1A: Reasoning and Logic: An Introduction

1. (a) **False statement**. A statement is a sentence that is either true or false, but not both.

(b) **False statement.**

(c) **Not a statement.**

(d) **True statement.**

2. (a) **There exists at least one** natural number n such that $n + 8 = 11$.

(b) **There exists at least one** natural number n such that $n^2 = 4$.

(c) For **all** natural numbers n, $n + 3 = 3 + n$.

(d) For **all** natural numbers n, $5n + 4n = 9n$.

3. (a) For **all** natural numbers n, $n + 8 = 11$.

(b) For **all** natural numbers n, $n^2 = 4$.

(c) There is **no** natural number x such that $x + 3 = 3 + x$.

(d) There is **no** natural number x such that $5x + 4x = 9x$.

4. (a) The book **does not have** 500 pages.

(b) $3 \cdot 5 \neq 15$.

(c) **Some** dogs **do not have** four legs.

(d) **No** rectangles are squares.

(e) **All** rectangles are squares.

(f) **Some** dogs have fleas.

5. (a) If $n = 4$, or $n = 5$, then $n < 6$ *and* $n > 3$, so the statement is **true**, since it can be shown to work for some natural numbers n..

(b) If $n = 10$, then $n > 0$; *or* if $n = 1$, then $n < 5$, so the statement is **true**.

6. (a)

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

(b)

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

(c) **Yes**. The truth table entries are the same.

(d) **No**. The truth table entries are not the same.

7. (a)

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	
T	T	T	F	T	
T	F	F	F	F	
F	T	T	T	T	
F	F	T	T	T	

(b) Answers will vary. Here are two possible examples:1. Let p be "the Bobcats win" and q be "the Bobcats make the playoffs". Then Column 3 would read "If the Bobcats win, then the Bobcats make the playoffs". Column 5 would read "The Bobcats lose or the Bobcats make the playoffs."2. Let p be "it is summer vacation" and q be "I am at home". Then Column 3 would read "If it is summer vacation, then I am at home". Column 5 would read "It is not summer vacation or I am at home."

(c) In this problem, p is the statement "2 + 3 = 5" and q is the statement "4 + 6 = 10". That would make the statement in this problem be in the form of $\sim p \vee q$. So, it will be logically equivalent to a statement in the form of $p \rightarrow q$ or "if 2 + 3 = 5, then 4 + 6 = 10."

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8. (a) $q \wedge r$. Both q and r are true.

(b) $r \vee \sim q$. r is true or q is not true.

(c) $\sim(q \wedge r)$. q and r are not both true.

(d) $\sim q$. q is not true.

9. (a) **False**. The statement is a conjunction. The two parts could be stated as such: p is the statement $2 + 3 = 5$ and q is the statement $4 + 7 = 10$. In this situation, p is true, but q is false. In order for a conjunction to be true, both p and q must be true; otherwise, the conjunction is false.

(b) **True**. This statement is false, since Barack Obama was president in 2013.

(c) **False**. The United States Supreme Court currently has nine justices.

(d) **True**. The only triangles that have three sides of the same length are equilateral triangles. In every case, an equilateral triangle will have two sides the same length as well.

(e) **False**. Isosceles triangles have two sides equal in length, but the third side is not equal to the other two.

10. (a) By DeMorgan's Laws, the negation of $p \wedge q$ is $\sim p \vee \sim q$. Therefore, the answer is $2 + 3 \neq 5$ or $4 + 7 \neq 10$.

(b) The president of the United States in 2013 was Barack Obama.

(c) With every seat filled, the Supreme Court of the United States does not have 12 justices.

(d) The triangle has three sides of the same length and the triangle does not have two sides of the same length.

(e) The triangle has two sides of the same length or the triangle does not have three sides of the same length.

In both (d) and (e) above, the negation of a conditional statement $p \rightarrow q$ is $p \wedge \sim q$.

11. (a)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \vee q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	T

Since the truth values for $\sim p \vee \sim q$ are not the same as for $\sim(p \vee q)$, the statements are **not logically equivalent**.

(b)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Since the truth values for $\sim(p \wedge q)$ are not the same as for $\sim p \wedge \sim q$, the statements are **not logically equivalent**.

12. Dr. No is a male spy. He is not poor, and he is not tall.

13. If $p =$ "it is raining" and $q =$ "the grass is wet":

(a) $p \rightarrow q$. **(b)** $\sim p \rightarrow q$.

(c) $p \rightarrow \sim q$.

(d) $p \rightarrow q$. The hypothesis is "it is raining;" the conclusion is "the grass is wet."

(e) $\sim q \rightarrow \sim p$. **(f)** $p \leftrightarrow q$.

14. (a) **Converse**: If a triangle has no two sides of the same length, then the triangle is scalene. **Inverse**: If a triangle is not scalene, then the triangle has (at least) two sides of the same length. **Contrapositive**:If a triangle does not have two sides of the same length, then the triangle is not scalene. Note that this statement is a biconditional.

(b) **Converse:** If an angle is a right angle, then it is not acute. **Inverse**: If an angle is acute, then it is not a right angle. **Contrapositive:** If an angle is not a right angle, then it is acute. Note

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that the original statement and the contrapositive are not true, while the converse and inverse are true.

(c) **Converse**: If Mary is not a citizen of Cuba, then she is a U.S. citizen. **Inverse**: If Mary is not a U.S. citizen, then she is a citizen of Cuba. **Contrapositive**: If Mary is a citizen of Cuba, then she is not a U.S. citizen. Note that the original statement and the contrapositive are true, while the converse and inverse are not true

(d) **Converse**: If a number is not a natural number, then it is a whole number. **Inverse**: If a number is not a whole number, then it is a natural number. **Contrapositive**: If a number is a natural number, then it is not a whole number.

15. The statements are negations of each other.

p	q	$\sim q$	$p \wedge \sim q$	$\sim (p \wedge \sim q)$	$p \rightarrow q$	$\sim (p \rightarrow q)$
T	T	F	F	T	T	F
T	F	T	T	F	F	T
F	T	T	F	T	T	F
F	F	F	F	T	T	F

16. The contrapositive is logically equivalent: "If a number is not a multiple of 4 then it is not a multiple of 8."

17. **(a)** **Valid.** This is valid by the transitivity property. "All squares are quadrilaterals" is $p \rightarrow q$ "all quadrilaterals are polygons" is $q \rightarrow r$; and "all squares are polygons" is $p \rightarrow r$.

(b) **Invalid.** We do not know what will happen to students who are not freshman. There is no statement "sophomores, juniors, and seniors do not take mathematics."

18.

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

19. In the set of teachers, some have IQ's of 145 or more, and some are in Mensa. Notice that those two subsets intersect; their intersection represents the group of teachers who belong in both categories. Therefore, the argument is valid: some teachers with IQ's of 145 or more are in Mensa.

20. **(a)** Since all students in Integrated Mathematics I make A's, and some of those students are in Beta Club, then **some Beta Club students make A's**.

(b) Let $p =$ I study for the final, $q =$ I pass the final, $r =$ I pass the course, $s =$ I look for a teaching job. Then $p \rightarrow q$, if I study for the final, then I will pass the final. $q \rightarrow r$, if I pass the final, then I will pass the course. $r \rightarrow s$, if I pass the course, I will look for a teaching job. So $p \rightarrow s$, **if I study for the final I will look for a teaching job**.

(c) The first statement could be rephrased as "If a triangle is equilateral, then it is isosceles." Let $p =$ equilateral triangle and $q =$ isosceles triangle. So the first statement is $p \rightarrow q$, . The second statement is simply p; then the conclusion should be q or **there exist triangles that are isosceles**.

21. **(a)** If a figure is a square, then it is a rectangle.

(b) If a number is an integer, then it is a rational number.

(c) If a polygon has exactly three sides, then it is a triangle.

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22. **(a)** If $\sim p \vee \sim q \equiv \sim(p \wedge q)$, then
$\mathbf{3 \cdot 2 \neq 6}$ ***or*** $\mathbf{1 + 1 = 3}$.

(b) If $\sim p \wedge \sim q \equiv \sim(p \vee q)$, then you cannot pay me now and you cannot pay me later.

Assessment 2-1B

1. **(a)** **Not a statement**. A statement is a sentence that is either true or false.

(b) **Not a statement**. A statement must be either true of false; this could be either.

(c) True statement.

(d) **False statement**. $2 + 3 \neq 8$.

(e) Not a statement.

2. **(a)** For **all** natural numbers n, $n + 0 = n$.

(b) There exists **no** natural number n such that $n + 1 = n + 2$.

(c) **There exists at least one** natural number n such that $3 \cdot (n + 2) = 12$.

(d) **There exists at least one** natural number n such that $n^3 = 8$.

3. **(a)** There is **no** natural number n such that $n + 0 = n$.

(b) **There exists at least one** natural number n such that $n + 1 = n + 2$.

(c) For **all** natural numbers n, $3 \cdot (n + 2) \neq 12$.

(d) For **all** natural numbers n, $n^3 \neq 8$.

4. **(a)** Six is **greater than** or equal to 8. Another way to express this would be to say 6 **is not less than** 8.

(b) **All** cats **have** nine lives. Another way to express this would be to say that **no** cats **do not have** nine lives.

(c) There exists **a** square that is **not** a rectangle. Another way to express this would be to say **some** squares **are not** rectangles.

(d) **All** numbers **are** positive.

(e) **No** people **have** blond hair.

5. **(a)** If $n = 10$, then $n > 5$ and $n > 2$, so the statement is **true**.

(b) x could equal 5, so the statement is **false**.

6. **(a)** $p \vee q$ is false only if both p and q are false, so if p is true the statement is **true** regardless of the truth value of q.

(b) An implication is false only when p is true and q is false, so if p is false then the statement is **true** regardless of the truth value of q.

7. **(a)** $\boldsymbol{q \wedge r}$.

(b) $\boldsymbol{q \wedge \sim r}$.

(c) $\boldsymbol{\sim r \vee \sim q}$.

(d) $\boldsymbol{\sim(q \wedge r)}$.

8. **(a)** **True**. This statement is a disjunction. The two parts could be stated as such: p is the statement "4 + 6 = 10", while q is the statement "2 + 3 = 5". In this situation p is true, and q is true. In order for a disjunction to be true, either p or q (or both) have to be true; the only way a disjunction can be false is if both p and q are false. So therefore, this statement is true.

(b) **False**. If a team has more than 11 players on the field, it is a penalty and the play will not count.

(c) True.

(d) **False**. To see, sketch a drawing where three sides are the same length, but with the two angles where the sides intersect being different measures (in fact, make one a right angle, the other an obtuse angle). You should easily make a quadrilateral with a side length different from the other three.

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(e) **True**. If a rectangle has four sides of the same length, then by default it must have three sides the same length. Of course, a rectangle with four equal sides is a square!.

9. **(a)** By DeMorgan's Laws, the negation of the disjunction $p \vee q$ is $\sim p \wedge \sim q$. So, the statement would be $4 + 6 \neq 10$ and $2 + 3 \neq 5$

(b) A National Football League team cannot have more than 11 players on the field while a game is in progress.

(c) The first president of the United States was not George Washington.

(d) A quadrilateral has three sides of the same length and the quadrilateral does not have four sides of the same length.

(e) A rectangle has four sides of the same length and that rectangle does not have three sides of the same length.

In both (d) and (e), the negation of the conditional statement $\boldsymbol{p \rightarrow q}$ is $\boldsymbol{p \wedge \sim q}$.

10. **(a)**

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Since the truth values for $\sim(p \vee q)$ are the same as for $\sim p \wedge \sim q$, the statements **are logically equivalent**.

(b)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Since the truth values for $\sim(p \wedge q)$ are the same as for $\sim p \vee \sim q$, the statements are **logically equivalent**.

11. Ms. Makeover is not single. She has straight blond hair.

12. **(a)** $\boldsymbol{p \rightarrow q}$.

(b) $\boldsymbol{\sim p \rightarrow q}$.

(c) $\boldsymbol{p \rightarrow \sim q}$.

(d) q if p, or $\boldsymbol{p \rightarrow q}$.

(e) $\boldsymbol{\sim p \rightarrow \sim q}$.

(f) $\sim q \rightarrow \sim p$.

13. **(a)** Converse: **If $\boldsymbol{x^2 = 9}$, then $\boldsymbol{x = 3}$.**

Inverse: **If $\boldsymbol{x \neq 3}$, then $\boldsymbol{x^2 \neq 9}$.**

Contrapositive: **If $\boldsymbol{x^2 \neq 9}$, then $\boldsymbol{x \neq 3}$.**

(b) Converse: If classes are canceled, then it snowed.

Inverse: If it does not snow, then classes are not canceled.

Contrapositive: If classes are not canceled, then it did not snow.

14. **No**. This is the inverse; i.e., if it does not rain then Iris can either go to the movies or not without making her statement false.

15. **(a)** **Valid**. Use modus ponens: Hypatia was a woman → all women are mortal → Hypatia was mortal.

(b) **Valid**. Since *Dirty Harry* was not written by J.K. Rowling, and she wrote all the *Harry Potter* books, then *Dirty Harry* cannot be a *Harry Potter* book.

(c) **Not valid**. There exist some whole numbers that are not natural numbers. It might be easier to understand if the word seven is replaced by a variable "x". So, it reads: Some whole numbers are not natural numbers. "x" is a whole number. Conclusion: "x" is a natural number.

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16. **(a)** Since all students in Integrated Mathematics I are in Kappa Mu Epsilon, and Helen is in Integrated Mathematics I, then the conclusion is that **Helen is in Kappa Mu Epsilon**.

(b) Let p = all engineers need mathematics and q = Ron needs mathematics.

Then $p \rightarrow q$, or if all engineers need mathematics then Ron needs mathematics.

p is true, but q is false, Ron does not need mathematics.

So Ron is not an engineer.

(c) Since all bicycles have tires and all tires use rubber, then the conclusion is **all bicycles use rubber.**

17. **(a)** **If** a number is a natural number, **then** it is a real number.

(b) **If** a figure is a circle, **then** it is a closed figure.

18. DeMorgan's Laws are that:

$\sim(p \wedge q)$ is the logical equivalent of $\sim p \vee \sim q$.

$\sim(p \vee q)$ is the logical equivalent of $\sim p \wedge \sim q$.

Thus:

(a) The negation is $\mathbf{3 + 5 = 9}$ **or** $\mathbf{3 \cdot 5 \neq 15}$.

(b) The negation is I am not going and she is not going.

19. Therefore, a square is a parallelogram.

Assessment 2-2A: Describing Sets

1. **(a)** Either a list or set-builder notation may be used: $\{\mathbf{a,s,e,m,n,t}\}$ or $\{\boldsymbol{x} \mid \boldsymbol{x}$ **is a letter in the word** ***assessment***$\}$.

(b) $\{\mathbf{21,22,23,24,\ldots}\}$ or $\{\boldsymbol{x} \mid \boldsymbol{x}$ **is a natural number and** $\boldsymbol{x > 20}\}$ or $\{x \mid x \in N$ **and** $\boldsymbol{x > 20}\}$.

2. **(a)** $P = \{p, q, r, s\}$.

(b) $\{\mathbf{1,2}\} \subset \{\mathbf{1,2,3}\}$. The symbol $\subset$ refers to a proper subset.

(c) $\{\mathbf{0,1}\} \not\subseteq \{\mathbf{1,2,3}\}$. The symbol $\subseteq$ refers to a subset.

3. **(a)** **Yes**. $\{1,2,3,4,5\} \sim \{m,n,o,p,q\}$ because both sets have the same number of elements and thus exhibit a one-to-one correspondence.

(b) **Yes**. $\{a,b,c,d,e,f,...,m\} \sim \{1,2,3,\ldots,13\}$ because both sets have the same number of elements.

(c) **No**. $\{x \mid x$ is a letter in the word *mathematics*$\}$ $\not\sim \{1,2,3,4,\ldots,11\}$; there are only eight unduplicated letters in the word *mathematics*.

4. **(a)** The first element of the first set can be paired with any of the six in the second set, leaving five possible pairings for the second element, four for the third, three for the fourth, two for the fifth, and one for the sixth. Thus there are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{720}$ one-to-one correspondences.

(b) There are $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = n!$ possible one-to-one correspondences. The first element of the first set can be paired with any of the n elements of the second set; for each of those n ways to make the first pairing, there are $n - 1$ ways the second element of the first set can be paired with any element of the second set; which means there are $n - 2$ ways the third element of the first set can be paired with any element of the third set; and so on. The Fundamental Counting Principle states that the choices can be multiplied to find the total number of correspondences.

5. **(a)** If x must correspond to 5, then y may correspond to any of the four remaining elements of $\{1,2,3,4,5\}$, z may correspond to any of the three remaining, etc. Then $1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{24}$ **one-to-one correspondences**.

(b) There would be $1 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = \mathbf{6}$ **one-to-one correspondences**.

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(c) The set $\{x, y, z\}$ could correspond to the set $\{1,3,5\}$ in $3 \cdot 2 \cdot 1 = 6$ ways. The set $\{u, v\}$ could correspond with the set $\{2,4\}$ in $2 \cdot 1 = 2$ ways. There would then be $6 \cdot 2 = \mathbf{12}$ **one-to-one correspondences**.

6. **(*i*)** $\boldsymbol{A = C}$. The order of the elements does not matter.

(*ii*) $\boldsymbol{E = H}$; they are both the null set.

(*iii*) $\boldsymbol{I = L}$. Both represent the numbers $1,3,5,7,\ldots$.

7. **(a)** Assume an arithmetic sequence with $a_1 = 201, a_n = 1100$, and $d = 1$. Thus $1100 = 201 + (n-1) \cdot 1$; solving, $n = 900$. The cardinal number of the set is therefore **900**.

(b) Assume an arithmetic sequence with $a_1 = 1, a_n = 101$, and $d = 2$. Thus $101 = 1 + (n-1) \cdot 2$; solving, $n = 51$. The cardinal number of the set is therefore **51**.

(c) Assume a geometric sequence with $a_1 = 1, a_n = 1024$, and $r = 2$. Thus $1024 = 1 \cdot 2^{n-1} \Rightarrow 2^{10} = 2^{n-1} \Rightarrow n - 1 = 10 \Rightarrow n = 11$. The cardinal number of the set is therefore **11**.

(d) If $k = 1,2,3,\ldots,100$, the cardinal number of the set $\{x | x = k^3, k = 1,2,3,\ldots,100\} = \mathbf{100}$, since there are 100 elements in the set.

8. $\overline{A}$ represents all elements in U that are not in A, or the set of all college students with at least one grade that is not an A.

9. **(a)** A proper subset must have at least one less element than the set, so the maximum $\boldsymbol{n(B) = 7}$.

(b) Since $B \subset C$, and $n(B) = 8$ then C could have any number of elements in it, so long as it was greater than eight.

10. **(a)** The sets arc equal, so $\boldsymbol{n(D) = 5}$.

(b) Answers vary.For example, the sets are equal; the sets are also equivalent

11. **(a)** A has 5 elements, thus $2^5 = \mathbf{32}$ **subsets**.

(b) Since A is a subset of A and A is the only subset of A that is not proper, A has $2^5 - 1 =$ **31 proper subsets**.

(c) Let $B = \{b,c,d\}$. Since $B \subset A$, the subsets of B are all of the subsets of A that do not contain a and e. There are $2^3 = 8$ of these subsets. If we join (union) a and e to each of these subsets there are still **8 subsets**.

Alternative. Start with $\{a,e\}$. For each element b, c, and d there are two options: include the element or don't include the element. So there are $2 \cdot 2 \cdot 2 = 8$ ways to create subsets of A that include a and e.

12. If there are n elements in a set, 2^n subsets can be formed. This includes the set itself. So if there are 127 proper subsets, then there are 128 subsets. Since $2^7 = 128$, the set has **7 elements**.

13. In roster format, $A = \{3,6,9,12,\ldots\}, B = \{6,12,18,24,\ldots\}$, and $C = \{12,24,36,\ldots\}$. Thus, $C \subset A, C \subset B$, and $B \subset A$.

Alternatively: $12n = 6(2n) = 3(4n)$. Since $2n$ and $4n$ are natural number $\boldsymbol{C \subset A, C \subset B,}$ and $\boldsymbol{B \subset A}$.

14. **(a)** $\notin$. There are no elements in the empty set.

(b) $\in$. $1024 = 2^{10}$ and $10 \in N$.

(c) $\in$. $3(1001) - 1 = 3002$ and $1001 \in N$.

(d) $\notin$. For example, $x = 3$ is not an element because for $3 = 2^n$, $n \notin N$.

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15. **(a)** $\not\subseteq$. 0 is not a set so cannot be a subset of the empty set, which has only one subset, $\varnothing$.

(b) $\not\subseteq$. 1024 is an element, not a subset.

(c) $\not\subseteq$. 3002 is an element, not a subset.

(d) $\not\subseteq$. x is an element, not a subset.

16. **(a)** **Yes**. Any set is a subset of itself, so if $A = B$ then $A \subseteq B$.

(b) **No**. A could equal B; then A would be a subset but not a proper subset of B.

(c) **Yes**. Any proper subset is also a subset.

(d) **No**. Consider $A = \{1, 2\}$ and $B = \{1,2,3\}$.

17. **(a)** Let $A = \{1, 2, 3, \ldots, 100\}$ and $B = \{1, 2, 3\}$. Then $n(A) = 100$ and $n(B) = 3$. Since $B \subset A$, $\boldsymbol{n(B) = 3 < 100 = n(A)}$.

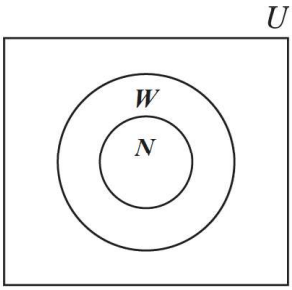
(b) $n(\emptyset) = 0$. Let $A = \{1, 2, 3\} \Rightarrow n(A) = 3$. $\emptyset \subset A$, which implies that there is at least one more element in A than in $\emptyset$. Thus $\mathbf{0 < 3}$.

18. There are seven senators to choose from and 3 will be chosen. Consider the ways to form subsets with only three members. If we pick the first member, there are 7 senators to choose from. To pick the second member, there are only 6 to choose from, since 1 member has already been chosen. For the third seat, there are 5 to choose from. This yields $7 \cdot 6 \cdot 5$. However, this calculation counts {Able, Brooke, Cox} as a different committee that {Brooke, Able, Cox}. In fact, for any 3 names, there are $3 \cdot 2 \cdot 1$ ways to arrange the names. Thus, the number of unique committees is $7 \cdot 6 \cdot 5 / 3 \cdot 2 \cdot 1 = 7 \cdot 5 = \mathbf{35}$.

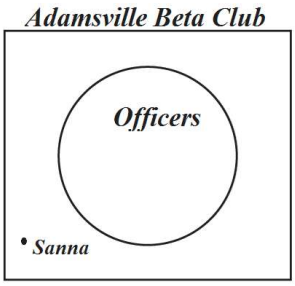
19. Answers vary. For example, the set of all odd natural numbers and the set of all even natural numbers are two infinite sets that are equivalent but not equal.. Another possibility is the set of all natural numbers and the set of all whole numbers.

20. Each even natural number $2n$ can be paired with each odd natural number $2n - 1$ in a one-to-one correspondence.

21.



22.



23. Answers vary. Example: All members of the Adamsville Beta Club are officers.

Assessment 2-2B

1. **(a)** Either a list or set-builder notation may be used: $\boldsymbol{\{a,l,g,e,b,r\}}$ or $\boldsymbol{\{x \mid x}$ **is a letter in the word** ***algebra*****}**

(b) $\boldsymbol{\{1,2,3,4,5,6,7,8,9\}}$ or $\boldsymbol{\{x \mid x}$ **is a natural number and** $\boldsymbol{x<10\}}$ or $\boldsymbol{\{x \mid x \in N}$ **and** $\boldsymbol{x<10\}}$.

2. **(a)** $\boldsymbol{Q = \{q,r,s\}}$

(b) $\boldsymbol{\{1,3\} = \{3,1\}}$. The symbol $=$ refers to the sets being equal (containing the same elements).

(c) $\boldsymbol{\{1,3\} \not\subset \{1,4,6\}}$. The symbol $\not\subset$ refers to "not a proper subset."

3. **(a)** **Yes.** $\{1, 2, 3, 4\} \sim \{w, c, y, z\}$ because both sets have the same number of elements and thus exhibit a one-to-one correspondence.

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(b) **Yes**, because both sets have the same number of elements.

(c) **No**. $\{x \mid x$ is a letter in the word *geometry*$\} \neq \{1,2,3,4,...,8\}$; there are only seven unduplicated letters in geometry.

4. **(a)** The first element of the first set can be paired with any of the eight in the second set, leaving seven possible pairings for the second element, six for the third, five for the fourth, etc. Thus, there are $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ **40,320** one-to-one correspondences.

(b) There are $\boldsymbol{(n-1)\cdot(n-2)\cdot(n-3)\cdot\ldots\cdot 3\cdot 2\cdot 1}$ possible one-to-one correspondences. The first element of the first set can be paired with any of the $n-1$ elements of the second set; there are $n-2$ ways the second element of the first set can be paired with any element of the second set; which means there are $n-3$ ways the third element of the first set can be paired with any element of the third set; and so on. The Fundamental Counting Principle states that the choices can be multiplied to find the total number of correspondences

5. **(a)** If b must correspond to 3, then a may correspond to any of the three remaining elements of $\{1,2,3,4\}$, and c may correspond to any of the two remaining, etc. Then $1 \cdot 3 \cdot 2 \cdot 1 =$ **6 one-to-one correspondences.**

(b) **T**here would be a $1 \cdot 1 \cdot 2 \cdot 1 =$ **2 one-to-one correspondences.**

(c) The set $\{a,c\}$ could correspond to the set $\{2,4\}$ in $2 \cdot 1 = 2$ ways. The set $\{b,d\}$ could correspond with the set $\{1,3\}$ in $2 \cdot 1 = 2$ ways. There would then be $2 \cdot 2 =$ **4 one-to-one correspondences.**

6. $\boldsymbol{A = C}$.

7. **(a)** Assume an arithmetic sequence with $a_1 = 19,\ a_n = 99,$ and $d = 1$. Thus $99 = 19 + (n-1) \cdot 1;$ solving, $n = 81$. The cardinal number of the set is therefore **81**.

(b) Assume an arithmetic sequence with $a_1 = 2,\ a_n = 1002,$ and $d = 2$. Thus $1002 = 2 + (n-1) \cdot 2;$ solving, $n = 501$. The cardinal number of the set is therefore **501**.

(c) Assume an arithmetic sequence with $a_1 = 1,\ a_n = 99,$ and $d = 2$. Thus $99 = 1 + (n-1) \cdot 2;$ solving, $n = 50$. The cardinal number of the set is therefore **50**.

(d) There are no natural numbers that have the property $x = x + 1,$ so the set is empty and the cardinal number is **0**.

8. $\bar{G}$ represents all elements in U that are not in G, or the set of all women who are not alumni of Georgia State University. In set-builder notation, $\bar{G} = \{x \mid x$ is a woman who is not a graduate of Georgia State University$\}$.

9. **(a)** Since the empty set is a subset of any set, A could be the empty set; then the minimum number of elements in A would be **0**.

(b) **Yes**. Since A is not assumed to be a proper subset, A and B could be equal . Thus, both sets could be empty.

10. To be subsets of each other, the two sets must be **equal** and **equivalent**.

11. **(a)** Since $A = \{1,2,3,4,5,6,7,8,9\}$ has 9 elements, A has $2^9 =$ **512 subsets**.

(b) Since A is a subset of A and not proper, A has $2^9 - 1 =$ **511 subsets**.

12. If there are n elements in a set, 2^n subsets can be formed. Thus, $16 = 2^n$ and $n =$ **4 elements**.

13. In roster format, $A = \{4,7,10,13,16,...\}$, $B = \{7,13,18,...\},$ and $C = \{13,25,38,...\}$. Thus, $C \subset A,\ C \subset B,$ and $B \subset A$.

Alternative: $12n + 1 = 6(2n) + 1 = 3(4n) + 1$. Since $2n$ and $4n$ are natural numbers $\boldsymbol{C \subset A}$, $\boldsymbol{C \subset B}$, and $\boldsymbol{B \subset A}$.

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14. **(a)** $\in$. The set containing the empty set has one element; the empty set.

(b) $\in$. $1022 = 2^{10} - 2$ and $10 \in N$.

(c) $\in$. $3(1001) + 1 = 3004$ and $1001 \in N$.

(d) $\in$. 17 is an element of the natural numbers.

15. **(a)** **No**. For example, if $A = \{1\}$ and $B = \{1, 2\}$, then $A \subseteq B$ but $A \neq B$.

(b) **No**. $A \subset B$ implies that A must have at least one less element than B.

(c) **No**. A and B must have the same number of elements, but not necessarily be equal; for example, if $A = \{1, 2\}$ and $B = \{a, b\}$.

(d) **No**. See part (c).

16. **(a)** $n(\emptyset) = 0$. Let $A = \{1, 2\} \Rightarrow n(A) = 2$. $\emptyset \subset A$, which implies that there is at least one more element in A than in $\emptyset$. Thus **0 < 2**.

(b) Let $A = \{1, 2, 3, \ldots 99\}$ and $B = \{1, 2, 3, \ldots, 100\}$. $n(A) = 99$ and $n(B) = 100$, but $A \subset B$ so $\boldsymbol{n(A) = 99 < 100 = n(B)}$.

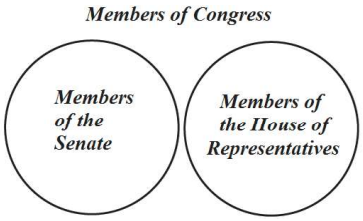
17. **(a)** There are 4 flavors from which to pick the first scoop, leaving 3 from which to pick the second, 2 from which to pick the third, and only 1 from which to pick the last. Thus there are $4 \cdot 3 \cdot 2 \cdot 1 =$ **24 ways to pick the four flavors**.

(b) Each of the four scoops may be picked in four different ways, thus there are $4^4 =$ **256 ways to pick the four flavors**.

18. 6. Note that $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. The Fundamental Counting Principle states that the product of choices gives the number of ways that the one-to-one correspondence can happen; since there are 720 one-to-one correspondences, then there must be six elements in each set.

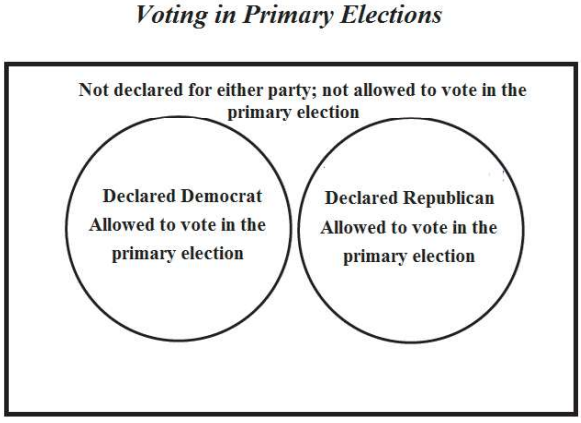
19. Let every natural number n be paired with the number $n - 1$ from the set of whole numbers. The pairing is a one-to-one correspondence between the set of natural numbers and the set of whole numbers.

20. The Congress of the United States consists of all members of the Senate, and all members of the House of Representatives; however, no member of the Senate is also a member of the House of Representatives, and vice-versa. Therefore, a Venn diagram consisting of two disjoint circles will work for this problem, as shown below:



21. Since both sets are equal, that means both sets contain all of the same elements, by defintion of equal sets. So, you can infer that every swimmer in the 100 meter butterfly race is from the Maryville Swim Team.

22. For voting in a primary election, some states make voters declare a party affiliation (so that you vote in that parties' primary only). No state would allow a voter to declare as both a Republican and a Democrat, so the Venn diagram would consist of two disjoint circles, as shown below:


Voting in Primary Elections
Not declared for either party; not allowed to vote in the primary election
Declared Democrat Allowed to vote in the primary election
Declared Republican Allowed to vote in the primary election

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23. Answers will vary. Some possibilities: Since set B has one more element in it than set A, set A could be a subset of set B. However, the number of elements doesn't necessarily imply that the set with the smaller number of elements is a subset of the set with the larger number of elements, so set A might not be a subset of set B. Similar reasoning could be used to state that set A could be a proper subset of set B, or not. Assuming the sets are finite, the sets are not equivalent nor equal.

24. **(a)** Answers will vary. For example, let the universal set be the whole numbers. Now, let set A consist of all whole numbers greater than ten. That set is infinite. Its complement, $\overline{A}$, would be the natural numbers less than or equal to ten. That set is finite.

(b) Answers will vary. For example, let the universal set be the set of whole numbers. Let A be the set of odd whole numbers. This set is infinite. Then $\overline{A}$ would be the set of even whole numbers, also an infinite set.

Mathematical Connections 2-2: Review Problems

16. **(a)** False. In order for a conjunction to be true, both parts of the conjunction must be true. Since p is false, $p \wedge q$ is false.

(b) False. Since q is true, $\sim q$ is false.

(c) True. Since p is false, $\sim p$ is true. This makes both parts of the conjunction true, so therefore the conjunction $\sim p \wedge q$ is true.

(d) True. In part (a) above, we found out that $p \wedge q$ was false. Therefore, $\sim (p \wedge q)$ must be true.

(e) False. In order for a conjunction to be true, both parts of the conjunction must be true. Since q is true, $\sim q$ is false; so $\sim q \wedge \sim p$ is false.

17. **(a)** False. In order for a conjunction to be true, both parts of the conjunction must be true. Since q is false, $p \wedge q$ is false.

(b) True. Since q is false, $\sim q$ is true.

(c) False. In order for a conjunction to be true, both parts of the conjunction must be true. In this conjunction, neither side is true; so, $\sim p \wedge q$ is false.

(d) True. In part (a) above, we found out that $p \wedge q$ was false. Therefore, $\sim (p \wedge q)$ must be true.

(e) False. In order for a conjunction to be true, both parts of the conjunction must be true. Since p is true, $\sim p$ is false; so $\sim q \wedge \sim p$ is false.

18.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	**T**
T	F	T	**T**
F	T	F	**F**
F	F	T	**T**

Assessment 2-3A: Other Set Operations and Their Properties

1. $A = \{1,3,5,\ldots\}; B = \{2,4,6,\ldots\}; C = \{1,3,5,\ldots\}$

(a) ***A*** or ***C***. Every element in C is either in A or C.

(b) ***N***. Every natural number is in either A or B.

(c) $\emptyset$. There are no natural numbers in both A and B.

2. For example, let $U = \{1,2,3,4,5,6,7\}$, $A = \{1,5,6\}, B = \{1,4,5,6,7\}$, and $C = \{1,2,3,4\}$

(a) **Yes**. $A - A$ means the set of all elements that are in A that are also not in A; or more formally, $A - A = \{x | x \in A \text{ and } x \notin A\} = \varnothing$.

(b) **Yes**. $B - A = \{4,7\}$; $\overline{A} = \{2,3,4,7\}$ which means $B \cap \overline{A} = \{4,7\}$ so the sets are equal.

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(c) **No**. $B - A = \{4,7\}$;
$B \cap A = \{1,5,6\}$
so the sets are not equal.

(d) **Yes**. $A \cup A = \{1,5,6\}$; $A \cup \emptyset = \{1,5,6\}$, so the sets are equal. In general, combining a set to itself via set union will not change the elements in the set; combining a set via set union to the empty set will not change the elements in the set.

3. **(a)** **True**. Let $A = \{1,2\}$. $A \cup \varnothing = \{1,2\}$. In general, combining any set with the empty set via set union contributes no extra elements to the set than what is already in the set.

(b) **False**. Let $A = \{1,2\}$ and $B = \{2,3\}$.
$A - B = \{1\}; B - A = \{3\}$.

(c) **False**. Let $U = \{1,2\}; A = \{1\}; B = \{2\}$.
$\overline{A \cap B} = \{1,2\}; \overline{A} \cap \overline{B} = \varnothing$.

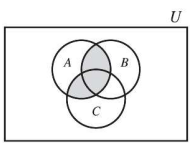
(d) **False**. Let $A = \{1,2\}; B = \{2,3,4\}$.
$(A \cup B) - A = \{1,2,3,4\} - \{1,2\} = \{3,4\} \neq B$.

(e) **False**. Let $A = \{1,2\}; B = \{2,3\}$.
$(A - B) \cup A = \{1\} \cup \{1,2\} = \{1,2\}$;
$(A - B) \cup (B - A) = \{1\} \cup \{3\} = \{1,3\}$.

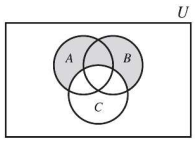
4. **(a)** If $B \subseteq A$, all elements of B must also be elements of A, but there may be elements of A that are not elements of B, so $\boldsymbol{A \cap B = B}$.

(b) If $B \subseteq A$, all elements of B must also be elements of A, but there may be elements of A that are not elements of B, so $\boldsymbol{A \cup B = A}$.

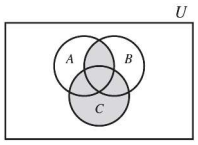
5. **(a)** $(A \cap B) \cup (A \cap C)$



(b) $(A \cup B) \cap \overline{C}$



(c) $(A \cap B) \cup C$



6. **(a)** $S \cup \overline{S} = \{x | x \in S \text{ or } x \in \overline{S}\} = \boldsymbol{U}$.

(b) If U is the universe the complement of U can have no elements, thus $\boldsymbol{\overline{U} = \emptyset}$.

(c) There are no elements common to S and $\overline{S}$, so $\boldsymbol{S \cap \overline{S} = \varnothing}$.

(d) Since there are no elements in the empty set there are none common to it and S, so $\boldsymbol{\varnothing \cap S = \varnothing}$.

7. **(a)** If $A \cap B = \varnothing$ then A and B are disjoint sets and any element in A is not in B, so $A - B = \{x | x \in A \text{ and } x \notin B\} = \boldsymbol{A}$.

(b) Since B is the empty set, there are no elements to remove from A, so $A - B = \boldsymbol{A}$.

(c) If $B = U$ there are no elements in A which are not in B, so $A - B = \varnothing$.

8. **Yes**. By definition, $A - B$ is the set of all elements in A that are not in B. If $A - B = \varnothing$ this means there are no elements in A that are not in B, thus $A \subseteq B$.

More formally, suppose $A \not\subseteq B$. Then there must be an element in A that is not in B, which implies that it is in $A - B$. This implies that $A - B$ is not empty, which is a contradiction. Thus $A \subseteq B$.

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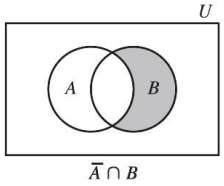
9. Answers may vary.

(a) $\boldsymbol{B \cap \overline{A}}$ or $\boldsymbol{B - A}$; i.e., $\{x | x \in B$ but $x \notin A\}$.

(b) $\boldsymbol{\overline{A \cup B}}$ or $\boldsymbol{\overline{A} \cap \overline{B}}$; i.e., $\{x | x \notin A$ or $B\}$.

(c) $\boldsymbol{(A \cap B) \cap \overline{C}}$ or $\boldsymbol{(A \cap B) - C}$; i.e., $\{x | x \in A$ and B but $x \notin C\}$.

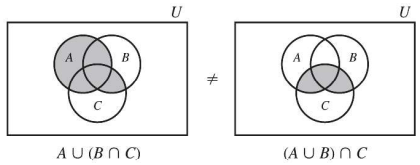
10. **(a)** $\overline{A}$ is the set of all elements in U that are not in A. $\overline{A} \cap B$ is the set of all elements common to $\overline{A}$ and B:



(b) Answers vary. $\overline{A} \cap B = B - (A \cap B)$. or $\overline{A} \cap B = B - A$. are two common responses.

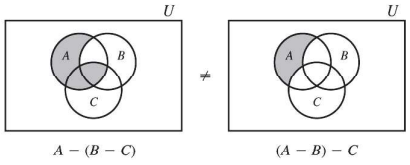
11. **(a)** **False**:

$A \cup (B \cap C) \neq (A \cup B) \cap C$



(b) **False**:

$A - (B - C) \neq (A - B) - C$



12. **(a)** Yes. Answers will vary. Note that in general, $\boldsymbol{A \cap B \subseteq A \cup B}$ because all elements of $A \cap B$ are included in $A \cup B$. Example: Let $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8\}$. Then $A \cap B = \{4,5\}$ and $A \cup B = \{1,2,3,4,5,6,7,8\}$.

(b) Yes. Answers will vary. Example: Let $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8\}$, where the universal set is $\{1,2,3,4,5,6,7,8\}$ Then $A - B = \{1,2,3\}$ which means $\overline{A - B} = \{4,5,6,7,8\}$, which does contain two elements of A, namely 4 and 5.

13. **(a)** ***(i)*** Greatest $n(A \cup B) = n(A) + n(B) = \mathbf{5}$ if A and B are disjoint.

(ii) Greatest $n(A \cap B) = n(B) = \mathbf{2}$ if $B \subseteq A$.

(iii) Greatest $n(B - A) = n(B) = \mathbf{2}$ if A and B are disjoint.

(iv) Greatest $n(A - B) = n(A) = \mathbf{3}$ if A and B are disjoint.

(b) ***(i)*** Greatest $n(A \cup B) = \boldsymbol{n + m}$ if A and B are disjoint.

(ii) Greatest $n(A \cap B) = \boldsymbol{m}$, if $B \subseteq A$, or $\boldsymbol{n}$, if $A \subseteq B$.

(iii) Greatest $n(B - A) = \boldsymbol{m}$ if A and B are disjoint.

(iv) Greatest $n(A - B) = \boldsymbol{n}$ if A and B are disjoint.

14. **(a)** ***(i)*** Greatest $n(A \cup B \cup C) = n(A) + n(B) + n(C) = 4 + 5 + 6 = \mathbf{15}$, if A, B, and C are disjoint.

(ii) Least $n(A \cup B \cup C) = n(C) = \mathbf{6}$, if $A \subset B \subset C$.

(b) ***(i)*** Greatest $n(A \cap B \cap C) = n(A) = \mathbf{4}$, if $A \subset B \subset C$.

(ii) Least $n(A \cap B \cap C) = \mathbf{0}$, if A, B, and C are disjoint.

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15. **(a)** 319.2 million non-Muslims. Subtract the Muslim population (2.6 million) away from the total population (321.8 million) of the United States.

(b) 148 million. Subtract the Muslim population (2.6 million) away from the number of people identified as being of some faith (150.6 million).

16. Constructing a Venn diagram will help in visualization:

(a) $B \cap S$ is the set of college **basketball players more than 200 cm tall**.

(b) $\overline{S}$ is the set of humans who are **not college students** or who are **college students less than or equal to 200 cm tall**.

(c) $B \cup S$ is the set of humans who are **college basketball players** or who are **college students taller than 200 cm**.

(d) $\overline{B \cup S}$ is the set of all humans who are **not college basketball players and who are not college students taller than 200 cm**.

(e) $\overline{B} \cap S$ is the set of all **college students taller than 200 cm who are not basketball players**.

(f) $B \cap \overline{S}$ is the set of all **college basketball players less than or equal to 200 cm tall**.

17. Use a three-set Venn diagram, labeling the sets B (for basketball), V (volleyball), and S (soccer):

(i) Enter 2 in the region representing $B \cap V \cap S$ (i.e., there were two who played all three sports);

(ii) Enter 1 in the region representing $(B \cap V) - S$ (i.e., there was one who played basketball and volleyball but not soccer);

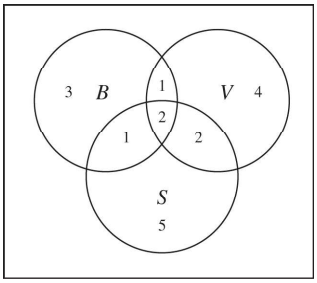
(iii) Enter 1 in the region representing $(B \cap S) - V$ (i.e., there was one who played basketball and soccer but not volleyball);

(iv) Enter 2 in the region representing $(V \cap S) - B$ (i.e., there were two who played volleyball and soccer but not basketball);

(v) Enter $7 - (1 + 1 + 2) = 3$ in the region representing $B - (V \cup S)$ (i.e., of the seven who played basketball, one also played volleyball, one also played soccer, and two also played both volleyball and soccer—leaving three who played basketball only);

(vi) Enter $9 - (1 + 2 + 2) = 4$ in the region representing $V - (B \cup S)$ (i.e., of the nine who played volleyball, one also played basketball, two also played soccer, and two also played both basketball and soccer—leaving four who played volleyball only);

(vii) Enter $10 - (1 + 2 + 2) = 5$ in the region representing $S - (B \cup V)$ (i.e., of the ten who played soccer, one also played basketball, two also played volleyball, and two also played both basketball and volleyball—leaving five who played soccer only).



There are then $3 + 4 + 5 + 1 + 1 + 2 + 2 = \mathbf{18}$ who played one or more sports.

18. In the Venn diagram below:

(i) There were 5 members who took both biology and mathematics;

(ii) Of the 18 who took mathematics 5 also took biology, leaving 13 who took mathematics only;

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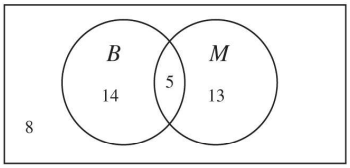
(*iii*) 8 took neither course, so of the total of 40 members there were $40 - (5 + 13 + 8) = \mathbf{14}$ who took biology but not mathematics.



19. **(a)** If all bikes needing new tires also need gear repairs, i.e., if $\{TIRES\} \subset \{GEARS\}$, then $n[\{TIRES\} \cap \{GEARS\}] = \mathbf{20\ bikes}$.

(b) Adding the separate repairs gives $20 + 30 = 50$ bikes, which is the total number of bikes; so it is possible that **zero bikes** needed both repairs, if every bike there needed exactly one repair.

(c) If the maximum number of bikes needed both repairs then all 20 receiving tires would also receive gear work. That would leave 10 additional bikes needing gear work only, leaving **20 bikes** that needed no service.

20. Generate the following Venn diagram in this order:

(*i*) The 4 who had A, B, and Rh antigens;

(*ii*) The $5 - 4 = 1$ who had A and B antigens, but who were Rh negative;

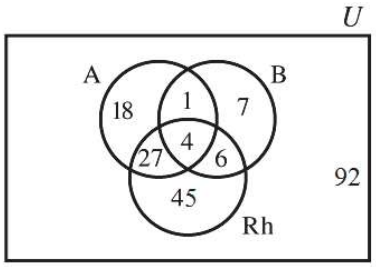
(*iii*) The $31 - 4 = 27$ who had A antigens and were Rh positive;

(*iv*) The $10 - 4 = 6$ who had B antigens and were Rh positive;

(*v*) The $50 - 27 - 4 - 1 = 18$ who had A antigens only;

(*vi*) The $18 - 6 - 4 - 1 = 7$ who had B antigens only, and;

(*vii*) The $82 - 27 - 4 - 6 = 45$ who were O positive.



$n(A \cup B \cup \text{Rh}) = 18 + 1 + 7 + 27 + 4 + 6 + 45 = 108$. Thus the set of people who are O-negative is $200 - 108 = \mathbf{92}$.

21. Let M be the set of students taking mathematics, C be the set of students taking chemistry, and P be the set of students taking physics. Generate the following Venn diagram in this order:

(*i*) The 8 who had all three subjects;

(*ii*) The $10 - 8 = 2$ students who took mathematics and physics, but not chemistry;

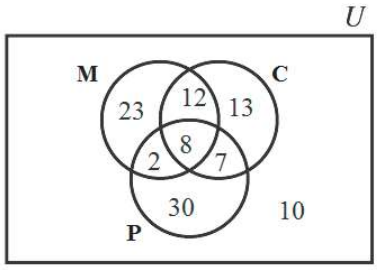
(*iii*) The $15 - 8 = 7$ students who took chemistry and physics, but not mathematics;

(*iv*) The $20 - 8 = 12$ students who took mathematics and chemistry, but not physics;

(*v*) The $45 - 12 - 8 - 2 = 23$ students who took mathematics only;

(*vi*) The $40 - 12 - 8 - 7 = 13$ students who took chemistry only;

(*vii*) The $47 - 2 - 8 - 7 = 30$ students who took physics only



Combined with the 10 students who didn't take any of the three courses, this Venn diagram yields a total number of students as 105, which contradicts John's reported total of 100. So, given that there must be errors somewhere in John's data gathering, John probably shouldn't be hired for the job.

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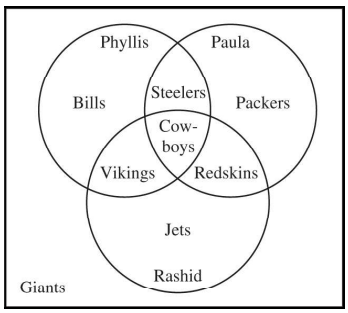
22. The following Venn diagram helps in isolating the choices:



All picked the Cowboys to win their game, so their opponent cannot be among any of the other choices; the only team not picked was the Giants.

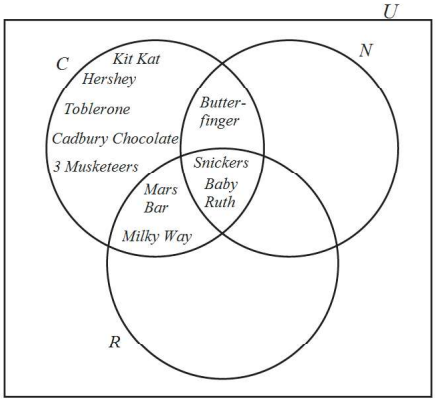
Phyllis and Paula both picked the Steelers, so their opponent cannot be among their other choices. This leaves the Jets.

Phyllis and Rashid both picked the Vikings which leaves the Packers as the only possible opponent.

Paula and Rashid both picked the Redskins which leaves the Bills as the only possible opponent.

Thus we have **Cowboys vs Giants**, **Vikings vs Packers**, **Redskins vs Bills**, **and Jets vs Steelers**.

23. (a)



(b) (*i*) **none**. Since all candy bars have chocolate, there aren't any in the set of bars without chocolate.

(b) (*ii*) **none**. Same reason as (i).

(b) (*iii*) **Chocolate** is the most popular ingredient; **nuts/peanut butter** is the least popular ingredient.

(b) (*iv*) Answers will vary.

24. **57.** Of the 324 first class passengers, you know there were $146 + 4 = 150$ total women and children, and 117 men who were lost. To find the number of men who survived, find $324 - 150 - 117 = 57$

25. **Abby, Harry, and Dick are in one family; Tom, Jane, and Mary are in the other family.** First, note that Abby and Harry have the same characteristics, blue eyes and blond hair. Also note that Mary is their opposite, with brown eyes and brown hair; therefore, Mary is certainly in a different family than Abby and Harry. Now note that Jane and Tom both have blue eyes and brown hair, while Dick has brown eyes and blond hair. Dick cannot be in the same family as Jane and Tom. So, since each family has 3 children, Dick is in the family with Abby and Harry (where they all have blond hair), while Tom, Jane, and Mary are in the other family (where they all have brown hair).

26. If a Cartesian product is the set of all ordered pairs such that the first element of each pair is an element of the first set and the second element of each pair is an element of the second set:

(a) $A \times B = \{(x,a),(x,b),(x,c),(y,a),(y,b),(y,c)\}$.

(b) $B \times A = \{(a,x),(a,y),(b,x),(b,y),(c,x),(c,y)\}$.

(c) **No**. For sets to be equal, all the elements of one must be elements of the other. But (a,x) for example is not a member of $A \times B$. For ordered pairs to be equal, their first coordinates must be equal and their second coordinates must be equal.

27. **(a)** The first element of each ordered pair is a, so $C = \{a\}$. The second elements in the ordered pairs are, respectively, b, c, d, and e, so $D = \{b,c,d,e\}$.

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(b) The first element in the first three ordered pairs is 1; in the second three is 2, so $\boldsymbol{C = \{1,2\}}$. The second element in the ordered pairs is, respectively, 1, 2, and 3, so $\boldsymbol{D = \{1,2,3\}}$.

(c) The numbers 0 and 1 appear in each ordered pair, so $\boldsymbol{C = D = \{0,1\}}$. (The order of the numbers in these sets is irrelevant.)

Assessment 2-3B

1. Given $W = \{0,1,2,3,\ldots\}$ and $N = \{1,2,3,\ldots\}$ then $A = \{1,3,5,7,\ldots\}$ and $B = \{0,2,4,6,\ldots\}$:

(a) $\boldsymbol{B}$, or the set of all elements in W that are not in A.

(b) $\varnothing$. There are no elements common to both A and B.

(c) $\boldsymbol{N}$. All elements are common to both except 0.

2. **(a)** **No**. For example, $A = \{l,i,t\}$ and $B = \{l,i\}$. Then $A - B = \{t\}$ and $B - A = \varnothing$.

(b) **Yes**. By definition $A - B = \{x | x \in A$ and $x \notin B\}$. Since $x \notin B$ describes $\overline{B}$, $A - B = \{x | x \in A$ and $x \in \overline{B}\} = A \cap \overline{B}$.

(c) **No**. By definition, $B - A = \{x | x \in B$ and $x \notin A\}$. So, $B - A$ cannot contain an element that is in A unless it is also in their intersection. But, $B \cup A = A$, if $B \subseteq A.$, which means everything in B must be in A. So, the two sets cannot be equal

(d) **Yes**. Since $\varnothing$ is empty, $B \cup \varnothing = B$. Since $B \cap B$ is all the elements common to B and B, $B \cap B = B$.

3. **(a)** **False**. Let $A = \{1,2\}$ and $B = \{2\}$. Then $A - B = \{1\}$, but $A - \varnothing = \{1,2\}$.

(b) **False**. Let $U = \{1,2,3\}, A = \{1,2\}$, and $B = \{2,3\}$. Then $A \cup B = \{1,2,3\}$ and $\overline{A \cup B} = \varnothing$. But $\overline{A} = \{3\}$ and $\overline{B} = \{1\}$, making $\overline{A} \cup \overline{B} = \{1,3\}$.

(c) **False**. Let $A = \{1,2\}, B = \{2,3\}$, and $C = \{1,2,4\}$. Then $B \cup C = \{1,2,3,4\}$ $\Rightarrow A \cap (B \cup C) = \{1,2\}$ but $A \cap B$ $= \{2\} \Rightarrow (A \cap B) \cup C = \{1,2,4\}$.

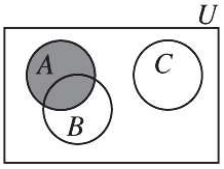
(d) **False**. Let $A = \{1,2\}$ and $B = \{2,3\}$. Then $A - B = \{1\} \Rightarrow (A - B) \cap A = \{1\}$, but $A = \{1,2\}$.

(e) **False.** Let $A = \{1,2,3\}, B = \{1,2,4\}$, and $C = \{1,2,3,4\}$. Then $B \cap C = \{1,2,4\}$, $A - B = \{3\}$, and $A - C = \varnothing$. Thus $A - (B \cap C) = \{3\}$ but $(A - B) \cap (A - C) = \varnothing$.

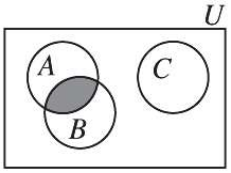
4. **(a)** Since $X \subseteq Y$, there are no elements in X, that are not also in Y so $X - Y = \varnothing$.

(b) If $X \subseteq Y$, then there are no elements in $\overline{Y}$ that are also in X, so $X \cap \overline{Y} = \varnothing$.

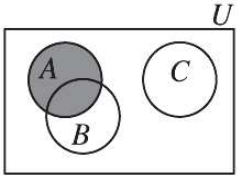
5. **(a)** $A \cap \overline{C}$



(b) $(A \cap B) \cup (B \cap C)$



(c) $A \cup (B \cap C)$



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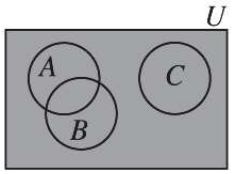
(d) $A \cup \overline{(B \cap C)}$



6. (a) $A \cup U = \boldsymbol{U}$. There are no elements in A that are not also in U.

(b) $U - A = \overline{\boldsymbol{A}}$. The set of all elements in U that are not in A is the complement of A.

(c) $A - \varnothing = \boldsymbol{A}$. There are no elements in $\varnothing$, so there are no elements which are not in A.

(d) $\overline{\varnothing} \cap A = \boldsymbol{A}$. $\overline{\varnothing} = U$, so $U \cap A = A$.

7. (a) If $A = B$ there are no elements in one set which are not also in the other, so $B - A = \varnothing$.

(b) If $B \subseteq A$ then all elements of B must also be in A, so $B - A = \{x | x \in B$ and $x \notin A\} = \varnothing$.

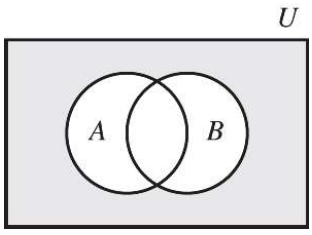
8. Answers may vary.

(a) $\boldsymbol{A \cap C}$; i.e., $\{x | x \in A$ and $C\}$.

(b) $\boldsymbol{(A \cup B) \cap C}$ or $\boldsymbol{C - (\overline{A \cup B})}$ or $\boldsymbol{(A \cap C) \cup (B \cap C)}$; i.e., $\{x | x \in A$ or B, and $x \in C\}$.

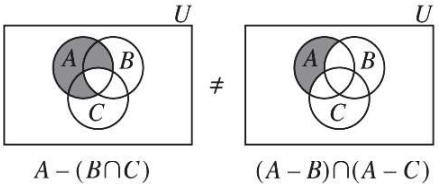
(c) In the diagram, the elements in the shaded region are in B or C but not in A. Thus, the shaded region is $\boldsymbol{(B \cup C) - A}$ **or** $\boldsymbol{(B \cup C) \cap \overline{A}}$

9.

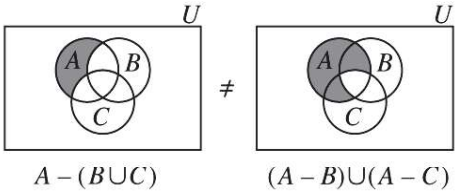


(b) Answers may vary. For example, $\overline{A \cup B} = \overline{A} \cap \overline{B}$; also $\overline{A \cup B} = U - (A \cup B)$

10. (a) False.



(b) False.



11. The first description should have fewer people fitting it. The first description is a subset of the second description, since it includes the fact that the suspect had a beard (in addition to having blond hair and green eyes). There would be less elements of that set, since it would be expected that some blond hair, green-eyed people do not have beards.

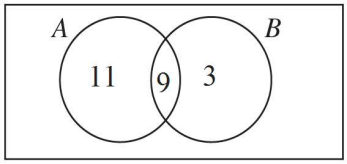
12. (a) Use a Venn diagram:

(i) Enter 9 as $n(A \cap B)$;

(ii) $n(B) = 12$, but 9 of these are in $A \cap B$, so there are 3 elements in B but not A;

(iii) $n(A \cup B) = 23$, but 12 are accounted for so there are 11 elements in A but not in B; so

(iv) $\boldsymbol{n(A) = 11 + 9 = 20.}$



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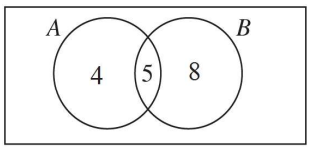
(b) Use a Venn diagram:

(i) Enter 5 as $n(A \cap B)$;

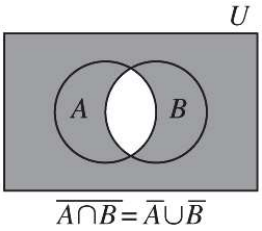
(ii) $n(A) = 9$, but 5 of these are in $A \cap B$, so there are 4 elements in A but not B;

(iii) $n(B) = 13$, but 5 of these are in $A \cap B$, so there are 8 elements in B but not in A, so;

(iv) $\boldsymbol{n(A \cup B) = 4 + 5 + 8 = 17.}$



13. (a) $\overline{A \cap B} = \overline{A} \cup \overline{B}$



(b) Let $U = \{a,b,c,d\}, A = \{a,b\}, B = \{b,c\}$:

$\overline{A \cap B} = \overline{\{b\}} = \{a,c,d\};$
$\overline{A} \cup \overline{B} = \{c,d\} \cup \{a,d\} = \{a,c,d\}.$

14. (a) The set of all Paxson 8th graders who are **members of the band but not the choir**, or $B - C$.

(b) The set of all Paxson 8th graders who are **members of both the band and the choir**, or $B \cap C$.

(c) The set of all Paxson 8th graders who are **members of the choir but not the band**, or $C - B$.

(d) The set of all Paxson 8th graders who are **neither members of the band nor of the choir**, or $\overline{B \cup C}$.

15. Enter numbers in each region in the following order:

(i) $n(A \cap B \cap C) = 3$;

(ii) $n(A \cap B) = 10$; removing the 3 from $(A \cap B \cap C)$ leaves 7 in the region $n(A \cap B \cap \overline{C})$

(iii) Similarly, $n(A \cap C \cap \overline{B}) = 8 - 3 = 5$; and

(iv) $n(B \cap C \cap \overline{A}) = 12 - 3 = 9$;

So, now what is left inside the Venn Diagram is to find (a) the number of elements in A, but not in B and C; (b) the number of elements in B, but not in A and C; and (c) the number of elements in C that are not in A and B. Those calculations are shown below

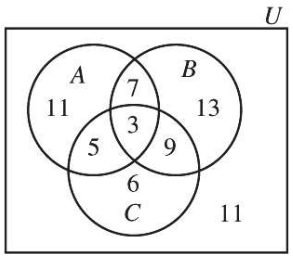
(v) $(a) = 26 - 7 - 3 - 5 = 11$;

(vi) $(b) = 32 - 7 - 3 - 9 = 13$;

(vii) $(c) = 23 - 5 - 3 - 9 = 6$;

Now, $11 + 7 + 3 + 5 + 13 + 9 + 6 = 54$ represents the total number of elements inside the Venn Diagram. Since $n(U) = 65$, find the number of elements outside the Venn Diagram but in the Universe:

(viii) $65 - 54 = 11$.



16. In the Venn diagram below:

(i) "I" is the only letter contained in the set $A \cap B \cap C$ (i.e., the only letter common to *Iowa*, *Hawaii*, and *Ohio*);

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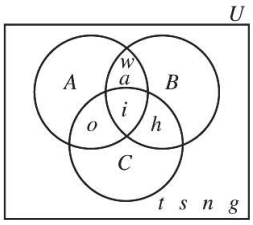
(ii) "W" and "A" are the only letters contained in the set $(A \cap B) - C$ (i.e., the letters contained in both *Iowa* and *Hawaii* other than "I");

(iii) "O" is the only letter contained in the set $(A \cap C) - B$ (i.e., the letter contained in both *Iowa* and *Ohio* other than "I");

(iv) "H" is the only letter contained in the set $(B \cap C) - A$ (i.e., the letter contained in both *Hawaii* and *Ohio* other than "I");

(v) "T", "S", "N", and "G" are the letters in *Washington* not used in *Iowa*, *Hawaii*, or *Ohio*.



17. **(a)** Elements in regions $c, f, g,$ **and** h represent students who do not take algebra.

(b) Elements in regions $b, c, d, e, f,$ **and** g represent students who take biology or chemistry.

(c) Elements in regions b **and** e represent students who took both algebra and biology.

(d) Since (a) is the part of A with no elements in B or C, we could say **the students at Hellgate who took algebra but did not take biology and did not take chemistry**.

(e) Students at Hellgate who took biology and chemistry but did not take algebra.

(f) $(B \cap C) - A$ **or** $(B \cap C) \cap \overline{A}$

(g) Since regions (d) and (g) represent students who took chemistry, but did not take biology, we write $C - B$ **or** $C \cap \overline{B}$.

(h) Region (g) represents students who took chemistry but did not take biology and did not take algebra. We write $C - (A \cup B)$ **or** $C \cap \overline{A \cup B}$.

18. Generate the following Venn diagram in this order:

(i) The 50 who used all three cards;

(ii) The 60 who used **only** Super and Thrift cards;

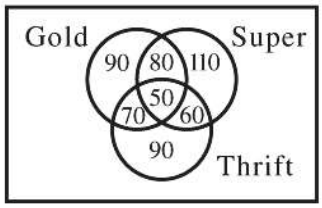
(iii) The 70 who used **only** Gold and Thrift cards;

(iv) The 80 who used **only** Gold and Super cards;

(v) The $290 - (80 + 50 + 70) = 90$ who used **only** Gold cards;

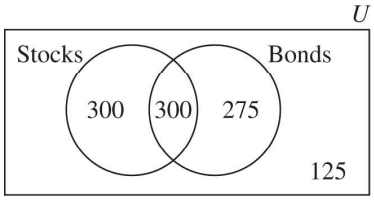
(vi) The $300 - (80 + 50 + 60) = 110$ who used **only** Super cards; and

(vii) The $270 - (70 + 50 + 60) = 90$ who used **only** Thrift cards.



The diagram indicates only 550 cardholders accounted for, so either there is some other type credit card used by the remaining 50 people or **the editor was right**.

19. Complete the Venn diagram by first completing the region that represents investors who invested in stocks and bonds.



(a) **300**

(b) **875**

(c) **125**

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20. **(a)** It is possible that no student took both algebra and biology. **0**

(b) It is possible that all **30** biology students also took algebra.

(c) If all 30 biology students also took algebra, then the numbers of students who took neither is $150 - 90 = \mathbf{60}$ students.

21. **(a)** **146.4 million.** Subtract those who identify themselves as religious Jews (4.2 million) away from the number of people identified as being of some faith (150.6 million).

(b) **171.2 million**. Subtract those who identify themselves as being of some faith (150.6 million) from the total population of the United States (321.8 million).

22. **93.** From the data given, we know there are 44 total Canadian citizens. We also know there were a total of 34 American women and males on the tour. So, all we need to find the number of American girls to complete the problem. Because there are 29 women total, and 17 are American, we know there were 12 Canadian women on the tour. Since the total number of Canadian females is 26, we can deduce that 14 Canadian females were girls. Since there were 29 girls on the trip, we know that 15 girls must be American girls. Therefore $44 + 34 + 15 = \mathbf{93}$

23. **(a)** **False**. These are ordered pairs, thus order is relevant.

(b) **False**. The left side is an ordered pair, while the right side is a set.

24. **(a)** Each of the five elements in A are paired with each of the four in B so there are $5 \cdot 4 = \mathbf{20}$ **elements**.

(b) Each of the m elements in A are paired with each of the n in B so there are $\boldsymbol{m \cdot n}$ **elements**.

(c) $A \times B$ has $m \cdot n$ elements, each of which are paired with the p elements in C so there are $\boldsymbol{m \cdot n \cdot p}$ **elements**.

Mathematical Connections 2-3: Review Problems

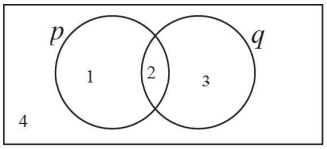
14. The contrapositive of the contrapositive of a statement is the statement itself. If the original statement is $p \rightarrow q$, then by definition its contrapositive is $\sim q \rightarrow \sim p$. To take the contrapositive of that statement, you would end up with $p \rightarrow q$.

15. **(a)** Therefore, Mary will change the lunch menu. (Modus Ponens, or law of detachment)

(b) Therefore, Samuel stays after school.

(c) Therefore, the lake is not frozen. (Modus Tollens)

16. Answers will vary. Consider the numbered Venn diagram below:



The area in numbered region 1 represents the area where p is true but q is false; the area in numbered region 3 represents the area where q is true but p is false; the area in numbered region 2 (the intersection) is where both p and q are true; and the area in numbered region 4 (outside the two circles) is the area where both p and q are false. All cases, therefore, are covered.

17. **(a)** Set-builder notation describes the elements of the set, rather than listing them; i.e., $\boldsymbol{\{x | x \in N \text{ and } 3 < x < 10\}}$, where N represents the set of natural numbers, allows the set to be built.

(b) This set has only three elements; i.e., $\{\mathbf{15, 30, 45}\}$ thus the elements may easily be listed.

18. **(a)** **6**; $c, o, m, n, r,$ and e are the elements in the set. c and m are duplicated once,and o is duplicated twice.

(b) **6**; c, o, m, i, t, e are the elements in the set. $m, t,$ and e are duplicated once.

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19. **(a)** These are all the subsets of $\{2,3,4\}$. There are $2^3 = \mathbf{8}$ such subsets.

(b) **8**. Every subset either contains 1 or it does not, so exactly half the $2^4 = 16$ subsets contain 1.

(c) **Twelve** subsets. There are 4 subsets of $\{3,4\}$. Each of these subsets can be appended with 1, 2, or 1 and 2 to each. By the Fundamental Counting Principle then, there are $3 \cdot 4 = 12$ possibilities. (It is also possible to simple systematically list all the possible subsets with 1 or 2 and count them.)

(d) There are **four** subsets containing neither 1 nor 2, since 12 do contain 1 or 2.

(e) **16**. B has $2^5 = 32$ subsets; half contain 5 and half do not.

20. Answers may vary. Some possibilities:

(a) The set of 18 holes on a golf course, and the set of 18 flagsticks on a golf course.

(b) The set of letters in the English alphabet (26) with the set of letters in the Greek alphabet (24).

21. The number of combinations is equivalent to the Cartesian product of $\{SLACKS\}$, $\{SHIRTS\}$, and $\{SWEATERS\}$ so the number of elements is $n(\{SLACKS\}) \cdot n(\{SHIRTS\}) \cdot n(\{SWEATERS\}) = 4 \cdot 5 \cdot 3 =$ **60 combinations**.

Chapter 2 Review

1. Answers will vary. Some statements: i). Arizona is a state; ii) The president of the United States is Barack Obama; iii) $2 + 12 = 24$; $2 \times 12 = 24$. Some non-statements: i) Las Vegas is a fun city; ii) Teaching is a tough job; iii) How old are you?; iv) Zane Grey is the best writer ever.

2. In statement (i), every student in the class earned an A, B, or C grade on the final exam; in statement (ii), there was at least one student who earned an A, B, or C grade on the final exam. In statement (i), no student could have received a grade of D or F on the final exam; while in statement (ii), it is possible that a student (or several students) did receive a grade of D or F on the final exam.

3. **(a)** **Yes**. Even though it is false, it is still a statement, by the definition of a statement.

(b) **No**. This sentence is neither true or false, since it depends upon the value of the variable n. Since its truth value cannot be definitely ascertained, it is not a statement.

(c) **Yes**. It can definitely be determined whether or not this statement is true or false.

4. **(a)** **Some women smoke**. The original statement means that there exists no women who smoke. So to negate it, one would have to write a statement that implied there exists at least one woman who smokes.

(b) $3 + 5 = 8$. The original statement is false; to negate it, write a statement that is true.

(c) **Bach wrote some music that was not classical**. The reasoning behind this answer is similar to the answer in part (a). The original statement implies that Bach only wrote classical music. To negate it, a statement must be written to imply that there exists at least one piece of Bach's music that was not classical.

5. The original statement is in the form $p \rightarrow q$.
Converse $(q \rightarrow p)$:If someone will read a tweet, the whole world is tweeting. Inverse $(\sim p \rightarrow \sim q)$: If the whole world is not tweeting, no one will read a tweet. Contrapositive $(\sim q \rightarrow \sim p)$: If no one will read a tweet, the whole world is not tweeting.

6.

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow q$	$\sim q \rightarrow p$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

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7. (a)

p	q	$\sim q$	$p \vee \sim q$	$p \vee q$	$(p \vee \sim q) \wedge (p \vee q)$
T	T	F	T	T	T
T	F	T	T	T	T
F	T	F	F	T	F
F	F	T	T	F	F

(b)

p	q	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge \sim q$	$[(p \vee \sim q) \wedge \sim q] \rightarrow p$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	T	T	F

8. (a) $\therefore$ Alfinia loves Mom and apple pie. This is true by direct reasoning. The first statement could be read as "If a person is a Eurasian, then that person loves Mom and apple pie". We can then make p be the statement "a person is a Eurasian" and q the statement "A person loves Mom and apple pie." So, the first statement is denoted in logical notation as $p \rightarrow q$; the second statement, Alfinia is a Eurasian, is denoted as p. So, it makes sense that the conclusion should be q.

(b) $\therefore$ The Washington Monument will eventually crack. This is true by direct reasoning, similar to (a). The first statement could be written in $p \rightarrow q$ form as "if an object is made of marble and granite, then it will eventually crack."

(c) $\therefore$ Therefore, Josef passed the math for elementary teachers course. The first sentence consists of two statements "Josef passed the math for elementary teachers course" is one statement; "Josef dropped out of school" is the other statement. The "or" between the two statements implies that one or the other is true. The second sentence "Josef did not drop out of school" would imply that the second part of that first sentence isn't true…so the first part must be.

9. Let p be the statement "you passed your classes"; q be the statement "your parents will allow you to go to the dance"; and r be the statement "you sit in the corner." So, the sentences can be written as follows: first sentence: $p \rightarrow q$; second sentence $q \rightarrow \sim r$. Third sentence: $\sim(\sim r) = r$; so the conclusion is $\sim p$. This is a valid argument by transitivity and modus tollens.

10. This argument is valid by modus tollens. Let p be the statement "Bob passes the course" and q be the statement "Bob scored at least a 75 on the final exam". Then the first sentence is $p \rightarrow q$; the second sentence is $\sim q$; therefore the conclusion is $\sim p$.

11. A set with n elements has 2^n subsets. $2^4 = 16$. This includes A. There are $2^4 - 1 =$ **15 proper subsets**.

12. There are 4 elements in the set, thus there are $2^4 = \mathbf{16}$ **subsets**: $\{\}, \{m\}, \{a\}, \{t\}, \{h\}, \{m,a\}, \{m,t\}, \{m,h\}, \{a,t\}, \{a,h\}, \{t,h\}, \{m,a,t\}, \{m,a,h\}, \{m,t,h\}, \{a,t,h\}, \{m,a,t,h\}$.

13. (a) $A \cup B = \{r,a,v,e\} \cup \{a,r,e\} = \{r,a,v,e\} = \boldsymbol{A}$.

(b) $C \cap D = \{l,i,n,e\} \cap \{s,a,l,e\} = \boldsymbol{\{l,e\}}$.

(c) $\overline{D} = \overline{\{s,a,l,e\}} = \boldsymbol{\{u,n,i,v,r\}}$.

(d) $A \cap \overline{D} = \{r,a,v,\text{e}\} \cap \{u,n,i,v,r\} = \boldsymbol{\{r,v\}}$.

(e) $\overline{B \cup C} = \overline{\{a,r,\text{e}\} \cup \{l,i,n,e\}}$
$= \overline{\{a,e,i,l,n,r\}}$
$= \boldsymbol{\{s,u,v\}}$.

(f) $\text{B} \cup \text{C} = \{a,e,i,l,n,r\} \Rightarrow (B \cup C) \cap D$
$= \{a,e,i,l,n,r\} \cap \{s,a,l,\text{e}\}$
$= \boldsymbol{\{a,l,e\}}$.

(g) $\overline{A} \cup B = \{u,n,i,s,l\} \cup \{a,r,e\}$
$= \{u,n,i,e,r,s,a,l\}$.

$C \cap \overline{D} = \{l,i,n,e\} \cap \{u,n,i,v,r\} =$
$\{i,n\}. \Rightarrow (\overline{A} \cup B) \cap (C \cap \overline{D}) = \boldsymbol{\{i,n\}}$.

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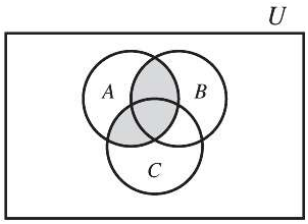
(h) $(C \cap D) \cap A = (\{l,i,n,e\} \cap \{s,a,l,e\}) \cap \{r,a,v,e\} = \{l,e\} \cap \{r,a,v,e\} = \{e\}$.

(i) $n(B - A) = \mathbf{0}$.

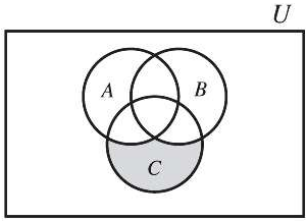
(j) $n(\overline{C}) = n\{u,v,r,s,a\} = \mathbf{5}$.

(k) $n(C \times D) = 4 \cdot 4 = \mathbf{16}$. Each of the four elements in C can be paired with each of the four in D.

14. (a) $A \cap (B \cup C)$ includes the elements in A that are common to the union of B and C:



(b) $(\overline{A \cup B}) \cap C$ is the same as $C - (A \cup B)$:

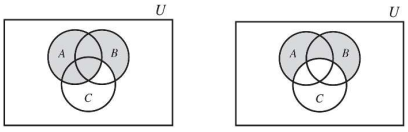


15. Since all 5 letters are distinct, consider seven "slots" in which to put the letters. There are 5 letters which could go in the first slot, then 4 left which could go in the second slot, and so on. So, the number of possible arrangements is then $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

16. (a) Answers may vary; **one possible** correspondence is $t \leftrightarrow e$, $h \leftrightarrow n$, **and** $e \leftrightarrow d$.

(b) There are three possible one-to-one correspondences between D and the e in E, two possible between D and the n in E, and then only one possible remaining for the d in E. Thus $3 \cdot 2 \cdot 1 = \mathbf{6}$ one-to-one correspondences are possible.

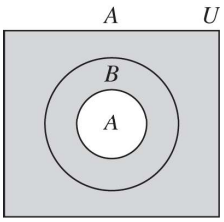
17. $A \cup (B - C) \neq (A \cup B) - C$



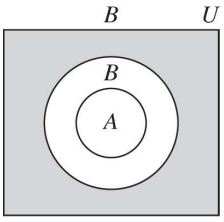
18. (a) **False**. Let $U = \{1,2,3,4\}$, $A = \{1,2\}$, and $B = \{1,2,3\}$ Then $\overline{A} = \{3,4\}$ and $\overline{B} = \{4\}$. So $\overline{A} \not\subseteq \overline{B}$.

(b) **True**.

$\overline{A}$



$\overline{B}$



(c) **True**. Since $A \subseteq B$, every element in A is in B, So combining A and B does not include new elements to B.

(d) **True**. Since $A \subseteq B$ and $A \cap B = \{x | x \in A \text{ and } x \in B\}$, the intersection, the elements common to A and B, is A.

(e) **True**. Since $\overline{B} \subseteq \overline{A}$, this property was established in d.

(f) **True,** This is because $\overline{B} \subseteq \overline{A}$, as established in b.

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19. (a) False. For example, let $U = \{1,2,3,4,5\}$; $A = \{1,2,3,4\}$; $B = \{3,4,5\}$ and $C = \{3,4\}$. Then $A - B = \{1,2\} = A - C$; but $B \neq C$

(b) True

20. (a) A**.** By the distributive property of set intersection over set union $(A \cap B) \cup (A \cap \overline{B})$ can be written as $A \cap (B \cup \overline{B})$. Since $(B \cup \overline{B}) = U$, we have $A \cap U$, which equals A.

(b) $A \cup \overline{B}$**.** By the distribution property of set union over set intersection, $(A \cap B) \cup \overline{B}$ can be rewritten as $(A \cup \overline{B}) \cap (B \cup \overline{B})$. Since $(B \cup \overline{B}) = U$, we have $(A \cup \overline{B}) \cap U$, which equals $A \cup \overline{B}$

21. (a) False. The sets could be disjoint.

(b) False. The empty set is not a proper subset of itself.

(c) False. $A \sim B$ only requires the same number of elements—not necessarily the same elements.

(d) False. The set is in one-to-one correspondence with the set of natural numbers, so it increases without limit.

(e) False. For example, the set $\{5,10,15,...\}$ is a proper subset of the natural numbers and is equivalent since there is a one-to-one correspondence.

(f) False. Let $B = \{1,2,3\}$ and $A =$ the set of natural numbers.

(g) True. If $A \cap B \neq \emptyset$, then the sets are not disjoint.

(h) False. The sets may be disjoint but not empty.

22. (a) True. Venn diagrams show that $A - B$, $B - A$, and $A \cap B$ are all disjoint sets, so $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$.

(b) True. Venn diagrams show that $A - B$ and B are disjoint sets, so $n(A - B) + n(B) = n(A \cup B)$. Likewise, Venn diagrams show that $(B - A)$ and A are disjoint, so $n(B - A) + n(A) = n(A \cup B)$.

23. (a) 17, if $P = Q$.

(b) 34, if P and Q are disjoint.

(c) 0, if P and Q are disjoint.

(d) 17, if $P = Q$.

24. $A \times B \times C$ is the set of ordered triples (a,b,c), where $a \in A$, $b \in B$, and $c \in C$. There are 3 possibilities for a, the first entry, 4 possibilities for the second, and 2 for the third. So $n(A \times B \times C) =$ **24**.

25. $n(\text{Crew}) + n(\text{Swimming}) + n(\text{Soccer}) = 57$. The 2 lettering in all three sports are counted three times, so subtract 2 twice, giving 53. $n(\text{Awards}) = 46$, so $53 - 46 = 7$ were counted twice; i.e., **7 lettered in exactly two sports**.

26. Use the following Venn diagram place values in appropriate areas starting with the fact that 3 students liked all three subjects; 7 liked history and mathematics, but of those 7, three also liked English so 4 is placed in the $\text{H} \cap \text{M}$ region; etc.

(a) A total of **36 students** were in the survey.

(b) 6 students liked only mathematics.

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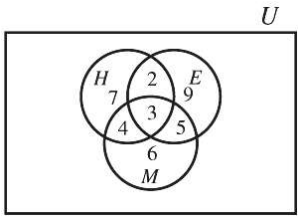
(c) **5 students** liked English and mathematics but not history.



27. Answers may vary. Possibilities are:

(a) The shaded areas show $\boldsymbol{B \cup (A \cap C)}$

(b) The shaded areas show $\boldsymbol{B - C}$ or $\boldsymbol{B \cap \overline{C}}$

28. 2 slacks $\times$ 3 blouses $\times$ 2 sweaters $=$ **12** outfits.

29. **(a)** Let $A = \{1, 2, 3, ..., 13\}$ and $B = \{1, 2, 3\}$. $B \subset A$, so B has fewer elements than A. Then $n(B) < n(A)$ and thus $3 < 13$.

(b) Let A $= \{1, 2, 3, ... , 12\}$ and $B = \{1, 2, 3, ... , 9\}$. $B \subset A$, so A has more elements than B. Then $n(A) > n(B)$ and thus $12 > 9$.

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CHAPTER 3

NUMERATION SYSTEMS AND WHOLE NUMBER OPERATIONS

Assessment 3-1A: Numeration Systems

1. Using the following place value table,

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units

we can write the numerals in words.

(a) Fifty-six million, two hundred eighty-three thousand, nine hundred fourteen.

(b) Five billion, three hundred sixty-five million, two hundred ninety-five thousand, two hundred thirty-four.

2. Using the place value table in (1):

(a) The hundreds place value is 5.

(b) The ten thousands place value is 3.

3. **(a)** **Hundreds**. From the decimal point moving to the left: units $\rightarrow$ tens $\rightarrow$ hundreds.

(b) **Tens**. From the decimal point moving to the left: units $\rightarrow$ tens.

4. **(a)** $3{,}000{,}000 + 4000 + 5 =$ **3,004,005**.

(b) $20{,}000 + 1 =$ **20,001**.

5. If $\square$, $\triangle$ and $\bigcirc$ each represent different non-zero base-ten digits then ordering them from least to greatest is based on the number of digits each number has.

(a) $\square$ (one-digit); $\square\ \triangle$ (two-digit); $\triangle\ \square\ \bigcirc$ (three-digit)

(b) $\square\ \square$ (two-digit); $\triangle\ \square\ \bigcirc$ (three-digit); $\square\ \square\ \triangle\ \triangle$ (four-digit)

6. There are 15 digits in the standard form of $625 \bullet 10^{12}$: $625{,}000{,}000{,}000{,}000$.

7. Either **811** or **910**. Each number satisfies the conditions that the hundreds digit must be 8 or 9, the tens digit must be odd, and the sum of the digits must be 10.

8. **(a)** $\overline{\overline{\text{MCDXXIV}}}$. The double bar over the M represents $1000 \cdot 1000 \cdot 1000$ while a single bar over the M would represent only $1000 \cdot 1000$.

(b) **46,032**. The 4 in 46,032 represents 40,000 while the 4 in 4632 represents only 4000.

(c) $<\ \blacktriangledown\blacktriangledown$. $<$ in the first number represents 10, while in the second number it represents $10 \cdot 60$ because of the space.

(d) $\text{𓎆}\cap\text{𓍢}$. 𓎆 has a place value of 1000 while 𓍢 has a place value of only 100.

(e) 𓂭. 𓂭 represents three groups of 20 plus zero l's, or 60, while 𓂝 represents three 5's and three l's, or 18.

9. **(a)** MCMXLIX represents 1949; thus one more is 1950, or **MCML**; one less is 1948, or **MCMXLVIII**.

(b) $<\ <\ <\nabla$ is $20 \cdot 60 + 11 = 1211$; thus one more is 1212, or $<\ <\ <\nabla\nabla$ one less is 1210, or $<<\ <$.

(c) 𓎆𓍢𓍢 is $1000 + 100 + 100 = 1200$; thus one more is 1201, or 𓎆𓍢𓍢𓏺; one less is 1199, or 𓎆𓍢 𓐍𓐍𓐍𓐍 𓏺𓏺𓏺𓏺.

(d) 𓂝 is $7 \cdot 20 + 13 \cdot 1 = 153$; thus one more is 154, or 𓂝; one less is 152, or 𓂝.

10. MCMXXII is $1000 + 900 + 20 + 2$, or the year **1922**.

11. **(a)** CXXI, or $100 + 20 + 1$.

(b) XLII, or $50 - 10 + 2$.

(c) XCI, or $100 - 10 + 1$.

(d) MMXIV, or $2000 + 10 + 5 - 1$.

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12. **(a)** Hindu-Arabic: **72**

Babylonian: $60 + 10 + 2$, or $\nabla \quad < \nabla\nabla$

Egyptian: $70 + 2$, or

Roman: **LXXII**

Mayan: $3 \cdot 20 + 12$, or

(b) Hindu-Arabic: **602**

Babylonian: $10 \cdot 60 + 2$, or $< \quad \nabla\nabla$

Egyptian: $6 \cdot 100 + 2$, or

Roman: **DCII**

Mayan: $1 \cdot 360 + 12 \cdot 20 + 2$, or

(c) Hindu-Arabic: **1223**

Babylonian: $2 \cdot 10 \cdot 60 + 23$, or

$< \; < \quad < \; < \nabla\nabla\nabla$

Egyptian:

Roman: **MCCXXIII**

Mayan: $3 \cdot 360 + 7 \cdot 20 + 3$, or

13. **(a)** $(3 \cdot 25) + (2 \cdot 5) + (1 \cdot 1) =$ **86**.

(b) $(1 \cdot 8) + (0 \cdot 4) + (1 \cdot 2) + (1 \cdot 1) =$ **11**.

14. The top row of blocks contains one set of 64 blocks plus four sets of 16 each, or $2 \cdot 4^3$ blocks; the second row of blocks contains one set of 16 blocks, or $1 \cdot 4^2$, plus four single blocks, or $1 \cdot 4^1$ blocks, plus two single blocks, or $2 \cdot 4^0 =$ $\mathbf{2112}_{\boldsymbol{four}}$.

15. **(a)** Place values represent powers of 2; e.g.,

$1 \cdot 2^0 = 1_{two}$

$1 \cdot 2^1 + 0 \cdot 2^0 = 10_{two}$

$1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 100_{two}$, etc.

Thus the first 15 counting numbers are: $(1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111)_{two}$.

(b) Place values represent powers of 4; e.g.,

$1 \cdot 4^0 = 1_{four}$

$1 \cdot 4^1 + 0 \cdot 4^0 = 10_{four}$

$1 \cdot 4^2 + 0 \cdot 4^1 + 0 \cdot 4^0 = 100_{four}$, etc.

Thus the first 15 counting numbers are: $(1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33)_{four}$.

16. **20** digits. One digit is needed for each of the units from 1 to 19, and one digit for 0.

17. $2032_{four} = (2 \cdot 10^3 + 0 \cdot 10^2 + 3 \cdot 10^1 + 2 \cdot 10^0)_{four}$. 2032_{four} may also be expanded as $(2 \cdot 4^3 + 0 \cdot 4^2 + 3 \cdot 4^1 + 2 \cdot 4^0)_{ten} =$ **142**.

18. **(a)** $\mathbf{111}_{\boldsymbol{two}}$. 1 is the largest units digit in base two.

(b) $\mathbf{EEE}_{\boldsymbol{twelve}}$. E is the largest units digit in base twelve.

19. **(a)** $EE0_{twelve} = 11 \cdot 12^2 + 11 \cdot 12^1 + 0 \Rightarrow$

(i) $(EE0 - 1)_{twelve} = 11 \cdot 12^2 + 10 \cdot 12^1 + 11 =$ $\boldsymbol{ETE}_{twelve}$.

(ii) $(EE0 + 1)_{twelve} = 11 \cdot 12^2 + 11 \cdot 12^1 + 1 =$ $\boldsymbol{EE1}_{twelve}$.

(b) ***(i)*** $(100000 - 1)_{two} = 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 - 1 = 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 =$ $\mathbf{11111}_{two}$ [analogous to $(100{,}000 - 1)_{ten} = 99{,}999_{ten}$].

(ii) $(100000 + 1)_{two} = 1 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^3 + 0 \cdot 2^1 + 0 + 1 =$ $\mathbf{100001}_{\boldsymbol{two}}$.

(c) ***(i)*** $(555 - 1)_{six} = 5 \cdot 6^2 + 5 \cdot 6^1 + 5 - 1 =$ $\mathbf{554}_{\boldsymbol{six}}$.

(ii) $(555 + 1)_{six} = 5 \cdot 6^2 + 5 \cdot 6^1 + 5 + 1 = 1 \cdot 6^3 + 0 \cdot 6^2 + 0 \cdot 6^1 + 0 = 1000_{six}$ [analogous to $(999 + 1)_{ten} = 1000_{ten}$].

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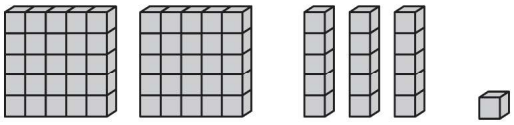
20. **(a)** There is **no numeral 4 in base four**. The numerals are 0, 1, 2, and 3.

(b) There are **no numerals 6 or 7 in base five**. The numerals are 0, 1, 2, 3, and 4.

21. There are 3 groups of 4^3, 1 group of 4^2, 1 group of 4^1, and 2 groups of 4^0 in 214, so $214 = 3112_{four}$. Thus, where a block is $4 \times 4 \times 4$, a flat is 4×4, and a long is 1×4, **three** blocks, **one** flat, **one** long, and **two** units is the least number of base four blocks.

22. 231_{five} is represented by two flats $(2 \cdot 5^2)$, three longs $(3 \cdot 5^1)$, and one unit.



23. **(a)** ***(i)*** 8 pennies may be changed into 1 nickel and 3 pennies. The total is now 2 quarters, 10 nickels, and 3 pennies.

(ii) 10 nickels may be changed into 2 quarters. The total is now **4 quarters, 0 nickels, and 3 pennies**.

(b) $73¢$, in the fewest number of coins, can be obtained with 2 quarters, 4 nickels, and 3 pennies, or $(2 \cdot 5^2 + 4 \cdot 5 + 3)¢$. This is $\mathbf{243}_{\boldsymbol{five}}$.

24. **(a)** Since both numerals have the same groups of 64s, we only need to compare lesser place values.

$100_{four} = (1 \cdot 4^2 + 0 \cdot 4 + 0 \cdot 1)_{ten} = 16$

$030_{four} = (0 \cdot 4^2 + 3 \cdot 4 + 0 \cdot 1)_{ten} = 12.$

So, 3030_{four} is the lesser.

(b) Again, we only need to compare TE_{twelve} to ET_{twelve}.

TE_{twelve} has ten groups of 12 and 11 groups of 1.

ET_{twelve} has 11 groups of 12 and 10 groups of 1.

Thus, $\boldsymbol{EOTE_{twelve}}$ **is lesser**.

25. **(a)** 10 flats $=$ 1 **block**. $1 \cdot 10^3 = \mathbf{1000}$.

(b) 20_{twelve} flats $= 12$ flats $+ 8$ flats $=$ **1 block + 8 flats**. When written in base twelve notation, it looks like $1 \cdot 12^3 + 8 \cdot 10^2 = \mathbf{1800}_{\boldsymbol{twelve}}$.

26. **(a)**

$$\begin{array}{r|l|l} 125 & 456 & 3 \\ & -375 & \\ \hline 25 & 81 & 3 \\ & -75 & \\ \hline 5 & 6 & 1 \\ & -5 & \\ \hline & 1 & \end{array}$$

$456 = \mathbf{3311}_{\boldsymbol{five}}$

How many groups of 125 in 456?
How many groups of 25 in 81?
How many groups of 5 in 6?

(b)

$$\begin{array}{r|l|l} 1728 & 1782 & 1 \\ & -1728 & \\ \hline 12 & 54 & 4 \\ & -48 & \\ \hline & 6 & \end{array}$$

$1782 = \mathbf{1046}_{\boldsymbol{twelve}}$

How many groups of $1728 = 12^3$ in 1782?
How many groups of 12 in 54?

(c)

$$\begin{array}{r|l|l} 32 & 32 & 1 \\ & -32 & \\ \hline & 0 & \end{array}$$

$32 = \mathbf{100000}_{\boldsymbol{two}}$

How many groups of $2^5 = 32$ in 32?

27. **(a)** $432_{five} = 4 \cdot 5^2 + 3 \cdot 5^1 + 2 = 100 + 15 + 2 = \mathbf{117}$.

(b) $101101_{two} = 1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 = 32 + 8 + 4 + 1 = \mathbf{45}$.

(c) $92E_{twelve} = 9 \cdot 12^2 + 2 \cdot 12 + 11 = 1296 + 24 + 11 = \mathbf{1331}$.

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28. Using the place value in base six,

6^3	6^2	6^1	1

we have $a \bullet 6^3 + b \bullet 6^2 + c \bullet 6^1 + d \bullet 1$.

29. To find out the base, add the units digits: $2_b + 6_b$. In base 10 that equals 8 but in base b it equals a numbers with units digit 1. Take out a group of b and have one left over. The only choice is base seven and then $2_7 + 6_7 = 11_7$ and

$$\begin{aligned} 12_7 + 26_7 &= (10_7 + 20_7) + (2_7 + 6_7) \\ &= (30_7) + (11_7) \\ &= 41_7 \end{aligned}$$

30. **(a)** Using the base sixteen place value:

$$\begin{aligned} 256_{sixteen} &= 2 \bullet 16^2 + 5 \bullet 16^1 + 6 \bullet 1 \\ &= 512 + 80 + 6 \\ &= 598 \end{aligned}$$

(b) Using the base sixteen place value:

$$\begin{aligned} 3ACD_{sixteen} &= 3 \bullet 16^3 + A \bullet 16^2 + C \bullet 16^1 + D \bullet 1 \\ &= 3 \bullet 16^3 + 10 \bullet 16^2 + 12 \bullet 16^1 + 13 \bullet 1 \\ &= 12,288 + 2560 + 192 + 13 \\ &= 15,053 \end{aligned}$$

31. To convert 584 from base ten to base sixteen, ask:
How many groups of 16^3(= 4096) are there in 584?
None 4096>584.
How many groups of 16^2(= 256) are there in 584?
2 groups ($2 \bullet 16^2 = 512$) with 72 left over
(584 – 512 = 72).
How many groups of 16^1(= 16) are there in 72?
4 groups ($4 \bullet 16 = 64$) with 8 left over (72 – 64 = 8).
How many groups of 16^0(= 1) are there in 8?
8 groups ($8 \bullet 16^0 = 8$) with none left over (8 – 8 = 0).
So $584_{10} = 248_{\text{sixteen}}$.

32. To give the fewest number of prizes, the dollar amount of each must be maximized. Thus $900 =

1 prize or \$625 with \$275 left over;
2 prizes of \$125 with \$25 left over; and
1 prize of \$25 with nothing left over.
I.e., $900 = 12100_{five}.

33. **(a)** There are 8 groups of 7 days (1 week) with 2 days left over, so 58 days = **8 weeks and 2 days** (or $58 = 82_{seven}$).

(b) There is 1 group of 24 hours (1 day) with 5 hours left over, so 29 hours = **1 day and 5 hours** (or $29 = 15_{twenty\ four}$).

34. **(a)** $b = \mathbf{6}$. There are 6 groups of 7 in 44.

(b) $b = \mathbf{1}$. Subtracting 5 groups of 144 from 734 leaves 14; there is 1 group of 12 in 14.

35. **(a)** $3 \cdot 5^4 + 3 \cdot 5^2 = 3 \cdot 5^4 + 0 \cdot 5^3 + 3 \cdot 5^2 + 0 \cdot 5^1 + 0 \cdot 5^0 = \mathbf{30300}_{\textbf{\textit{five}}}$.

(b) $2 \cdot 12^5 + 8 \cdot 12^3 + 12 = 2 \cdot 12^5 + 0 \cdot 12^4 + 8 \cdot 12^3 + 0 \cdot 12^2 + 1 \cdot 12^1 + 0 \cdot 12^0 = \mathbf{208010}_{\textbf{\textit{twelve}}}$.

36. Answers may vary; some possibilities might be:

(a) Subtract a number with a 2 in the thousands place and a 2 in the tens place, or 2020. Then $32,420 - 2020 = 30,400$.

(b) Subtract a number with a 5 in the tens place, or 50. Then $67,357 - 50 = 67,307$.

Assessment 3-1B

1. Using the following place value table,

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units

we can write the numerals in words.

(a) Two hundred fifty-three million, two hundred forty-three thousand, eight hundred ten.

(b) Sixty-two billion, three hundred fifty-five million, two hundred eighty-eight thousand, four hundred thirty-two.

2. Using the place value table in (1):

(a) The thousands place value is 5.

(b) The hundred thousands place value is 3.

3. **(a)** **Thousands**. From the decimal point moving to the left: units → tens → hundreds → thousands → ten thousands.

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(b) **Hundred thousands**. From the decimal point moving to the left: units → tens → hundreds → thousands → ten thousands → hundred thousands.

4. **(a)** $3000 + 500 + 60 = \mathbf{3560}$.

(b) $9{,}000{,}000 + 90 + 9 = \mathbf{9{,}000{,}099}$.

5. If □, △ and ○ each represent different non-zero base-ten digits then ordering them from least to greatest is based on the number of digits each number has.

(a) (one-digit); ○ ○ (two-digit); □ △ ○ (three-digit).

(b) △ △ (two-digit); □ △ ○ (three-digit); □ □ □ □ (four-digit)

6. There are 19 digits in the standard form of $5250 \bullet 10^{15}$: $5{,}250{,}000{,}000{,}000{,}000{,}000$.

7. Let u = units digit and t = tens digit.

Then $u = t - 4$

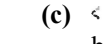
$t = 2u$

So $u = 2u - 4 \rightarrow u = 4$ and $t = 8$.

The number is **84**.

8. **(a)** $\overline{\mathbf{M}}$**DCXXIV**. D is greater than CD.

(b) **30**,**456**. The 3 in 30,456 represents 30,000 while the 3 in 3456 represents only 3000.

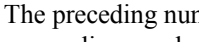
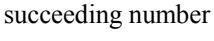
(c)  . The space indicates that < is multiplied by 60; the < without the space represents just 10.

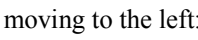
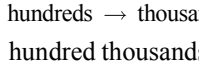
(d) , which represents 300; the other figure represents 211.

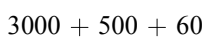
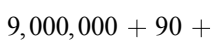
(e) , which represents $2 \cdot 20$. The other figure represents 13.

9. **(a)** $\overline{\mathbf{M}}$**I** represents 1,000,001. The preceding number is 1,000,000, or $\underline{\mathbf{M}}$; the succeeding number is 1,000,002, or **MII**.

(b) CMXCIX represents 999. The preceding number is 998, or **CMXCVIII**; succeeding number is 1000, or **M**.

(c) represents $10 \cdot 60 + 11 = 611$. The preceding number is 610, or  ; the succeeding number is 612, or  .

(d)  represents $1000 + 100 = 1100$. The preceding number is 1099, or ; the succeeding number is 1101, or  .

(e) represents $6 \cdot 20 + 8 = 128$. The preceding number is 127, or  ; the succeeding number is 129, or  .

10. Since no numeral of lesser value is to the left of a numeral of greater value, the numerals can be combined in an additive process.

MDCCLXXVI represents $1000 + 500 + 100 + 100 + 50 + 10 + 10 + 5 + 1 = \mathbf{1776}$.

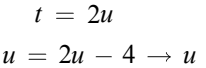
11. **(a)** LXXXIX or $50 + 30 + 9$.

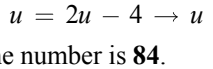
(b) $\overline{\text{V}}$CCII, or $1000 \cdot 5 + 200 + 2$.

(c) XXXII, or $30 + 2$.

(d) CII, or $100 + 2$

12. **(a)** Hindu-Arabic: 78

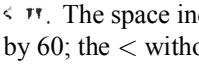
Babylonian: $60 + 10 + 8$, or 

Egyptian: 

Roman: **LXXVIII**

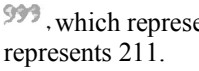
Mayan: $3 \cdot 20 + 18$, or

(b) Hindu-Arabic: **601**

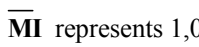
Babylonian: $10 \cdot 60 + 1$, or 

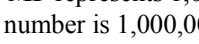
Egyptian:

Roman: **DCI**

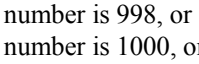
Mayan: $360 + 12 \cdot 20 + 1$, or 

(c) Hindu-Arabic: **1111**

Babylonian: $18 \cdot 60 + 31$, or 

Egyptian: 

Roman: **MCXI**

Mayan: $3 \cdot 360 + 20 + 11$, or 

13. **47**, or $1 \cdot 27 + 2 \cdot 9 + 2 \cdot 1 = 47$.

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14. The row of blocks contains one set of 27 blocks, or $1 \cdot 3^3$, plus two sets of 9 each, or $2 \cdot 3^2$ blocks, plus two sets of 3 each, or $2 \cdot 3^1$ blocks, plus two single blocks, or $2 \cdot 3^0$. The representation is thus $\mathbf{1222}_{\textbf{\textit{three}}}$.

15. **(a)** Place values represent powers of 3; e.g.,

$1 \cdot 3^0 = 1_{three}$

$1 \cdot 3^1 + 0 \cdot 3^0 = 10_{three}$

$1 \cdot 3^2 + 0 \cdot 3^1 + 0 \cdot 3^0 = 100_{three}$, etc.

Thus the first 10 counting numbers are: $\mathbf{(1, 2, 10, 11, 12, 20, 21, 22, 100, 101)_{\textit{three}}}$.

(b) Place values represent powers of 8; e.g.,

$1 \cdot 8^0 = 1_{eight}$

$1 \cdot 8^1 + 0 \cdot 8^0 = 10_{eight}$

$1 \cdot 8^2 + 0 \cdot 8^1 + 0 \cdot 8^0 = 100_{eight}$, etc.

Thus the first 10 counting numbers are: $\mathbf{(1, 2, 3, 4, 5, 6, 7, 10, 11, 12)_{\textit{eight}}}$.

16. **18** digits. One digit is needed for each of the units from 1 to 17, and one digit for 0.

17. $2022_{three} = \mathbf{(2 \cdot 10^3 + 0 \cdot 10^2 + 2 \cdot 10^1 + 2 \times 10^0)_{\textit{three}}}$. 2022_{three} may alternatively be expanded as $\mathbf{(2 \cdot 3^3 + 0 \cdot 3^2 + 2 \cdot 3^1 + 2 \cdot 3^0)_{\textit{ten}} = 62}$.

18. **(a)** $\mathbf{222_{\textit{three}}}$. 2 is the largest units digit in base three.

(b) 666_{seven}. 6 is the largest units digit in base seven.

19. **(a)** ***(i)*** $(100 - 1)_{seven} = 1 \cdot 7^2 + 0 \cdot 7^1 + 0 \cdot 7^0 - 1 = 6 \cdot 7^1 + 6 \cdot 7^0 = \mathbf{66_{\textit{seven}}}$

(ii) $(100 + 1)_{seven} = 1 \cdot 7^2 + 0 \cdot 7^1 + 0 \cdot 7^0 + 1 = \mathbf{101_{\textit{seven}}}$

(b) ***(i)*** $(1000 - 1)_{two} = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 - 1 = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = \mathbf{111_{\textit{two}}}$

(ii) $(1000 + 1)_{two} = 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 + 1 = \mathbf{1001_{\textit{two}}}$

(c) ***(i)*** $(101 - 1)_{two} = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 - 1 = 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = \mathbf{100_{\textit{two}}}$

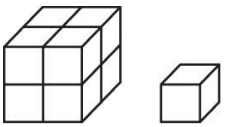
(ii) $(101 + 1)_{two} = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 + 1 = \mathbf{110_{\textit{two}}}$

20. **(a)** There is **no numeral 6** in base four. The numerals are 0, 1, 2, and 3.

(b) There are **no numerals 2 or 3** in base two. The numerals are 0 and 1.

21. There are 2 groups of 3^3, 2 groups of 3^2, 2 groups of 3^1, and 1 group of 3^0 in 79, so $79 = 2221_{three}$. Thus, where a block is $3 \times 3 \times 3$, a flat is 3×3, and a long is 1×3, **two** blocks, **two** flats, **two** longs, and **one** unit, so, **seven** is the fewest number of base three blocks.

22. 1001_{two} is represented by 1 block, 0 flats, 0 longs, and 1 unit, or



23. $277 = 1 \cdot 12^2 + 11 \cdot 12^1 + 1 \cdot 12^0$, or **1 gross, 11 dozens, and 1 unit**.

24. **(a)** $\mathbf{E0T9E_{\textit{twelve}} < EET9E_{\textit{twelve}}}$. $EE000_{twelve}$ is greater than $E0000_{twelve}$.

(b) $\mathbf{101011_{\textit{two}} < 1011011_{\textit{two}}}$. Any seven-digit number is greater than any six-digit number.

(c) $\mathbf{50555_{\textit{six}} < 51000_{\textit{six}}}$. 1000_{six} is greater than 555_{six}.

25. **(a)** 10_{four} longs $= 40$ units, which can be represented by **2 flats and 2 longs**, or four pieces.

(b) 10_{three} longs $= 30$ units, which can be represented by **1 block and 1 long**, or two pieces.

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26. **(a)** Place values in base 4 are $4^3 = 64, 4^2 = 16$, $4^1 = 4$, and $4^0 = 1$; the largest needed here is 4^3.

(i) There are 3 groups of 64 (or 4^3) in 234, with remainder 42;

(ii) There are 2 groups of 16 (or 4^2) in 42, with remainder 10;

(iii) There are 2 groups of 4 (or 4^1) in 10, with remainder 2;

(iv) There are 2 groups of 1 in 2, with remainder 0.

Thus $234 = \mathbf{3222}_{\textit{four}}$.

(b) Place values in base 12 are $12^3 = 1728$, $12^2 = 144$, $12^1 = 12$, and $12^0 = 1$.

(i) There is 1 group of 1728 in 1876, with remainder 148;

(ii) There is 1 group of 144 in 148, with remainder 4;

(iii) There are 0 groups of 12 in 4, with remainder 4;

(iv) There are 4 groups of 1 in 4, with remainder 0.

Thus $1876 = \mathbf{1104}_{\textit{twelve}}$.

(c) Place values in base 3 are $3^5 = 243$, $3^4 = 81, 3^3 = 27, 3^2 = 9, 3^1 = 3$, and $3^0 = 1$.

(i) There is 1 group of 243 in 303, with remainder 60;

(ii) There are 0 groups of 81 in 60, with remainder 60;

(iii) There are 2 groups of 27 in 60, with remainder 6;

(iv) There are 0 groups of 9 in 6, with remainder 6;

(v) There are 2 groups of 3 in 6, with remainder 0;

(vi) There 0 groups of 1 in 0, with remainder 0.

Thus $303 = \mathbf{102020}_{\textit{three}}$.

(d) Place values in base 2 are $2^4 = 16, 2^3 = 8$, $2^2 = 4, 2^1 = 2$, and $2^0 = 1$.

(i) There is 1 group of 16 in 22, with remainder 6;

(ii) There are 0 groups of 8 in 6, with remainder 6;

(iii) There is 1 group of 4 in 6, with remainder 2;

(iv) There is 1 groups of 2 in 2, with remainder 0;

(v) There are 0 groups of 1 in 0, with remainder 0.

Thus $22 = \mathbf{10110}_{\textit{two}}$.

27. **(a)** $432_{six} = 4 \cdot 6^2 + 3 \cdot 6^1 + 2 \cdot 6^0 = \mathbf{164}$.

(b) $11011_{two} = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = \mathbf{27}$.

(c) $E29_{twelve} = 11 \cdot 12^2 + 2 \cdot 12^1 + 9 \cdot 12^0 = \mathbf{1617}$.

28. Using the place value in base eight,

8^3	8^2	8^1	1

we have $a \bullet 8^3 + b \bullet 8^2 + c \bullet 8^1 + d \bullet 1$.

29. To find out the base, add the units digits: $4_b + 5_b$. In base 10 that equals 9 but in base b it equals a numbers with units digit 2. Take out a group of b and have two left over. The only choice is base seven and then $4_7 + 5_7 = 12_7$ and

$$\begin{aligned} 54_7 + 45_7 &= (50_7 + 40_7) + (4_7 + 5_7) \\ &= (120_7) + (12_7) \\ &= 132_7 \end{aligned}$$

30. **(a)** Using the base sixteen place value:

$$\begin{aligned} 864_{sixteen} &= 8 \bullet 16^2 + 6 \bullet 16^1 + 4 \bullet 1 \\ &= 2,048 + 96 + 4 \\ &= 2,148 \end{aligned}$$

(b) Using the base sixteen place value:

$$\begin{aligned} 2CDE_{sixteen} &= 2 \bullet 16^3 + C \bullet 16^2 + D \bullet 16^1 + E \bullet 1 \\ &= 2 \bullet 16^3 + 12 \bullet 16^2 + 13 \bullet 16^1 + 14 \bullet 1 \\ &= 8,192 + 3072 + 208 + 14 \\ &= 11,486 \end{aligned}$$

31. To convert 256 from base ten to base sixteen, ask:
How many groups of 16^3(= 4096) are there in 256? None 4096>584.
How many groups of 16^2(= 256) are there in 256? 1 group ($1 \bullet 16^2 = 256$) with none left over (256 – 256 = 0). There are no groups of 16^1(= 16) and no groups of 16^0(= 1).
So $256_{10} = 100_{\text{sixteen}}$.

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32. To give the fewest number of prizes, the dollar amount of each must be maximized. Thus $\$900 =$
1 prize of \$512 with \$388 left over;
1 prize of \$256 with \$132 left over;
1 prize of \$128 with \$4 left over, and
1 prize of \$4, with nothing left over,
i.e., $\$900 = \$1110000100_{two}$. Thus, the minimum number of prizes is **four**.

33. There are $4 = 2^2$ cups per quart and $2 = 2^1$ cups per pint. Thus 1 cup, 1 pint, and 1 quart $= \mathbf{111_{two}}$.

34. **(a)** **7**. $b3_{four} = 4b + 3 = 31$
$\Rightarrow 4b = 28 \Rightarrow b = 7$.
But there is no digit 7 in base 4 so this is not possible.

(b) Not possible. $1534_{six} = 418_{ten}$ and
$1b2_{twelve} = (1 \cdot 10^2 + b + 2)_{twelve} =$
$(1 \cdot 12^2 + b \cdot 12 + 2)_{ten} = 146 + 12b$.
Then $418 = 146 + 12b \Rightarrow b$ is not an integer, so no solution exists.

35. If each key may be used only once, the largest four-digit number would be **9876.**

36. Answers vary. For example,
$MID = 1000 + 500 - 1$.
$= 1499$

Assessment 3-2A: Addition and Subtraction of Whole Numbers

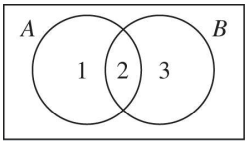
1. The student is adding 13 and 4.Start with thirteen and add one each time until seventeen then notice that four numbers were added.

2. **(a)** **True**.
$n(A) = 3; n(B) = 2; n(A) + n(B) = 5$.
$A \cup B = \{a,b,c,d,e\}; n(A \cup B) = 5$.
$\boldsymbol{n(A) + n(B) = n(A \cup B)}$ because the sets are disjoint.

(b) **False**.
$n(A) = 3; n(B) = 2; n(A) + n(B) = 5$.
$A \cup B = \{a, b, c\}; n(A \cup B) = 3$.
$\boldsymbol{n(A) + n(B) \neq n(A \cup B)}$ because the sets are not disjoint.

(c) **True**.
$n(A) = 3; n(B) = 0; n(A) + n(B) = 3$.
$A \cup B = \{a,b,c\}; n(A \cup B) = 3$.
$\boldsymbol{n(A) + n(B) = n(A \cup B)}$ because the sets are disjoint.

3. The sets are not disjoint because $n(A) + n(B) \neq n(A \cup B)$. $n(A \cap B)$ must therefore equal $n(A) + n(B) - n(A \cup B) = 3 + 5 - 6 = 2$. In the following Venn diagram:



$n(A) = 3, n(B) = 5, n(A \cup B) = 6,$ and
$\boldsymbol{n(A \cap B) = 2}$.

4. If A and B are not disjoint:
Let $A = \{1, 2\}, B = \{2, 3\}$, so $A \cup B = \{1, 2, 3\}$.
Then $n(A) = 2, n(B) = 2, n(A \cup B) = 3$.
But $n(A) + n(B) = 4 \neq 3 = n(A \cup B)$.

5. **(a)** **(*i*)** $\boldsymbol{n(B) = 3}$ if the sets are disjoint.
(*ii*) $\boldsymbol{n(B) = 4}$ if $n(A \cap B) = 1$.
(*iii*) $\boldsymbol{n(B) = 5}$ if $n(A \cap B) = 2$.
(*iv*) $\boldsymbol{n(B) = 6}$ if $n(A \cap B) = 3$.

(b) If $n(A \cap B) = \emptyset$ the sets are disjoint and $\boldsymbol{n(B) = 3}$.

6. **(a)** **Closed**. $0 + 0 = 0$, and $0 \in \{0\}$.

(b) **Closed**. Assuming the arithmetic sequence $0, 3, 6, \ldots,$ any element of T added to any other element in T is whole and divisible by 3.

(c) **Closed**. N is the set of natural numbers and any natural number added to any other natural number is an element of N.

(d) **Not Closed**. $3 + 7 \notin \{3,5,7\}$.

(e) **Closed**. $\{W\}$ is the set of whole numbers and any whole number greater than 10 added to any other whole number greater than 10 is an element of W.

(f) **Not Closed.** $1 + 1 \notin \{0,1\}$.

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7. Answers may vary. A possible answer is $2 + 2 = \mathbf{4}$, $4 + 2 = \mathbf{6}$, $4 + 4 = \mathbf{8}$, $2 + 3 = \mathbf{5}$, $5 + 2 = \mathbf{7}$, and $7 + 2 = \mathbf{9}$.

8. If set A is closed under addition then all whole numbers greater than or equal to 2 are in A. The only possible missing numbers are 0 and 1.

9. **(a)** 1, commutative property of addition.

(b) 7, commutative property of addition.

(c) 0, additive identity.

(d) 7, associative and commutative properties of addition.

10. **(a)** **Commutative property of addition**; i,e., if a and b are any whole numbers, then $a + b = b + a$.

(b) **Associative property of addition**; i.e., if $a, b,$ and c are any whole numbers, then $(a + b) + c = a + (b + c)$.

(c) **Commutative property of addition**; i.e., $(6 + 3) = (3 + 6)$.

(d) **Identity property of whole numbers**; i.e., for any whole number $a, a + 0 = a$.

(e) **Commutative property of addition**.

(f) **Associative property of addition**.

(g) **Closure property of addition**.

11. **No**. If $k = 0$ (a whole number) and $a = b = 0$, then the $a = b + k \Rightarrow a < b$, which would imply that $0 < 0$, a contradiction.

12. ***(i)*** For any whole numbers a and $b, a < b$ if and only if there exists a natural number k such that $b - k = a$ (or equivalently, if and only if $b - a$ is a natural number).

(ii) For any whole numbers a and $b, a > b$ if and only if there exists a natural number k such that $a - k = b$ (or equivalently, if and only if $a - b$ is a natural number).

13. **(a)** Each term is found by adding 5 to the previous term. Thus the next three are $28 + 5 = \mathbf{33}$, $33 + 5 = \mathbf{38}$, $38 + 5 = \mathbf{43}$.

(b) Each term is found by subtracting 7 from the previous term. Thus the next three are $63 - 7 = \mathbf{56}$, $56 - 7 = \mathbf{49}$, $49 - 7 = \mathbf{42}$.

14. In a magic square each row, column, and diagonal must have the same sum. $8 + 5 + 2 = 15$ so each row, column, and diagonal must add to 15.

8	**3**	**4**
1	**5**	**9**
6	**7**	**2**

or

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

15. **(a)** **9**. A number greater than 9 would have two digits.

(b) **8**. If A were greater, C would have two digits.

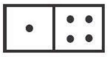
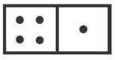
(c) **3**. A and B must be 1 and 2 (in any order); no lesser single digit numbers are available.

(d) **6 or 8**. A and B must be 2 and 4 or 2 and 6; greater even numbers would make C have two digits.

(e) **5**. If $A + B = C$ and C is 5 more than A, then $A + 5 = C$.

(f) **4 or 8**. B could be 1 and A must then be 3; or B could be 2 and then A must be 6.

(g) **9**. B must be 2 and A must be 7.

16. Since the way the domino is positioned doesn't matter, i.e.,  is the same domino as  , each number put on the left side gets paired with each of the 9 choices for the right

Number printed on left	Number of choices for right
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1

Sum the right column and we have $\frac{10(9)}{2} = 45$ dominos.

17. **(a)** Order the players as follows:

Kent < Mischa < Sally < Vera

Kent is shortest; Vera is tallest.

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(b) Answers may vary, as long as the player's heights increase in the order:

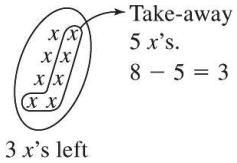
Kent → Mischa → Sally → Vera.

18. (a) Add 7 to each side: $\mathbf{9 = 7 + x}$.

(b) Add 6 to each side: $\boldsymbol{x = 3 + 6}$

(c) Add x to each side: $\mathbf{9 = x + 2}$.

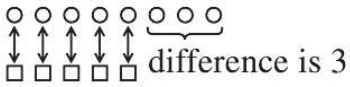
19. (a) Take away:



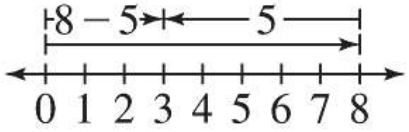
(b) Missing addend:

$\square + 5 = 8 \Rightarrow \square = 3.$

(c) Comparison:



(d) Number line:



20. (a) **Answers vary.** For example, "Kaden has 12 cookies and Tristan takes away 7 cookies. How many cookies does Kaden have left?"

(b) **Answers vary.** For example, "Jasmine has 7 yards of fabric and makes a blanket for her sister Ebony with 3 yards of **fabric. How much fabric is left over?"**

(c) **Answers vary.** For example, "Luke has seventeen marbles. His brother Zach has 12 marbles. How many more marbles does Luke have than Zach?"

(d) **Answers vary.** For example, "Isabelle has $25 and is saving for a video game called Minecraft. If the game costs $30, how much more does Isabelle need to save?"

21. (a) Solve for x:

i. $15 - 9 = 15 - 10 + x$

$6 = 5 + x$

$1 = x$

$27 - 9 = 27 - 10 + x$

ii. $18 = 17 + x$

$1 = x$

$32 - 9 = 32 - 10 + x$

iii. $23 = 22 + x$

$1 = x$

(b) A strategy for subtracting 9 from a whole number would be subtracting 10 then adding 1.

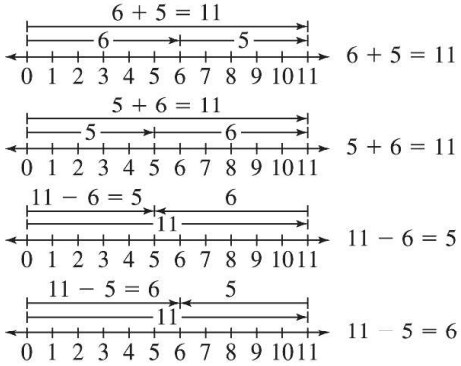
22. (a) The other three members of the fact family are:

$5 + 6 = 11$

$11 - 6 = 5$

$11 - 5 = 6$

(b)



23. (a) $\boldsymbol{a \geq b}$; if $b > a$ then $a - b$ would not be a whole number.

(b) $a - (b - c) \geq 0 \Rightarrow a \geq (b - c) \Rightarrow$ $a + c \geq b$ and $b \geq c$ so that each difference is a whole number.

24. (a) By the associative property of addition, $x + (y + z) = (x + y) + z$. By the commutative property of addition, $(x + y) + z = z + (x + y)$.

(b) By the commutative property of addition, $x + (y + z) = (y + z) + x$.

By the associative property of addition, $(y + z) + x = y + (z + x)$.

By the commutative property of addition, $y + (z + x) = y + (x + z)$.

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25. **(a)** $3 + (4 + 7) = (3 + x) + 7$
$(3 + 4) + 7 = (3 + x) + 7$
$3 + 4 = 3 + x$
$\boldsymbol{4 = x}$.

(b) $8 + 0 = x \Rightarrow \boldsymbol{x = 8}$.

(c) $5 + 8 = 8 + x$
$5 + 8 = x + 8$
$\boldsymbol{5 = x}$.

(d) $x + 8 = 12 + 5$
$x + 8 = 17$
$x + 8 - 8 = 17 - 8$
$\boldsymbol{x = 9}$.

(e) $x + 8 = 5 + (x + 3)$
$x + 8 = x + 8$
$\boldsymbol{x = x}$.
The solution is all whole numbers.

(f) $x - 2 = 9$
$\boldsymbol{x = 11}$.

(g) $\boldsymbol{x} - 3 = \boldsymbol{x} + 1$
$-3 = 1$
There are no solutions
in set of whole numbers.

(h) $0 + x = x + 0$
$\boldsymbol{x = x}$.
All whole numbers
are solutions.

26. Kelsey has $\boldsymbol{a - (b + c)}$ more marbles than Gena and Noah combined.

27. Yes. Both sets are closed with respect to addition. Each set represents the multiples of 5. A multiple of 5 plus a multiple of 5 is always a multiple of 5.

Assessment 3-2B

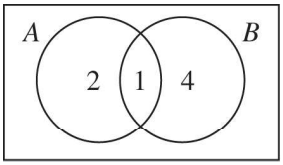
1. The student is adding 15 and 3.Start with fifteen and add one each time until eighteen then notice that three numbers were added.

2. **(a)** **True**.
$n(A) = 2; n(B) = 2; n(A) + n(B) = 4$.
$A \cup B = \{a, b, d, e\}; n(A \cup B) = 4$.
$\boldsymbol{n(A) + n(B) = n(A \cup B)}$ because the sets are disjoint.

(b) **False**.
$n(A) = 3; n(B) = 3; n(A) + n(B) = 6$.
$A \cup B = \{a, b, c, d\}; n(A \cup B) = 4$.
$\boldsymbol{n(A) + n(B) \neq n(A \cup B)}$ because the sets are not disjoint.

(c) **True**.
$n(A) = 1; n(B) = 0; n(A) + n(B) = 1$.
$A \cup B = \{a\}; n(A \cup B) = 1$.
$\boldsymbol{n(A) + n(B) = n(A \cup B)}$ because the sets are disjoint.

3. The sets are not disjoint because $n(A \cap B) \neq 0$. $n(A \cup B)$ must therefore equal $n(A) + n(B) - n(A \cap B) = 3 + 5 - 1 = \mathbf{7}$. In the following Venn diagram:



$n(A) = 3, n(B) = 5, n(A \cap B) = 1,$ and $\boldsymbol{n(A \cup B) = 7}$.

4. $A - B =$ all the elements in A that are not in B. Then if $n(A) - n(B)$ is defined as $n(A - B)$ there can be no elements in B that are also not in A. So, that would mean that $n(A - B) = 0$, or $A \cap B = B$.

5. **(a)** Since $n(A \cup B) = 6$ and $n(B) = 4$, A could contain 2 elements not in B or A could contain some or all of the elements of B. For example, if $B \subset A$, then A would have 6 elements. So there could be $\mathbf{2, 3, 4, 5, or\ 6}$ elements in A.

(b) If $n(A \cap B) = 0$, then $A \cap B = \emptyset$. So A would have only **2** elements.

6. **(a)** **Not Closed**. $1 + 1 = 2$, and $2 \notin \{0, 1\}$.

(b) **Closed**. Assuming the arithmetic sequence $0, 4, 8, \ldots$, any element of T added to any other element in T is whole and divisible by 4.

(c) **Closed.** Any element of E added to any other element of E is > 5 and is thus $\in E$.

(d) **Closed**. W is the set of whole numbers and any whole number greater than 100 added to any other whole number greater than 100 is $\in W$.

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7. **(a)** Answers may vary. For example, if $A = \{2, 5, 7, 8, 9, 10, 11, 12, 13, ...\}$, then possible sums of 2, 5, and 8 would be elements of the set.

(b) The set must contain **the rest of the whole numbers greater than 1** to be closed.

8. If set A is closed under addition and 2 and 4 are included then all other even whole numbers must be included for the set to be closed.

9. **(a)** $3 + 4 = \mathbf{4} + 3$; **commutative property of addition.**

(b) $5 + (4 + 3) = (4 + 3) + \mathbf{5}$; **commutative property of addition**.

(c) $8 + \mathbf{0} = 8$; **identity property of addition**.

(d) $3 + (4 + 5) = (3 + \mathbf{4}) + 5$; **associative property of addition**.

10. **(a)** **Commutative property of addition**; i.e., if a and b are any whole numbers, then $a + b = b + a$.

(b) **Identity property of whole numbers**; i.e., for any whole numbers $a + b, (a + b) + 0 = a + b$.

(c) **Commutative property of addition**; i.e., $(6 + 8) = (8 + 6)$.

(d) **Associative property of addition**; i.e., if $a, b,$ and c are any whole numbers, then $(a + b) + c = a + (b + c)$.

11. In this chapter we are only considering whole numbers a and b. So k must be a **whole** number.

12. $a \geq b$ if and only if $a - b$ is a whole number. Alternative: $a \geq b$ if and only if $a - k = b$ for some whole number k.

13. **(a)** Each term is found by adding 7 to the previous term. Thus the next three are $33 + 7 = \mathbf{40}$, $40 + 7 = \mathbf{47}$, $47 + 7 = \mathbf{54}$.

(b) Each term is found by subtracting 4 from the previous term. Thus the next three are $47 - 4 = \mathbf{43}$, $43 - 4 = \mathbf{39}$, $39 - 4 = \mathbf{35}$.

14. In a magic square each row, column, and diagonal must have the same sum. $6+1+8=15$ so each row, column, and diagonal must add the 12.

6	**7**	**2**
1	**5**	**9**
8	**3**	**4**

15. **(a)** **1**. Addition of any two different single-digit numbers will result in a sum no greater than 17.

(b) **No**. $C = 1$ and D must be different.

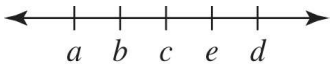
(c) **9 or 8**. If A is 9. B must be 8; if A is 8, B must be 9.

(d) **2**. A must be 9; B must be 3.

16. **(a)** The number 0 can be paired with all digits from 0 to 6 (7 pairs), the number 1 can additionally be paired with all digits from 1 to 6 (6 pairs), then 2 can be paired with all digits from 2 to 6, . . ., 6 is paired with 6. The total of unique pairings is thus $7 + 6 + 5 + 4 + 3 + 2 + 1 = \mathbf{28}$.

(b) If the domino is horizontal and the sum of the dots in the right square plus the sum of the dots in the left square is known, and the domino is then turned 180°, the sum is the same.

17. A strategy would be to construct a number line with the information.



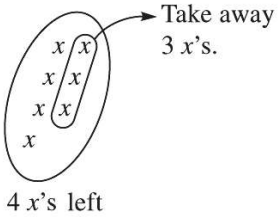
So, ***a, b, c, e, d*** are ordered from least to greatest.

18. **(a)** Add 3 to each side: $\mathbf{9 = x + 3}$.

(b) Add 5 to each side: $\boldsymbol{x = 8 + 5}$.

(c) Add x to each side: $\mathbf{11 = 2 + x}$.

19. **(a)** Take-away:



(b) Missing addend:

$\square + 3 = 7 \Rightarrow \square = 4$

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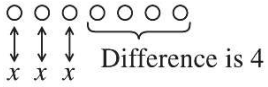
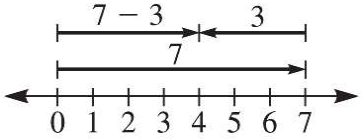
(c) Comparison:



(d) Number line:



20. **(a)** **Answers vary.** For example, "Kaden has 12 cookies and Tristan takes away 7 cookies. How many cookies does Kaden have left?"

(b) **Answers vary.** For example, "Jasmine has 7 yards of fabric and makes a blanket for her sister Ebony with 3 yards of **fabric. How much fabric is left over?"**

(c) **Answers vary.** For example, "Luke has seventeen marbles. His brother Zach has 12 marbles. How many more marbles does Luke have than Zach?"

(d) **Answers vary.** For example, "Isabelle has $25 and is saving for a video game called Minecraft. If the game costs $30, how much more does Isabelle need to save?"

21. Using the doubles plus strategy:

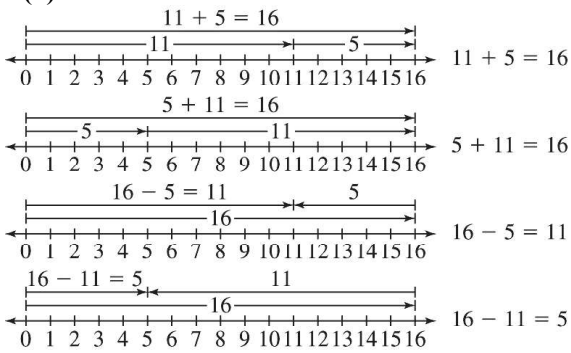
(a) $11+7=7+11$
$=7+(7+4)$
$=(7+7)+4$
$=14+4$
$=18$

(b) $24+28=24+(24+4)$
$=(24+24)+4$
$=48+4$
$=52$

(c) $35+37=35+(35+2)$
$=(35+35)+2$
$=70+2$
$=72$

22. **(a)** The other three members of the fact family are:
$5+11=16$
$16-11=5$
$16-5=11$

(b)



23. **(a)** For $b-a$ to be a whole number, $\boldsymbol{b \geq a}$.

(b) If we assert that the expression $a-3$ must be a whole number, then $\boldsymbol{a \geq 3}$. Then, b must be greater than or equal to $a-3$, $\boldsymbol{b \geq a-3}$.

24. **(a)** $a+(b+c)=(a+b)+c$: Associative property of addition.
$(a+b)+c=c+(a+b)$: Commutative property of addition.

(b) $a+(b+c)=a+(c+b)$: Commutative property of addition.
$a+(c+b)=(c+b)+a$: Commutative property of addition.

25. **(a)** $12-x=x+6$
$12-x+x=x+x+6$
$12=2x+6$
$12-6=2x+6-6$
$6=2x \Rightarrow \frac{6}{2}=\frac{2}{2}x \Rightarrow \boldsymbol{x=3}$.

(b) $9-x-6=1$
$3-x=1$
$3-x+x=1+x$
$3=1+x$
$3-1=1-1+x \Rightarrow \boldsymbol{x=2}$.

(c) $3+x=x+3$
$x+3=x+3 \Rightarrow$
All whole numbers.

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(d) $11 - x = 0$
$11 - x + x = 0 + x$
$11 = x \Rightarrow \boldsymbol{x = 11}$

(e) $14 - x = 7 - x$
$14 - x + x = 7 - x + x$
$14 = 7 \Rightarrow$ **No solution**.

(f) $x - 3 = 17$
$x - 3 + 3 = 17 + 3$
$\boldsymbol{x = 20}$

(g) $x + 3 = x - 1$
$x + 3 - x = x - 1 - x$
$3 = -1 \Rightarrow$ **No solution**.

(h) $0 + x = x - 0$
$x = x \Rightarrow$ **All whole numbers**.

26. The number of Kelly's pencils must be subtracted from the number of Rob's pencils, or ***(iii)***, $11 - 5 = 6$.

27. Yes. Both sets are closed with respect to addition. Each set represents the multiples of 7. A multiple of 7 plus a multiple of 7 is always a multiple of 7.

Mathematical Connections 3-2: Review Problems

25. In base 10, $5280 = \mathbf{5 \cdot 10^3 + 2 \cdot 10^2 + 8 \cdot 10^1 + 0 \cdot 10^0}$.

26. **1410**. $\boldsymbol{M} = 1000, \boldsymbol{CD} = 500 - 100$, and $\boldsymbol{X} = 10$.

27. **(a)** $E0T_{twelve} = 11 \cdot 12^2 + 0 \cdot 12^1 + 10 \cdot 12^0 = \mathbf{1594}$.

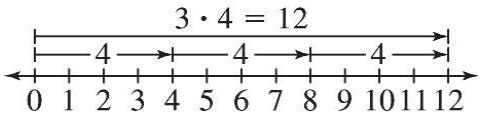
(b) $1011_{two} = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = \mathbf{11}$.

(c) $43_{five} = 4 \cdot 5^1 + 3 \cdot 5^0 = \mathbf{23}$.

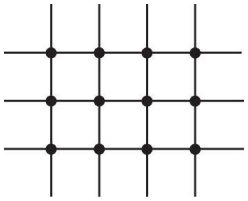
28. $12^4 + 12^2 + 13 = 1 \cdot 12^4 + 0 \cdot 12^3 + 1 \cdot 12^2 + 1 \cdot 12 + 1 = \mathbf{10111_{twelve}}$.

Assessment 3-3A: Multiplication and Division of Whole Numbers

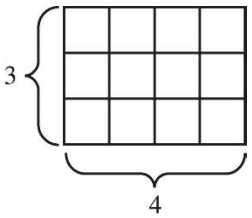
1. **(a)**



(b)



(c)



(d) $A = \{a, b, c\}$
$B = \{w, x, y, z\}$
$A \times B = \{(a,w),(a,x),(a,y),(a,z),$
$(b,w),(b,x),(b,y),(b,z),$
$(c,w),(c,x),(c,y),(c,z)\}$
$n(A) = 3, n(B) = 4$
$n(A \times B) = 3 \bullet 4 = 12.$

2. **(a)** $A = \{a, b\}$
$B = \{x, y, z\}$
$A \times B = \{(a,x),(a,y),(a,z),$
$(b,x),(b,y),(b,z)\}$

(b) $n(A) = 2, n(B) = 3$
$n(A \times B) = 2 \bullet 3 = 6$

(c) $2 \bullet 3 = 6$ **.**

3. **(a)** Illustrated is 4 groups of 2 x*s*: $\mathbf{4 \cdot 2} = 8$.

(b) Illustrated is a 4 by 2 array or a 2 by 4 array: $\mathbf{2 \cdot 4 = 8}$ **or** $\mathbf{4 \cdot 2 = 8}$.

4. **(a)** Use the repeated addition model: $3 + 3 + 3 + 3 + 3 = 15$; there are five threes, so $3 \cdot \boxed{5} = 15$.

(b) $18 - 6 = 6 - 6 + 3 \cdot \square$ (note the order of operations specifying that $6 + 3$ is not permitted) $\Rightarrow 12 = 3 \cdot \square$. Now use the

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repeated addition model: $3+3+3+3=12$; there are four threes, so $18 = 6 + 3 \cdot \boxed{4}$.

(c) The distributive property of multiplication over addition, where n is **any whole number,** specifies that $a(b + c) = ab + ac$. Thus $\boxed{n} \cdot (5 + 6) = \boxed{n} \cdot \mathbf{5} + \boxed{n} \cdot \mathbf{6}$.

5. (a) **Closed**. $0 \cdot 0 = 0, 0 \cdot 1 = 0, 1 \cdot 0 = 0,$ and $1 \cdot 1 = 1$ are all products contained in $\{0,1\}$.

(b) **Closed**. The product of any two even numbers is also an even number.

(c) **Closed**. Since the set can be written as $\{3n+1 \mid n \in W\}$, for any whole numbers m and $n, (3m+1)(3n+1) = 3(3mn+m+n)+1$ i s in the set since the whole numbers are closed under multiplication and addition.

6. (a) **No**. $2 + 3 = 5$.

(b) **Yes**. There will be no numbers in the set that will multiply to give a product of 5.

7. (a) Applying the distributive property gives $(a + b)(c + d) = a(c + d) + b(c + d) = \boldsymbol{ac + ad + bc + bd}$.

(b) $\square(\Delta + \mathrm{O}) = \square \cdot \boldsymbol{\Delta} + \square \cdot \mathbf{O}$.

(c) $a(b + c) - ac = \boldsymbol{ab + ac - ac} = ab$.

8. (a) $\mathbf{(5 + 6) \cdot 3} = 33$. Without parentheses, the result would be 23.

(b) **No parentheses** are needed; the order of operations specifies that addition and subtraction are performed in order from left to right.

(c) **No parentheses** are needed; the order of operations specifies that division is performed before addition or subtraction.

(d) $\mathbf{(9 + 6) \div 3} = 5$. Without parentheses, the result would be 11.

9. (a) $xy + y^2 = \boldsymbol{y(x + y)}$, factoring y from each term.

(b) $xy + x = \boldsymbol{x(y + 1)}$, factoring x from each term.

(c) $a^2b + ab^2 = \boldsymbol{ab(a + b)}$, factoring ab from each term.

10. (a) $18 \div 3 = \boxed{6}$ (since $3 \cdot 6 = 18$).

(b) $\boxed{0} \div 76 = 0$ (since $0 \cdot 76 = 0$).

(c) $28 \div \boxed{4} = 7$ (since $7 \cdot 4 = 28$).

11. Use the Cartesian-product model, where s is shirts, p is pants, and v is vests. Then $n(s) = 6,\ n(p) = 4,$ and $n(v) = 3$. The Fundamental Counting Principle indicates that the number of ordered triplets in $s \times p \times v = n(s) \cdot n(p) \cdot n(v) = 6 \cdot 4 \cdot 3 = \mathbf{72}$ **possible outfits**.

12. (a) Associative property of multiplication of whole numbers; i.e., $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

(b) Commutative property of multiplication of whole numbers; i.e., $a \cdot b = b \cdot a$.

(c) Commutative property of multiplication.

(d) Identity property of multiplication of whole numbers; i.e., $1 \cdot a = a$.

(e) Zero multiplication property of whole numbers; i.e., $a \cdot 0 = 0$.

(f) Distributive property of multiplication over addition for whole numbers; i.e., $(a + b)(c + d) = (a + b) \cdot c + (a + b) \cdot d$.

13. (a) Closure property of multiplication of whole numbers; i.e., for any whole numbers a and b, $a \cdot b$ is a *unique* whole number.

(b) Zero multiplication property of whole numbers; i.e., $a \cdot 0 = 0$.

(c) Identity property of multiplication of whole numbers; i.e., $a \cdot 1 = a$.

14. (a) Distributive property of multiplication over addition for whole numbers; i.e., $a(b + c) = ab + ac$.

(b) ***(i)*** $12 \cdot 32 = 12(30 + 2) = 12 \cdot 30 + 12 \cdot 2 = 360 + 24 = \mathbf{384}$, or

(ii) $32 \cdot 12 = 32(10 + 2) = 32 \cdot 10 + 32 \cdot 2 = 320 + 64 = \mathbf{384}$.

15. (a) $9(10 - 2) = 9 \cdot 10 - 9 \cdot 2 = 90 - 18 = \mathbf{72}$.

(b) $20(8 - 3) = 20 \cdot 8 - 20 \cdot 3 = 160 - 60 = \mathbf{100}$.

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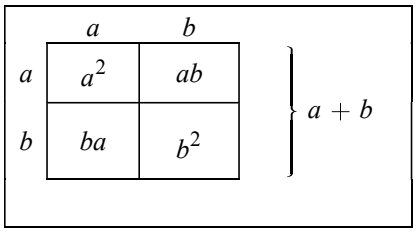
16. **(a)** $(a + b)^2 = (a + b)(a + b) = a(a + b) + b(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$.

(b)



The area of a square with sides $a + b$ can be expressed as $(a + b) \cdot (a + b)$, and also as the sum of areas of the four regions: two squares, $a \cdot a$ and $b \cdot b$, and two rectangles $a \cdot b$ and $b \cdot a$. Thus:

$$(a + b)^2 = a \cdot a + a \cdot b + b \cdot a + b \cdot b$$
$$= a^2 + 2ab + b^2.$$

17. The question is really to show that the area of the large square minus the area of the small square is the same as the area of the four rectangles.

The area of the large square is $(a + b)^2$. The area of the small square is $(a - b)^2$. The difference between the two areas is the four rectangles, each with area ab; the total area of the four is $4ab$.

Therefore $(a + b)^2 - (a - b)^2 = 4ab$.

18. **(a)** $51^2 = (50 + 1)(50 + 1)$
$= 50^2 + 2 \cdot 50 \cdot 1 + 1^2$
$= 2500 + 100 + 1 = 2601$

(b) $102^2 = (100 + 2)(100 + 2)$
$= 100^2 + 2 \cdot 100 \cdot 2 + +2^2$
$= 10{,}000 + 400 + 4 = 10{,}404$

19. **(a)** ***(i)*** $(ab)c = c(ab)$ by the commutative property of multiplication of whole numbers.

(ii) $c(ab) = (ca)b$ by the associative property of multiplication of whole numbers.

(b) ***(i)*** $(a + b)c = c(a + b)$ by the commutative property of multiplication.

(ii) $c(a + b) = c(b + a)$ by the commutative property of addition.

20. To factor is to reverse the process of the distributive property of multiplication over addition or subtraction. The result is to find the factors which, when multiplied together, will yield a given product.

(a) $xy - y^2 = \boldsymbol{y(x - y)}$; i.e., if y and $x - y$ were to be multiplied, using the distributive property of multiplication over subtraction, the product would be $xy - y^2$.

(b) $47 \cdot 101 - 47 = \mathbf{47(101 - 1)}$. i.e., 47 is a factor of both $47 \cdot 101$ and $47 \cdot 1$.

(c) $ab^2 - ba^2 = \boldsymbol{ab(b - a)}$.

21. **(a)** $40 \div 8 = 5 \Rightarrow \mathbf{40 = 8 \cdot 5}$.

(b) $326 \div 2 = x \Rightarrow \mathbf{326 = 2 \cdot x}$.

22. If $108 / a = b$ then $108 = a \cdot b$ and $108 / b = a$.

23. The complete fact family:
$72 / 8 = 9$
$72 / 9 = 8$
$8 \cdot 9 = 72$
$9 \cdot 8 = 72$

24. The process always results in the original numbers.
Step 1: "Think of a number." Name this number x.
Step 2: $5x$
Step 3: $5x + 5$
Step 4: $\frac{5x+5}{5} = x + 1$
Step 5: $x + 1 - 1 = x$.

25. Answers may vary, but for example:

(a) There is no associative property in division; e.g., $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$.

(b) There is no distributive property in division; e.g., $8 \div (2 + 2) \neq (8 \div 2) + (8 \div 2)$.

26. Suppose there are two bags of marbles containing a marbles in one and b marbles in the other. It is desired to divide the marbles equally among c boys. Then the number of marbles that each boy receives can be found in two different ways:
Put all the marbles in one bag. There will be $a + b$ marbles and each boy will receive $(a + b) \div c$ marbles.

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Divide the marbles in the first bag first and then the second. Each boy would receive $(a \div c) + (b \div c)$ marbles.

Let $a \div c = x$ and $b \div c = y$.

Then $a = cx$ and $b = cy$.

So $a + b = cx + cy = c(x + y)$.

By the definition of division:

$$(x + y) = (a + b) \div c.$$

Substituting for x and y:

$$(a \div c) + (b \div c) = (a + b) \div c.$$

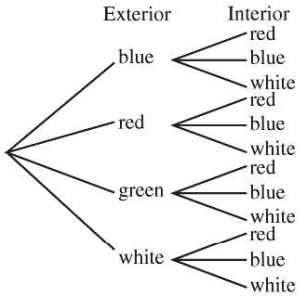
27. **(a)** $5x + 2 = 22 \Rightarrow 5x + 2 - 2 = 22 - 2 \Rightarrow 5x = 20 \Rightarrow \frac{5}{5}x = \frac{20}{5} \Rightarrow \boldsymbol{x = 4}$.

(b) $3x + 7 = x + 13 \Rightarrow 3x - x + 7 = x - x + 13 \Rightarrow 2x + 7 = 13 \Rightarrow 2x + 7 - 7 = 13 - 7 \Rightarrow 2x = 6 \Rightarrow \frac{2}{2}x = \frac{6}{2} \Rightarrow \boldsymbol{x = 3}$.

(c) $3(x + 4) = 18 \Rightarrow 3x + 12 = 18 \Rightarrow 3x + 12 - 12 = 18 - 12 \Rightarrow 3x = 6 \Rightarrow \frac{3}{3}x = \frac{6}{3} \Rightarrow \boldsymbol{x = 2}$.

(d) $(x - 5) \div 10 = 9 \Rightarrow \left(\frac{x-5}{10}\right)10 = 9(10) \Rightarrow x - 5 = 90 \Rightarrow x - 5 + 5 = 90 + 5 \Rightarrow \boldsymbol{x = 95}$.

28. Answers may vary; one possibility is:



Resulting in $4 \cdot 3 = \mathbf{12}$ **color schemes**.

29. **Yes**. The natural numbers must be $14, 24, 34, \ldots, 94$ to leave a remainder of 4 when divided by 10. Then $\mathbf{64} \div 47 = 1$ with a remainder of 17.

30. There were 10 teams with 12 on each team. So, there were $10(12) = 120$ people. Divide them into teams of 8 people and there are $120 \div 8 = \mathbf{15}$ **teams**.

31. **(a)** **Subtract 18 from 45**. Order of operations specifies operations within parenthese first, then multiplication before addition or subtraction.

(b) **Divide 54 by 9**. Order of operations specifies operations within parentheses first.

(c) **Add 11 to 48**. Operations within parentheses first.

(d) **Add 8 to 61**. Multiplication or division before addition.

32. These numbers will be multiples of 4 plus 1, $\{4n + 1 | n \in W\}$ or $\{1, 5, 9, 13, \ldots\}$.

33. **(a)** **Yes**, since only elements at $\{a, b, c\}$ appear in the table.

(b) **Yes**, $a \odot c = c \odot a; a \odot b = b \odot a$; and $b \odot c = c \odot b$.

(c) **Yes**, the first row and first column tell us that $a \odot b$, for example, is b.

(d) Answers vary. For example $(a \odot b) \odot c = b \odot c = a$ and $a \odot (b \odot c) = a \odot a = a$. All combinations would have to be checked to prove the operation is associative.

34. If $a \# b = a + b + 4$ then, for example, $3 \# 2 = 3 + 2 + 4 = 9$ and $2 \# 3 = 2 + 3 + 4 = 9$, so it is commutative.

In general $a \# b = a + b + 4 = b + a + 4 = b \# a$.

35. **(a)** The number of people who have entered after the 25th ring of the bell is the sum of the first 25 terms. It is an arithmetic sequence 1, 3, 5, … with the n^{th} term equal to $2n$-1. So the 25th term is $2(25) - 1 = 49$. And the sum is $1 + 3 + 5 + \ldots + 49 = 625$. So 625 people entered.

(b) After the n^{th} ring $\frac{(1 + (2n - 1)) \bullet n}{2} = \frac{(2n) \bullet n}{2} = \frac{2n^2}{2} = n^2$ people have entered.

(c) After $n^2 = 1000 \Rightarrow n = \sqrt{1000} \Rightarrow n \approx 32$. After 32 rings there will be at least 1000 people.

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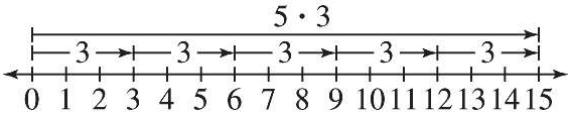
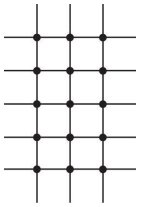
Assessment 3-3B

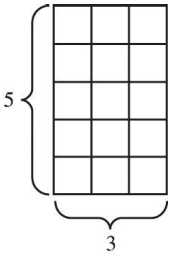
1. (a)



(b)



(c)



(d) $A = \{a, b, c, d, e\}$
$B = \{x, y, z\}$
$A \times B = \{(a,x),(a,y),(a,z),$
$(b,x),(b,y),(b,z),$
$(c,x),(c,y),(c,z),$
$(d,x),(d,y),(d,z),$
$(e,x),(e,y),(e,z)\}$
$n(A) = 5, n(B) = 3$
$n(A \times B) = 5 \cdot 3 = 15.$

2. (a) $A = \{a, b, c, d\}$
$B = \{x, y, z\}$
$A \times B = \{(a,x),(a,y),(a,z),$
$(b,x),(b,y),(b,z),$
$(c,x),(c,y),(c,z),$
$(d,x),(d,y),(d,z)\}$

(b) $n(A) = 4, n(B) = 3$
$n(A \times B) = 4 \cdot 3 = 12$

(c) $4 \cdot 3 = 12$ **.**

3. (a) Illustrated is 4 groups of 3 *xs*: $4 \cdot 3 = 12$.

(b) Illustrated is a 3 by 7 array or a 7 by 3 array: $3 \cdot 7 = 21$ *or* $7 \cdot 3 = 21$.

4. (a) Use the repeated addition model: $8 + 8 + 8 = 24$; there are three eights, so $8 \cdot \boxed{3} = 24$.

(b) $28 - 4 = 4 - 4 + 6 \cdot \square$ (note the order of operations indicates that multiplication is performed before addition, so that $4 + 6$ is not permitted) $\Rightarrow 24 = 6 \cdot \square$. Now use the repeated addition model: $6 + 6 + 6 + 6 = 24$; there are four sixes, so $28 = 4 + 6 \cdot \boxed{4}$.

(c) The distributive property of multiplication over addition, where n is any whole number, specifies that $a(b + c) = ab + ac$. Thus $\boxed{n} \cdot (8 + 6) = \boxed{n} \cdot 8 + \boxed{n} \cdot 6$.

5. (a) **No**, $2 \cdot 2 = 4$ which is not in $\{1, 2\}$.

(b) **Yes**, the expression $2k + 1, k \in W$, represents any odd number.

Let $2\ell + 1$ be another odd number, then $(2k + 1)(2\ell + 1) = 4k\ell + 2k + 2\ell + 1 = 2(2k\ell + k + \ell) + 1$ which is an odd number.

(c) **Yes**, a power of 2 multiplied by a power of 2 is still a power of 2.

6. (a) **No**, $1 + 1 = 2$.

(b) **Yes**, the product of two whole numbers greater than 1 is greater than one and still a whole number.

7. (a) $3(x + y + 5) = 3x + 3y + 3 \cdot 5 =$ $\mathbf{3x + 3y + 15}$.

(b) $(x + y)(x + y + z) = x(x + y + z) + y(x + y + z) = xx + xy + xz + yx + yy + yz = \mathbf{x^2 + 2xy + xz + y^2 + yz}$. (Note: $xy = yx$.)

(c) $x(y + 1) - x = \mathbf{xy + x - x} = xy$.

8. (a) $\mathbf{(4 + 3) \cdot 2 = 14}$. Without parentheses the result would be $4 + 6 = 10$.

(b) $\mathbf{9 \div 3 + 1 = 4}$. Parentheses are unnecessary.

(c) $\mathbf{(5 + 4 + 9) \div 3 = 6}$. Without parentheses the result would be $5 + 4 + 3 = 12$.

(d) $\mathbf{3 + 6 - 2 \div 1 = 7}$. Parentheses are unnecessary.

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9. **(a)** $47 \cdot 99 + 47 = 47 \cdot 99 + 47 \cdot 1 =$ $\mathbf{47 \cdot (99 + 1)}$

(b) $(x + 1)y + (x + 1) = (x + 1)y + (x + 1)1$ $= \mathbf{(x + 1)(y + 1)}$

(c) $x^2y + zx^3 = \mathbf{x^2(y + zx)}$

10. **(a)** $27 \div 9 = \boxed{3}$ (since $3 \cdot 9 = 27$).

(b) $\boxed{52} \div 52 = 1$ (since $1 \cdot 52 = 52$).

(c) $13 \div \boxed{1} = 13$ (since $13 \cdot 1 = 13$).

11. 5 exterior colors $\times 3$ interior colors $\Rightarrow 5 \cdot 3 =$ **15** possibilities.

12. **(a)** Zero multiplication property of whole numbers; i.e., $a \cdot 0 = 0$.

(b) Commutative property of multiplication of whole numbers; i.e., $a \cdot b = b \cdot a$.

(c) Commutative property of multiplication; i.e., $a(bc) = (bc)a$.

(d) Identity property of multiplication of whole numbers; i.e., $a \cdot 1 = a$.

(e) Distributive property of multiplication over addition for whole numbers; i.e. $(a + b)c = ac + bc$.

(f) Distributive property of multiplication over addition for whole numbers; i.e., $(a + b)(c + d) = (a + b) \cdot c + (a + b) \cdot d$.

13. **(a)** Commutative property of multiplication of whole numbers; i.e., $a \cdot b = b \cdot a$.

(b) Distributive property of multiplication over addition for whole numbers; i.e., $9 \cdot 6 = 9 \cdot 5 + 9 \cdot 1 = 45 + 9 = 54$.

14. **(a)** Distributive property of multiplication over addition for whole numbers; i.e., $a(b + c) = ab + ac$.

(b) $8 \cdot 34 = 8(30 + 4) = 8 \cdot 30 + 8 \cdot 4 = 240 + 32 =$ **272**.

15. **(a)** $15(10 - 2) = 15 \cdot 10 - 15 \cdot 2$ $= 150 - 30 =$ **120**.

(b) $30(9 - 2) = 30 \cdot 9 - 30 \cdot 2$ $= 270 - 60 =$ **210**.

16. The area of the shaded region is $a(b - c)$, but it can also be expressed as the area of the large rectangle minus the area of the small rectangle $\Rightarrow a(b - c) = ab - ac$.

17. The first two figures on the left illustrates a^2, viewed as the area of the large square, minus b^2, viewed as the area of the small square on the bottom right. The figure on the right illustrates how the pieces can be rearranged to form a rectangle with height $a + b$ and length $a - b$.

18. **(a)** $19 \cdot 21 = 21 \cdot 19 = (20 + 1)(20 - 1)$ $= 20^2 - 1 = 400 - 1 =$ **399**.

(b) $25 \cdot 15 = (20 + 5)(20 - 5) = 20^2 - 5^2$ $= 400 - 25 =$ **375**.

(c) $99 \cdot 101 = 101 \cdot 99 = (100 + 1)(100 - 1)$ $= 100^2 - 1^2 = 10000 - 1$ $=$ **9999**.

(d) $101^2 - 99^2 = (101 + 99)(101 - 99) =$ $(200)(2) =$ **400**.

19. **(a)** ***(i)*** $(ab)c = (ba)c$ by the commutative property of multiplication of whole numbers.

(ii) $(ba)c = b(ac)$ by the associative property of multiplication of whole numbers.

(b) ***(i)*** $a(b + c) = ab + ac$ by the distributive property of multiplication over addition.

(ii) $ab + ac = ac + ab$ by the commutative property of addition.

20. To factor is to reverse the process of the distributive property of multiplication over addition or subtraction. The result is to find the factors which, when multiplied together, will yield a given product.

(a) $xy - y = \mathbf{y(x - 1)}$. Note that $xy - y$ can be written as $xy - 1 \cdot y$.

(b) $(x + 1)y - (x + 1) = \mathbf{(x + 1)(y - 1)}$. $(x + 1)$ is a factor of both $(x + 1)y$ and $(x + 1) \cdot 1$.

(c) $a^2b^3 - ab^2 = \mathbf{ab^2(ab - 1)}$. a and b^2 are factors of each term.

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21. **(a)** $48 \div x = 16 \Rightarrow \mathbf{48 = x \cdot 16}$.
(b) $x \div 5 = 17 \Rightarrow \mathbf{x = 5 \cdot 17}$.

22. If $64 / a = b$ then $64 = a \bullet b$ and $64 / b = a$.

23. The complete fact family:
$30 / 6 = 5$
$30 / 5 = 6$
$5 \bullet 6 = 30$
$6 \bullet 5 = 30$

24. **(a)** **The original number is returned**.
(b) **Yes**. Let n be the original number.
Then $\frac{2n+2}{2} - 1 = \frac{2(n+1)}{2} - 1 =$
$n + 1 - 1 = n$.
So n is returned, regardless of the value of n.

25. Answers may vary, but for example:
(a) There is no commutative property in division; e.g., $(8 \div 4) \neq (4 \div 8)$.
(b) There is no commutative property in subtraction; e.g., $8 - 4 \neq 4 - 8$.

26. Write $(a - b) \div c = d$, where d is a whole number. This can be written as a multiplication problem $a - b = cd$. Similarly, $b \div c$ can be written as a multiplication problem $b = ck$. Combining this, we have $a - b = cd \Rightarrow a = cd + b \Rightarrow a = cd + ck \Rightarrow a = c(d + k)$
$\Rightarrow a \div c = d + k$.
Recall that $b \div c = k$. So,
$a \div c = d + k \Rightarrow a \div c - k = d$
$\Rightarrow a \div c - b \div c = d = (a - b) \div c$.

27. **(a)** $5x + 8 = 28 \Rightarrow 5x + 8 - 8 = 28 - 8 \Rightarrow$
$5x = 20 \Rightarrow \frac{5}{5}x = \frac{20}{5} \Rightarrow \mathbf{x = 4}$.
(b) $5x + 6 = x + 14 \Rightarrow 5x - x + 6 = x - x +$
$14 \Rightarrow 4x + 6 = 14 \Rightarrow 4x + 6 - 6 = 14 - 6$
$\Rightarrow 4x = 8 \Rightarrow \frac{4}{4}x = \frac{8}{4} \Rightarrow \mathbf{x = 2.}$
(c) $5(x + 3) = 35 \Rightarrow 5x + 15 = 35 \Rightarrow$
$5x + 15 - 15 = 35 - 15 \Rightarrow 5x = 20$
$\Rightarrow \frac{5}{5}x = \frac{20}{5} \Rightarrow \mathbf{x = 4}$.
(d) $(x - 6) \div 3 = 1 \Rightarrow x - 6 = 3 \Rightarrow \mathbf{x = 9}$.

28. Look for a pattern:
2 nails on each axis $\rightarrow$ 1 intersection.
3 nails on each axis $\rightarrow$ 3 intersections $\rightarrow$ 2 new.
4 nails on each axis $\rightarrow$ 6 intersections $\rightarrow$ 3 new.
5 nails on each axis $\rightarrow$ 10 intersections $\rightarrow$ 4 new.
$\vdots$
6 nails $\rightarrow$ 15 intersections.
7 nails $\rightarrow$ 21 intersections.
8 nails $\rightarrow$ 28 intersections.
9 nails $\rightarrow$ 36 intersections.
10 nails $\rightarrow$ **45 intersections**.

29. If Jonah adds the borrowed 5 marbles to his last row, he will have a row of 13 marbles. Then $13 - 5 = \mathbf{8}$, **the remainder** when his original number is divided by 13.

30. 8 teams $\times$ 9 players per team $=$ 72 players $\Rightarrow$ 72 players $\div$ 6 players per team $=$ **12 teams**.

31. **(a)** **Add 2** to $30 - 12$. Order of operations specifies multiplication before addition or subtraction.
(b) **Add 7 to 3**. Order of operations specifies multiplication and division before addition or subtraction.
(c) **Subtract 12 from** 15. $6 \div 2 \cdot 4 = 12$.
(d) **Add 18 to 5**. Operations within parentheses first, then multiplication.

32. Given any whole numbers a and $b (b \neq 0)$, there exist unique whole numbers q (quotient) and r (remainder) such that $a = bq + r \Rightarrow 5n + 3 \in W$. Those numbers are **3,8,13,**….

33. **(a)** **Yes**. $x \odot y \in S$ for any values of $x \in S$ and $y \in S$.
(b) **Yes**. $a \odot b = b \odot a, a \odot c = c \odot a, \ldots$ for all possible combinations of terms.
(c) **Yes**. Since $x \odot a = x$ and $a \odot x = x$ for all x in S, a is the identity on S.
(d) **Yes**. $a \odot (b \odot c) = (a \odot b) \odot c$ for all possible combinations of terms.

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34. Yes, if both a and b are odd then 3#5=0=5#3, if one is odd then 3#2=0=2#3, and if both are even then 2#4=1=4#2.

35. In rows of 5 one member remained so it is a number of rows multiplied by.5 plus 1: 6, 11, 16, 21, 26, 31, … In rows of 6 one member remained so it is the number of rows multiplied by 6 plus 1: 7, 13, 19, 25, … In rows of 7 no member remained so it is the number of rows multiplied by 7: 7, 14, 21, 28, 35, … The least number of members is then 91.

Mathematical Connections 3-3: Review Problems

19. Answers may vary; an example would be $\{2, 6, 10, 14, \ldots\}$.

20. **No**. For example, $5 - 2 \neq 2 - 5$.

21. **No**. The number "two" exists in base two as 10_{two} but there is no single symbol representing "two."

22. $\mathbf{10{,}000}_{three} = 3^4 = 81$. Any power of 3 in base ten can be written as a 1 followed by an appropriate number of zeros in base three.

Assessment 3-4A: Addition and Subtraction Algorithms, Mental Computation, and Estimation

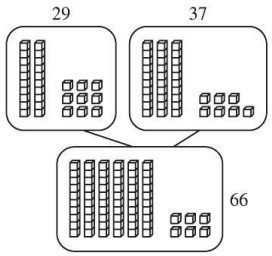
1. **(a)** In the units column, $1 + \underline{1} = 2$.
In the tens column, $\underline{8} + 2 = 10$ (regroup).
In the hundreds column, $1 + \underline{9} + 4 = 14$.

$$\begin{array}{r} \underline{\mathbf{9}}\ \underline{\mathbf{8}}\ 1 \\ +\ 4\ 2\ \underline{1} \\ \hline \underline{\mathbf{1}}\ 4\ 0\ 2 \end{array}$$

(b) In the units column, $5 + 6 + 8 = 19$ (regroup).
In the tens column, $1 + 2 + \underline{9} + 4 = 16$ (regroup).
In the hundreds column, $1 + 0 + 1 + 1 = \underline{3}$.
In the thousands column, $\underline{2} + 1 + 3 = 6$.

$$\begin{array}{r} \underline{\mathbf{2}}\ 0\ 2\ 5 \\ 1\ 1\ \underline{\mathbf{9}}\ 6 \\ +\ 3\ 1\ 4\ 8 \\ \hline 6\ \underline{\mathbf{3}}\ 6\ \underline{\mathbf{9}} \end{array}$$

2. Base ten blocks:



3. **(a)** Answers may vary: for the unique greatest sum, the larger numbers must be in the hundreds column:

$$\begin{array}{r} \boxed{7}\ \boxed{6}\ \boxed{2} \\ +\ \boxed{8}\ \boxed{5}\ \boxed{3} \\ \hline 1\ 6\ 1\ 5 \end{array}$$

(b) Answers may vary; for the unique least sum the smaller numbers must be in the hundreds column:

$$\begin{array}{r} \boxed{2}\ \boxed{6}\ \boxed{7} \\ +\ \boxed{3}\ \boxed{5}\ \boxed{8} \\ \hline 6\ 2\ 5 \end{array}$$

4. The ones digit must be a sum of 9, so the options are 3-6, 4-5, or 8-1. There is no way that a regrouping will take place, so the tens column must sum to 5 or 15. But 15 is not an option with the given digits, so it must sum to 5; the only options are 4-1 and 2-3. Since 15 was not an option, there is no regrouping into the hundreds column, meaning the sum must be 10. The only option for the hundreds column is 4-6. Placing these values in the correct boxes eliminates the options for the ones and tens columns, so the ones column must be 8-1 and the tens column must be 2-3. This leaves only 3-5 for the thousands column.

$$\begin{array}{r} \boxed{3}\ \boxed{4}\ \boxed{2}\ \boxed{8} \\ +\ \boxed{5}\ \boxed{6}\ \boxed{3}\ \boxed{1} \\ \hline 9\ 0\ 5\ 9 \end{array}$$

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5. **(a)**
$$\begin{array}{r} 93 \\ -\ 37 \end{array} \Rightarrow \begin{array}{r} 93+3 \\ -\ 37+3 \end{array} \Rightarrow \begin{array}{r} 96 \\ -\ 40 \\ \hline 56 \end{array}$$

(b)
$$\begin{array}{r} 321 \\ -\ 38 \end{array} \Rightarrow \begin{array}{r} 321+2 \\ -\ 38+2 \end{array} \Rightarrow \begin{array}{r} 323 \\ -\ 40 \end{array}$$

$$\Rightarrow \begin{array}{r} 323+60 \\ -\ 40+60 \end{array} \Rightarrow \begin{array}{r} 383 \\ -\ 100 \\ \hline 283 \end{array}$$

6. **(a)** ***(i)***
$$\begin{array}{r} 687 \\ +\ 549 \\ \hline 16 \\ 12 \\ 11 \\ \hline 1236 \end{array}$$

(ii)
$$\begin{array}{r} 359 \\ +\ 673 \\ \hline 12 \\ 12 \\ 9 \\ \hline 1032 \end{array}$$

(b) The algorithm works because the placement of partial sums under their addends accounts for place value.

7. Answers may vary; some possibilities are:

(a) $8+5=13$ and 13 was written down with no regrouping. $2+7=9$ and 9 was simply placed in front of the 13.

(b) $8+5=13$, but instead of writing 3 and regrouping with the 1, the 1 was written and the 3 was regrouped.

(c) Only the difference in the units $(9-5=4)$, tens $(5-0=5)$, and the hundreds $(3-2=1)$ was recorded, without taking into account the signs of the numbers.

(d) Three hundreds was regrouped as 2 hundreds and 10 tens, but 10 tens was not regrouped as $9 \cdot 10 + 15$ in order to obtain $15-9=6$ in the ones place.

8. By dinner time Tom had consumed $90+120+119+185+110+570=1194$ calories. Subtracting 1194 from 1500 gives $1500-1194=306$. Tom may have fish or salad, but **not both**. (He may have tea with either.)

9. Step 1 → Expanded form;
Step 2 → Commutative and associative properties of addition;
Step 3 → Distributive property of multiplication over addition;
Step 4 → Closure property of addition
Step 5 → Expanded form condensed.

10. **(a)**
$$\begin{aligned} 66+23 &= (6\cdot 10+6)+(2\cdot 10+3) \\ &= (6\cdot 10+2\cdot 10)+(6+3) \\ &= (6+2)\cdot 10+(6+3) \\ &= 8\cdot 10+9 \\ &= 89. \end{aligned}$$

(b)
$$\begin{aligned} 124+235 &= (1\cdot 100+2\cdot 10+4) \\ &\quad +(2\cdot 100+3\cdot 10+5) \\ &= (1\cdot 100+2\cdot 100) \\ &\quad +(2\cdot 10+3\cdot 10)+(4+5) \\ &= (1+2)\cdot 100+(2+3)\cdot 10 \\ &\quad +(4+5) \\ &= 3\cdot 100+5\cdot 10+9 \\ &= \mathbf{359}. \end{aligned}$$

11. **(a)**
$$\begin{array}{r} 4358 \\ +\ 3864 \\ \hline \boxed{{}^{0}\!/_{7}}\ \boxed{{}^{1}\!/_{1}}\ \boxed{{}^{1}\!/_{1}}\ \boxed{{}^{1}\!/_{2}} \\ 8222 \end{array}$$

(b)
$$\begin{array}{r} 4923 \\ +\ 9897 \\ \hline \boxed{{}^{1}\!/_{3}}\ \boxed{{}^{1}\!/_{7}}\ \boxed{{}^{1}\!/_{1}}\ \boxed{{}^{1}\!/_{0}} \\ 14820 \end{array}$$

12. Remember that in base five the number 11, for example, means $1\cdot 5+1$, corresponding to 6 in base ten.

(a)
$$\begin{array}{r} {\scriptstyle 1\ \ \ \ } \\ 43_{\textit{five}} \\ +\ 23_{\textit{five}} \\ \hline 121_{\textit{five}} \end{array}$$

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Alternative approach: An example of a different algorithm for checking our answers would be to write in expanded form.

$$\begin{aligned} 43_{five} + 23_{five} &= (4 \cdot 5 + 3 \cdot 5^0) \\ &\quad + (2 \cdot 5^1 + 3 \cdot 5^0) \\ &= 6 \cdot 5^1 + 6 \cdot 5^0 \\ &= (5+1)5^1 + (5+1)5^0 \\ &= 5^2 + 5 + 5 + 1 \\ &= 1 \cdot 5^2 + 2 \cdot 5 + 1 \cdot 5^0 \\ &= (121)_{five} \end{aligned}$$

(b)
$$\begin{array}{rrr} & 4 & 3 \quad five \\ - & 2 & 3 \quad five \\ \hline & 2 & 0 \quad five \end{array}$$

(c)
$$\begin{array}{rrrr} & \overset{1}{4} & \overset{1}{3} & 2 \quad five \\ + & & 2 & 3 \quad five \\ \hline 1 & 0 & 1 & 0 \quad five \end{array}$$

(d)
$$\begin{array}{rrr} & \overset{3}{\cancel{4}} & \overset{12}{\cancel{2}} \quad five \\ - & 2 & 3 \quad five \\ \hline & 1 & 4 \quad five \end{array}$$

(e)
$$\begin{array}{rrrr} & \overset{1}{1} & 1 & 0 \quad two \\ + & & 1 & 1 \quad two \\ \hline 1 & 0 & 0 & 1 \quad two \end{array}$$

(f)
$$\begin{array}{rrrrrr} & & \overset{1}{\cancel{10}} & \overset{1}{\cancel{10}} & 10 & \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & 1 & two \\ - & & & 1 & 1 & 1 \quad two \\ \hline & & 1 & 0 & 1 & 0 \quad two \end{array}$$

13. For an example of how to use the table, move down the rows in the $+$ column to 3 and then across that row to the column headed by 6. This will give the sum of $3 + 6$, or 11_{eight}.

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

(a)
$$\begin{array}{rrrr} & \overset{4}{\cancel{5}} & \overset{16}{\cancel{7}} & \overset{13}{\cancel{3}} \quad eight \\ - & & 7 & 7 \quad eight \\ \hline & 4 & 7 & 4 \quad eight. \end{array}$$

We used the table to complete the subtraction. For example $13_{eight} - 7_{eight}$ can be found by observing that $4_{eight} + 7_{eight} = 13_{eight}$ in the table.

(b)
$$\begin{array}{rrrr} & \overset{6}{\cancel{7}} & \overset{15}{\cancel{6}} & \overset{15}{\cancel{5}} \quad eight \\ - & & 7 & 6 \quad eight \\ \hline & 6 & 6 & 7 \quad eight \end{array}$$

14. **(a)**
$$\begin{array}{rrrrrrr} & \overset{1}{3} & \text{hours} & \overset{1}{36} & \text{minutes} & 58 & \text{seconds} \\ + & 5 & \text{hours} & 56 & \text{minutes} & 27 & \text{seconds} \\ \hline & 9 & \text{hours} & 33 & \text{minutes} & 25 & \text{seconds} \end{array}$$

(b)
$$\begin{array}{rrrrrrr} & \overset{4}{\cancel{5}} & \text{hours} & \overset{95}{\overset{35}{\cancel{36}}} & \text{minutes} & \overset{98}{\cancel{38}} & \text{seconds} \\ - & 3 & \text{hours} & 56 & \text{minutes} & 58 & \text{seconds} \\ \hline & 1 & \text{hour} & 39 & \text{minutes} & 40 & \text{seconds} \end{array}$$

15. **(a)** The sum of each row, column, and diagonal is **34**. For example, the sum of the entries in row 1 is $1 + 15 + 14 + 4 = \mathbf{34}$.

(b) $6 + 7 + 10 + 11 = \mathbf{34}$.

(c) $1 + 4 + 16 + 13 = \mathbf{34}$.

(d) **Yes**. Adding five to each number in the square will increase the sum of any four numbers in the square by 20.

(e) **Yes**. Subtracting 1 from each number in the square will decrease the sum of any four numbers in the square by 4.

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16. **(a)** Using the scratch algorithm, whenever a sum is 10 or more, scratch a line through the last digit added and write the number of units next to the scratched digit; count the number of scratches in each column and add to the column to the left.

$$\begin{array}{rcccc} & {\scriptstyle 1} & {\scriptstyle 1} & & \\ & 4 & 3 & 2 & \\ & \not{9}_{4} & \not{7}_{1} & 6 & \\ + & {}^{1}1 & 4 & 1 & \not{8}_{6} \\ \hline & 2 & 8 & 2 & 6 \end{array}$$

(b) A base five addition table might be helpful:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

$$\begin{array}{rcccc} & & {\scriptstyle 3} & & \\ 3 & & \not{3}_{1} & 2 & \textit{five} \\ & & 1 & \not{3}_{0} & \textit{five} \\ & & 2 & 2 & \textit{five} \\ & & \not{4}_{3} & \not{3}_{0} & \textit{five} \\ & & \not{2}_{0} & 3 & \textit{five} \\ & + & 1 & \not{2}_{0} & \textit{five} \\ \hline & & 3 & 1 & 0 \quad \textit{five} \end{array}$$

17. There is **no numeral 5** in base five; $2_{five} + 3_{five} = 10_{five}$; $22_{five} + 33_{five} = 110_{five}$.

18. **(a)** Subtract 2132_{five} from 3423_{five}.

$$\begin{array}{rcccc} & 3 & 4 & 2 & 3 \quad \textit{five} \\ - & 2 & 1 & 3 & 2 \quad \textit{five} \\ \hline & \mathbf{1} & \mathbf{2} & \mathbf{4} & \mathbf{1} \quad \textit{five} \end{array}$$

(b) Subtract 11011_{two} from $100,000_{two}$.

$$\begin{array}{rcccccc} & 1 & 0 & 0 & 0 & 0 & 0 \quad \textit{two} \\ - & & 1 & 1 & 0 & 1 & 1 \quad \textit{two} \\ \hline & & & \mathbf{1} & \mathbf{0} & 1 & \textit{two} \end{array}$$

(c) Subtract 1 from TEE_{twelve}:

$$\begin{array}{rccc} & T & E & E \quad \textit{twelve} \\ - & & & 1 \quad \textit{twelve} \\ \hline & \boldsymbol{T} & \boldsymbol{E} & \boldsymbol{T} \quad \textit{twelve} \end{array}$$

(d) Subtract 1000_{five} from $10,000_{five}$.

$$\begin{array}{rccccc} & 1 & 0 & 0 & 0 & 0 \quad \textit{five} \\ - & & 1 & 0 & 0 & 0 \quad \textit{five} \\ \hline & & \mathbf{4} & \mathbf{0} & \mathbf{0} & \mathbf{0} \quad \textit{five} \end{array}$$

19. The information in (a) and (b) complete the first column.

Teams	1
Hawks	14
Elks	$18 = 14 + 4$

The information in (c) and (d) complete the second column.

Teams	1	2
Hawks	14	$22 = 23 - 1$
Elks	18	$23 = 18 + 5$

The information in (g) and (h) complete the third column.

Teams	1	2	3
Hawks	14	31	$36 = 2(18)$
Elks	18	23	$45 = 14 + 31$

The information in (f) tells us that $14 + 22 + 36 + (4^{th}\text{ quarter score}) = 120$.

So the 4^{th} quarter score for the Hawks is 39. The information in (g) tells us that the Elks scored $48 + 6$ points in the 4^{th} quarter.

	Quarter				
Teams	1	2	3	4	Final
Hawks	14	31	36	39	120
Elks	18	23	45	45	131

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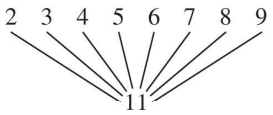
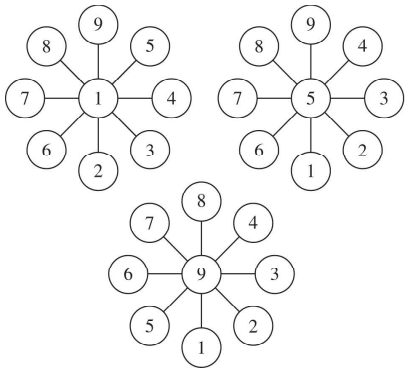
20. **(a)** If 1 is placed in the middle then we can pair the numbers 2 through 9 to sum to eleven so that the sum in each of the four directions is twelve.



Similar arrangements can be made for 5 and 9 in the middle.



(b) The three **{1,5,9}** are the only numbers that can be placed in the middle. To observe that 2, for example, can't be placed in the middle, pair 9 with each of the remaining numbers and observe that there is no way to pair the other numbers to form a common sum.

21. **(a)** $93 + 39 = 132; 132 + 231 = 363,$ which is a palindrome.

(b) $588 + 885 = 1473; 1473 + 3741 = 5214;$ $5214 + 4125 = 9339,$ which is a palindrome.

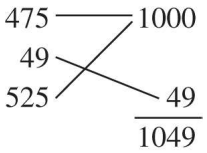
(c) $2003 + 3002 = 5005,$ which is a palindrome.

22. Answers may vary; possibilities include:

(a) $180 + 97 - 23 + 20 - 140 + 26$
$= 180 + 97 + 20 + 26 - 23 - 140$
$= (180 + 90 + 20 + 20 + 7 + 6)$
$\quad - 23 - 140$
$= (310 + 13) - 23 - 140$
$= 323 - 23 - 140 = 300 - 140 = \mathbf{160}.$

(b) $87 - 42 + 70 - 38 + 43$
$= 87 + 70 + 43 - 42 - 38$
$= (80 + 70 + 40 + 7 + 3)$
$\quad - 40 - 30 - 2 - 8$
$= (190 + 10) - 70 - 10$
$= 200 - 80 = \mathbf{120}.$

23. **(a)** Answers vary; possibilities include:
Compatible numbers



(b) Answers vary; possibilities include:
Breaking up and bridging.
$375 - 76 = 375 - 75 - 1$
$= 300 - 1 = 299.$

(c) Answers vary; possibilities include:
Compatible numbers
$230 + 60 + 70 + 44 + 6 =$
$(230 + 70) + (60 + 40) + (4 + 6) =$
$300 + 100 + 10 =$
410

24. **(a)** $567 + 38$:

(i) $567 + 30 = 597$ (add 567 to the tens value of 38).

(ii) $597 + 8 = \mathbf{605}$ (add 597 to the units value of 38).

(b) $418 + 215$:

(i) $418 + 200 = 618$ (add 418 to the hundreds value of 215).

(ii) $618 + 10 = 628$ (add 618 to the tens value of 215).

(iii) $628 + 5 = 633$ (add 628 to the units value of 215).

25. **(a)** $85 - 49 \Rightarrow (85 + 1) - (49 + 1) =$
$86 - 50 = \mathbf{36}.$

(b) $87 + 33 \Rightarrow (87 + 3) + (33 - 3) =$
$90 + 30 = \mathbf{120}.$

(c) $143 - 97 = (143 + 3) - (97 + 3) =$
$146 - 100 = \mathbf{46}.$

(d) $58 + 39 \Rightarrow (58 + 2) + (39 - 2) =$
$60 + 37 = \mathbf{97}.$

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26. **(a)** $28 + 2 = 30; 30 + 20 = 50; 50 + 3 = 53.$
Then $2 + 20 + 3 =$ **25**.

(b) $47 + 3 = 50; 50 + 10 = 60; 60 + 3 = 63.$
Then $3 + 10 + 3 =$ **16**.

27. **(a)** $52\underline{8}0$: The number is between 5200 and 5300;
The midpoint is 5250;
The number is greater than the midpoint;
So it rounds up to **5300**.

(b) $\underline{1}15,234$: The number is between 100,000 and 200,000;
The midpoint is 150,000;
The number is less than the midpoint;
So it rounds down to **100,000**.

(c) $11\underline{5},234$: The number is between 110,000 and 120,000;
The midpoint is 115,000;
The number is greater than the midpoint;
So it rounds up to **120,000**.

(d) $232\underline{5}$: The number is between 2320 and 2330;
The midpoint is 2325;
When the number is at the midpoint it is conventional to round up (to **2330**).

28. Answers may vary:

(a) $878 + 2340 \approx 900 + 2300 = 3200$

(b) $25,201 - 19,987 \approx 25,000 - 20,000 =$ **5000**.

(c) $2215 + 3023 + 5967 + 975 \approx 2000 + 3000 + 6000 + 1000 =$ **12,000**.

29. **(a)** $2215 + 3023 + 5987 + 975$:

(i) $2 + 3 + 5 + 0 = 10$ (add front-end digits);

(ii) $10,000$ (place value);

(iii) $215 + 23 + 987 + 975 \approx 200 + 0 + 1000 + 1000 = 2200$ (adjust);

(iv) $10,000 + 2200 =$ **12,200** (adjusted estimate).

(b) $234 + 478 + 987 + 319 + 469$:

(i) $2 + 4 + 9 + 3 + 4 = 22$ (add front-end digits)

(ii) 2200 (place value);

(iii) $34 + 78 + 87 + 19 + 69 \approx 30 + 80 + 90 + 20 + 70 = 290$ (adjust);

(iv) $2200 + 290 =$ **2490** (adjusted estimate).

30. **(a)** ***(i)*** **No**. The numbers are not clustered.

(ii) **Yes**. The numbers are clustered around 500.

(b) Estimate may vary:

(i) Front-end: $0 + 1 + 0 + 2 = 3$; place value 3000; adjust by $500 + 500 + 100 + 400 = 1500$; estimate $3000 + 1500 =$ **4500**.

Grouping to nice numbers: $474 + 1467 \approx 2000$; $64 + 2445 \approx 2500$; sum $\approx 2000 + 2500 =$ **4500**.

Rounding: Sum $\approx 500 + 1500 + 100 + 2400 =$ **4500**.

(ii) Front-end: $4 + 4 + 5 + 5 + 5 = 23$; place value 2300; adjust by $80 + 80 + 30 + 0 + 30 = 220$; estimate $2300 + 220 =$ **2520**.

Grouping to nice numbers: $483 + 475 \approx 1000$; $530 + 528 \approx 1000$; $503 \approx 500$; sum $\approx 1000 + 1000 + 500 =$ **2500**.

Rounding: sum $\approx 500 + 500 + 500 + 500 + 500 =$ **2500**.

31. **(a)** The range is $100 + 600 = 700$ to $200 + 700 = 900$. Then $700 < (145 + 678) < 900$.

(b) The range is $200 + 0 = 200$ to $300 + 100 = 400$. Then $200 < (278 + 36) < 400$.

32. Answers may vary; e.g., $3300 - 100 - 300 - 400 - 500 =$ **2000**. The estimate is **high** because the amounts were rounded to the hundreds place. \$8 was taken away from the check amounts while \$13 was added to the \$3287.

33. **(a)** **False**. Sweden would be estimated to be about 44,000 square miles larger than Finland.

(b) **False**. Twice Norway's size would be about 250,000 square miles.

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(c) **False**. France would be estimated to be about 85,000 square miles larger than Norway.

(d) **True**. About 195,000 square miles to about 174,000 square miles.

34. The clustering strategy yields $6 \cdot (\text{about } 70{,}000) \approx$ **420,000** in attendance.

Assessment 3-4B

1. **(a)** In the units column, $1\underline{3} - 9 = 4$ (trade from the tens column).

In the tens column, $7 - 5 = 2$, (1 has been traded from $\underline{8}$).

In the hundreds column, $3 - 1 = \underline{2}$.

$$\begin{array}{r} 3\ \ \mathbf{\underline{8}}\ \ \mathbf{\underline{3}} \\ -\ 1\ \ 5\ \ 9 \\ \hline \mathbf{\underline{2}}\ \ 2\ \ 4 \end{array}$$

(b) In the units column, 10 has been traded from the tens column.

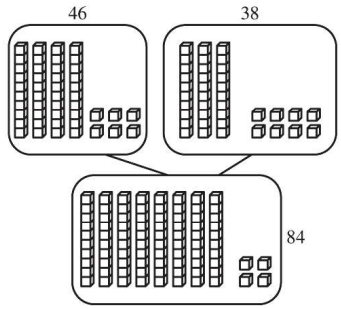
In the tens column, $8 - 0 = 8$ but 1 has been traded from $\underline{9}$.

In the hundreds column, $12 - 3 = 9$ (trade 1000 from the thousands column).

In the thousands column, $12 - 8 = 4$, (1 has been traded from $\underline{3}$).

$$\begin{array}{r} 1\ \ \mathbf{\underline{3}}\ \ \mathbf{\underline{2}}\ \ \mathbf{\underline{9}}\ \ 6 \\ -\ \ \ 8\ \ 3\ \ 0\ \ 9 \\ \hline \ \ \ 4\ \ 9\ \ 8\ \ 7 \end{array}$$

2. Base ten blocks:



3. If whole numbers are used:

(a)
$$\begin{array}{r} \boxed{8}\ \boxed{7}\ \boxed{6} \\ -\ \boxed{2}\ \boxed{3}\ \boxed{5} \\ \hline 6\ \ 4\ \ 1 \end{array}$$

(b)
$$\begin{array}{r} \boxed{6}\ \boxed{2}\ \boxed{3} \\ -\ \boxed{5}\ \boxed{8}\ \boxed{7} \\ \hline \ \ \ 3\ \ 6 \end{array}$$

4. 4 and 5 can be used in the units column to sum to 9. 8 and 4 can then be used in the tens column; they sum to 12 with the 1 regrouped to the hundreds column. The two 3's can be used in the hundreds column to sum, with the regrouped 1, to 7. The only remaining numbers, to be placed in the thousands column, are 5 and 6.

$$\begin{array}{r} 5\ \ 3\ \ 8\ \ 4 \\ +\ 6\ \ 3\ \ 4\ \ 5 \\ \hline 11\ \ 7\ \ 2\ \ 9 \end{array}$$

5. **(a)**
$$\begin{array}{rcrcr} 86 & \Rightarrow & 86 + 2 & \Rightarrow & 88 \\ -\ 38 & & -\ 38 + 2 & & -\ 40 \\ \hline & & & & \mathbf{48} \end{array}$$

(b)
$$\begin{array}{rcrcr} 582 & \Rightarrow & 582 + 6 & \Rightarrow & 588 \\ -\ \ 44 & & -\ \ 44 + 6 & & -\ \ 50 \\ \hline & & & & \mathbf{538} \end{array}$$

6. **(a)**
$$\begin{array}{r} 9\ \ 8\ \ 7 \\ +\ 3\ \ 5\ \ 6 \\ \hline \ \ \ \ \ 1\ \ 3 \\ \ \ \ 1\ \ 3 \\ 1\ \ 2 \\ \hline 1\ \ 3\ \ 4\ \ 3 \end{array}$$

(b)
$$\begin{array}{r} 4\ \ 1\ \ 5 \\ +\ 7\ \ 9 \\ \hline \ \ \ 1\ \ 4 \\ 4\ \ 8 \\ \hline 4\ \ 9\ \ 4 \end{array}$$

7. Answers may vary, for example:

(a) A tens digit was not regrouped as 1 ten when the sum of the units digits was more than 9.

(b) Partial sums are not in the correct place value position.

(c) The units minuend is subtracted from the units subtrahend.

(d) One tens value should have been traded from the 5 in the minuend's ten position.

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8. 75 minutes + 18 minutes + 45 seconds + 30 seconds = 93 minutes + 75 seconds = 93 minutes + 1 minute + 15 seconds = 94 minutes + 15 seconds = 1 hour + 34 minutes + 15 seconds. George's meal required **1 hour 34 minutes 15 seconds cooking time**. He must start at 2 : 25 : 45 pm.

9. Step 1 → Expanded form;
Step 2 → Commutative and associative properties of addition;
Step 3 → Distributive property of multiplication over addition;
Step 4 → Closure property of addition, one digit addition facts;
Step 5 → Expanded form condensed.

10. **(a)**
$$\begin{aligned} 46 + 32 &= (4 \cdot 10 + 6) + (3 \cdot 10 + 2) \\ &= (4 \cdot 10 + 3 \cdot 10) + (6 + 2) \\ &= (4 + 3) \cdot 10 + (6 + 2) \\ &= 7 \cdot 10 + 8 \\ &= \mathbf{78} \end{aligned}$$

(b)
$$\begin{aligned} 3214 + 783 &= (3 \cdot 10^3 + 2 \cdot 10^2 + 1 \cdot 10 + 4) \\ &\quad + (7 \cdot 10^2 + 8 \cdot 10 + 3) \\ &= (3 \cdot 10^3) + (2 + 7) \cdot 10^2 \\ &\quad + (1 + 8) \cdot 10 + (4 + 3) \\ &= 3 \cdot 10^3 + 9 \cdot 10^2 + 9 \cdot 10 + 7 \\ &= \mathbf{3997} \end{aligned}$$

11. **(a)**

	2	3	4	5	
+	8	8	8	8	
	$\boxed{1/0}$	$\boxed{1/1}$	$\boxed{1/2}$	$\boxed{1/3}$	
1	1	2	3	3	

(b)

	8	7	1	3	
+	4	2	1	4	
	$\boxed{1/2}$	$\boxed{0/9}$	$\boxed{0/2}$	$\boxed{0/7}$	
1	2	9	2	7	

12. Remember that in base five the number 11, for example, means $1 \cdot 5 + 1$, whereas in base ten it means $1 \cdot 10 + 1$.

(a)

	3	13	
	~~4~~	~~3~~	*five*
−	2	4	*five*
	1	4	*five*

(b)

	1	1		
	1	4	3	*five*
+		2	3	*five*
	2	2	1	*five*

(c)

	2	12	
	~~3~~	~~2~~	*five*
−	2	3	*five*
		4	*five*

(d)

	1	1		
	2	3	2	*five*
+		4	3	*five*
	3	3	0	*five*

(e)

1	1			
	1	1	0	*two*
+	1	1	1	*two*
1	1	0	1	*two*

(f)

	1				
	~~1~~	~~0~~	10		
~~1~~	~~0~~	~~0~~	0	1	*two*
−		1	0	1	*two*
	1	1	0	0	*two*

13. For an example of use of the table, moving down the rows in the + column to 3 and then across the row to the column headed by 4 will give the sum of $3 + 4$, or 11_{six}.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

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$$\begin{array}{rrrr} & {\scriptstyle 1} & {\scriptstyle 12} & {\scriptstyle 1} & \\ & \not{2} & \not{3} & \not{1} & six \\ - & 1 & 4 & 4 & six \\ \hline & & 4 & 3 & six \end{array}$$

Check:

$$\begin{array}{rrrr} & {\scriptstyle 1} & {\scriptstyle 1} & & \\ & 1 & 4 & 4 & six \\ + & & 4 & 3 & six \\ \hline & 2 & 3 & 1 & six \end{array}$$

$$\begin{array}{rrrr} & {\scriptstyle 2} & {\scriptstyle 13} & {\scriptstyle 12} & \\ & \not{3} & \not{4} & \not{2} & six \\ - & 1 & 4 & 4 & six \\ \hline & & 1 & 5 & 4 & six \end{array}$$

Check:

$$\begin{array}{rrrr} & {\scriptstyle 1} & {\scriptstyle 1} & & \\ & 1 & 5 & 4 & six \\ + & 1 & 4 & 4 & six \\ \hline & 3 & 4 & 2 & six \end{array}$$

14. (a) 2 cups $=$ 1 pint and 2 pints $=$ 1 quart. This is essentially operating in base two:

$$\begin{array}{rrrrrrr} & {\scriptstyle 1} & & & {\scriptstyle 1} & & \\ & 1 & \text{quart} & 1 & \text{pint} & 1 & \text{cup} \\ + & & & 1 & \text{pint} & 1 & \text{cup} \\ \hline & 2 & \text{quarts} & 1 & \text{pint} & 0 & \text{cup} \end{array}$$

(b) Again in base two; 2 pints must be traded from 1 quart in order to subtract 1 pint:

$$\begin{array}{rrrrrr} & & & {\scriptstyle 2} & & \\ & \not{1} & \text{quart} & 0 & \text{pint} & 1 & \text{cup} \\ - & & & 1 & \text{pint} & 1 & \text{cup} \\ \hline & & & 1 & \text{pint} & 0 & \text{cup} \end{array}$$

(c) 4 cups $=$ 1 quart and 4 quarts $=$ 1 gallon. This is essentially operating in base four:

$$\begin{array}{rrrrrrr} & & & {\scriptstyle 6} & & & \\ & {\scriptstyle 0} & & {\scriptstyle 2} & & {\scriptstyle 5} & \\ & \not{1} & \text{gallon} & \not{3} & \text{quarts} & \not{1} & \text{cup} \\ - & & & 4 & \text{quarts} & 2 & \text{cups} \\ \hline & & & 2 & \text{quarts} & 3 & \text{cups} \end{array}$$

(or 2 quarts, 1 pint, 1 cup)

15. (a) Each row, column, and diagonal sums to **34**.

(b) The sum is **34**.

(c) The sum is **34**.

(d) **Yes**. All rows, columns, and diagonals still contain four numbers; adding 11 to each adds 44 to all sums and they are still equal (to 78).

(e) **Yes**. This subtracts 44 from all sums and they are still equal (to $^{-}10$).

16. (a) Use the scratch algorithm. Whenever a sum is 10 or more, scratch a line through the last digit added and write the number of units next to the scratched digit; count the number of scratches in each column and add to the column to the left.

$$\begin{array}{rrrrr} & & {\scriptstyle 1} & {\scriptstyle 2} & \\ & 5 & 3 & 7 & \\ & 3 & 1 & \not{8}_5 & \\ + & {}^{1}2 & \not{3}_2 & \not{4}_0 & \not{5}_0 \\ \hline & 3 & 2 & 0 & 0 \end{array}$$

(b) A base six addition table might be helpful (see problem 13):

$$\begin{array}{rrrr} & {}^{2}\not{4}_0 & 1 & six \\ & 3 & 2 & six \\ & 2 & 2 & six \\ & \not{4}_3 & \not{3}_2 & six \\ & 2 & 2 & six \\ + & \not{5}_4 & \not{4}_2 & six \\ \hline 3 & 4 & 2 & six \end{array}$$

17. There is **no numeral 6 in base six**. $23_{six} + 43_{six} = 110_{six}$.

18. (a) Subtract 213_{five} from 342_{five}.

$$\begin{array}{rrrr} & 3 & 4 & 2 & five \\ - & 2 & 1 & 3 & five \\ \hline & \boldsymbol{1} & \boldsymbol{2} & \boldsymbol{4} & \boldsymbol{five} \end{array}$$

(b) Subtract 1011_{two} from 1101_{two}.

$$\begin{array}{rrrrr} & 1 & 1 & 0 & 1 & two \\ - & 1 & 0 & 1 & 1 & two \\ \hline & & & \mathbf{1} & \mathbf{0} & two \end{array}$$

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(c) Subtract 9 from $E08_{twelve}$:

$$\begin{array}{rcccc} & E & 0 & 8 & _{twelve} \\ - & & & 9 & _{twelve} \\ \hline & \boldsymbol{T} & \boldsymbol{E} & \boldsymbol{E} & \boldsymbol{_{twelve}} \end{array}$$

(d) Subtract 100_{two} from $10,000_{two}$.

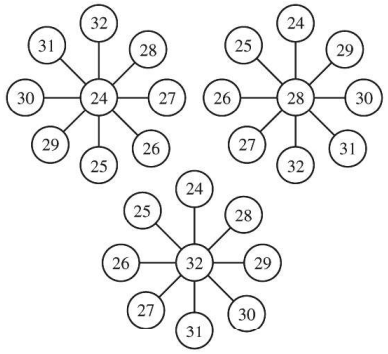
$$\begin{array}{rccccc} & 1 & 0 & 0 & 0 & _{two} \\ - & & & 1 & 0 & 0 & _{two} \\ \hline & \boldsymbol{1} & \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{_{two}} \end{array}$$

19.
- The Hawks scored 15 points in the first quarter, so 15 goes in that block.
- The Hawks were behind by 5 points at the end of the first quarter, so 20 goes to the Elks in the first.
- The Elks scored 5 more points in the second quarter than in the first, so 25 goes in that block.
- The Hawks scored 7 more points than the Elks in the second quarter, so 32 goes to the Hawks in the second.
- The Hawks scored twice as many points in the third quarter as the Elks did in the first, so 40 goes in that block.
- The Hawks scored 120 points in the game, so $120 - (15 + 32 + 40) = 33$ goes to the Hawks in the fourth quarter.
- The Elks outscored the Hawks by 6 points in the fourth quarter, so 39 goes in that block.
- The Elks scored as many points in the third quarter as the Hawks did in the first two; $15 + 32 = 47$ goes in that block.
- The Elks scored a total of $20 + 25 + 47 + 39 = 131$ in the game. Thus:

TEAMS	QUARTERS				FINAL SCORE
	1	2	3	4	
Hawks	**15**	**32**	**40**	**33**	**120**
Elks	**20**	**25**	**47**	**39**	**131**

20. Answers may vary

(a)



(b) **Three**. Let the sum of all three terms in a line be s. There are four lines in the completed shape, so adding them all is $4s$. This is the same, however, as adding all the terms together (i.e., $24, 25, \ldots, 32$) with one number used four times. Call the number in the middle $\boldsymbol{x}$. So $24 + 25 + \cdots + 32 + 3x = 4s$, where x is already used once in the sum of all the numbers 24 to 32. This reduces to $\frac{9 \cdot 56}{2} = 252$, making the equation $252 + 3x = 4s$. Solving for x yields: $3x = 4s - 252 = 4(s - 63)$. Thus x must be a multiple of 4, the only options for which are 24, 28, and 32.

21. The greatest addend possible to make 87 + 78 and answer with a 2 in the hundreds place would be 99. The largest 2-digit number is 99. Add 87 + 78 + 99 = 264 which is a number with a 2 in the hundreds place.

22. Answers may vary; possibilities include:

(a) $160 + 92 - 32 + 40 - 18$
$= 160 + 92 + 40 - 32 - 18$
$= (160 + 90 + 40 + 2) - 32 - 18$
$= (290 + 2) - 32 - 18$
$= 292 - 32 - 18 = 292 - 50 = \mathbf{242}$.

(b) $36 + 97 - 80 + 44$
$= 36 + 97 + 44 - 80$
$= (30 + 90 + 40 + 6 + 7 + 4) - 80$
$= (160 + 17) - 80$
$= 177 - 80 = \mathbf{97}$.

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23. Step 1: **Associative** property of addition.
Step 2: **Commutative** property of addition.
Step 3: **Associative** property of addition.
Step 4: **Closure property of whole-number addition**.

24. **(a)** $997 - 32$:
(i) $997 - 30 = 967$ (subtract the tens value of 32 from 997).
(ii) $967 - 2 =$ **965** (subtract the units value of 32 from 967).
(b) $560 + 136$:
(i) $560 + 100 = 660$ (add 560 to the hundreds value of 136).
(ii) $660 + 30 = 690$ (add 660 to the tens value of 136).
(iii) $690 + 6 = 696$ (add 690 to the untis value of 136).

25. **(a)** $75 - 38 \Rightarrow (75 + 2) - (38 + 2) = 77 - 40 =$ **37**.
(b) $57 + 35 \Rightarrow (57 + 3) + (35 - 3) = 60 + 32 =$ **92**.
(c) $137 - 29 \Rightarrow (137 + 1) - (29 + 1) = 138 - 30 =$ **108**.
(d) $78 + 49 \Rightarrow (78 + 2) + (49 - 2) = 80 + 47 =$ **127**.

26. **(a)** $63 + 7 = 70$; $70 + 4 = 74$. Then $7 + 4 =$ **11**.
(b) $57 + 3 = 60$; $60 + 10 = 70$; $70 + 3 = 73$. Then $3 + 10 + 3 =$ **16**.

27. **(a)** $3\underline{5}87$: The number is between 3500 and 3600;
The midpoint is 3550;
The number is greater than the midpoint;
So it rounds up to **3600**.
(b) $\underline{1}48{,}213$: The number is between 100,000 and 200,000;
The midpoint is 150,000;
The number is less than the midpoint;
So it rounds down to **100,000**.
(c) $2\underline{3}{,}785$: The number is between 23,000 and 24,000;
The midpoint is 23,500;
The number is greater than the midpoint;
So it rounds up to **24,000**.
(d) $23\underline{5}7$: The number is between 2350 and 2360;
The midpoint is 2355;
The number is greater than the midpoint;
So it rounds up to **2360**.

28. Estimates may vary.
(a) $937 + 28 \approx 940 + 30 = 970$
(b) $32{,}285 - 18{,}988 \approx 32{,}000 - 19{,}000 =$ **13,000**.
(c) $3215 + 3789 + 5987 \approx 3000 + 4000 + 6000 =$ **13,000**.

29. Estimates may vary.
(a) $2345 + 5250 + 4210 + 910$:
(i) $2 + 5 + 4 + 0 = 11$ (add front-end digits);
(ii) $11{,}000$ (place value);
(iii) $345 + 250 + 210 + 910 \approx 350 + 250 + 200 + 900 = 1700$ (adjust);
(iv) $11{,}000 + 1700 =$ **12,700** (adjusted estimate).
(b) $345 + 518 + 655 + 270$:
(i) $3 + 5 + 6 + 2 = 16$ (add front-end digits);
(ii) 1600 (place value);
(iii) $50 + 20 + 60 + 70 = 200$ (adjust);
(iv) $1600 + 200 = 1800$ (adjusted estimate).

30. **(a)** ***(i)*** **No**. The numbers are not clustered.
(ii) **Yes**. The numbers are clustered around 2000.
(b) Estimates may vary:
(i) Front-end: $0 + 2 + 0 + 3 = 5$; place value 5000; adjust by $300 + 300 + 100 + 500 = 1200$; estimate $5000 + 1200 =$ **6200**.

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Grouping to nice numbers:
$318 + 2314 \approx 2600$; $57 + 3489 \approx 3600$;
sum $\approx 2600 + 3600 =$ **6200**.

Rounding: sum $\approx 300 + 2300 + 100 + 3500 =$ **6200**.

(ii) Front-end:
$2 + 1 + 2 + 2 + 1 = 8$; place value 8000; adjust by $400 + 1000 + 0 + 100 + 900 = 2400$; estimate $8000 + 2400 =$ **10,400**.

Grouping to nice numbers:
$2350 + 1987 \approx 4000$; $2036 + 2103 \approx 4000$; $1890 \approx 2000$; sum $\approx 4000 + 4000 + 2000 =$ **10,000**.

Rounding: sum $\approx 2000 + 2000 + 2000 + 2000 + 2000 =$ **10,000**.

31. Estimates may vary:

(a) The range is $100 + 700 = 800$ to $200 + 800 = 1000$. Then $800 < (123 + 780) < 1000$.

(b) The range is $400 + 200 = 600$ to $500 + 300 = 800$. Then $600 < (482 + 246) < 800$.

32. Answers vary. For example

$\$65 + \$190 + \$45 + \$212 + \$420 \approx$
$\$70 + \$200 + \$50 + \$200 + \$400 = \$920$
$\$1237 \approx \$1200$
$\$1200 - \$920 \approx \$1200 - \$900 = \$300$

The remaining balance is about \$300.

33. **(a)** **Yes**. Rounding, Josh plans to write checks for about \$40, \$30, \$60, and \$250, or about \$380. Since the check amounts were rounded up, he will have enough.

(b) **Yes**. (assuming a non-negative beginning balance). $\$981 + \$1140$ is greater than $\$900 + \$1100 = \$2000$.

(c) **Alberto**. He received 10 more votes than Juan from the first district, but only 1 less from the second.

34. Answers may vary. Possibilities include:

Rounding	
	71,000
	65,000
	68,000
	73,000
	85,000
+	70,000
	432,000

Trading off

Observe that the high attendance on Friday distributed among the other day's attendance for approximately 72,000 visitors per day.
$72{,}000 \times 6 =$ **432,000**.

Mathematical Connections 3-4: Review Problems

27. **1410**. $\boldsymbol{M} = 1000, \boldsymbol{CD} = 500 - 100$, and $\boldsymbol{X} = 10$.

28. **(a)** $E0T_{twelve} = 11 \cdot 12^2 + 0 \cdot 12^1 + 10 \cdot 12^0 =$ **1594**.

(b) $1011_{two} = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 =$ **11**.

(c) $43_{five} = 4 \cdot 5^1 + 3 \cdot 5^0 =$ **23**.

29. **No**. For example, $2 + 3 = 5$, which is not an element of the set $\{1, 2, 3\}$.

30. For example, $(2 + 3) + 4 = 2 + (3 + 4)$.

31. **(a)** $20 - x = x$
$\Rightarrow 20 - x + x = x + x$
$\Rightarrow 20 = 2x$
$\Rightarrow 10 = x \Rightarrow \boldsymbol{x = 10}$.

(b) $20 - x - 6 = 0$
$\Rightarrow 20 - 6 = x$
$\Rightarrow 14 = x$
$\Rightarrow \boldsymbol{x = 14}$.

(c) $x + 4 = 3 + x + 1$
$\Rightarrow x + 4 = x + 4$
$\Rightarrow x = x$.
All whole numbers **solve the equation.**

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Assessment 3-5A: Multiplication and Division Algorithms, Mental Computation, and Estimation

1. (a) There are $7 + 12 = 19$ factors of 5.
Thus $5^7 \cdot 5^{12} = 5^{7+12} = \mathbf{5^{19}}$.

(b) $6^{10} \cdot 6^2 \cdot 6^3 = 6^{10+2+3} = \mathbf{6^{15}}$.

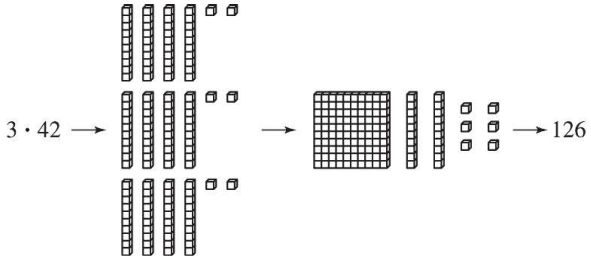
(c) $10^{296} \cdot 10^{17} = 10^{296+17} = \mathbf{10^{313}}$.

(d) $2^7 \cdot 10^5 \cdot 5^7 = 2^7 \cdot 5^7 \cdot 10^5$
$= (2 \cdot 5)^7 \cdot 10^5$
$= 10^7 \cdot 10^5$
$= 10^{7+5} = \mathbf{10^{12}}$.

2. (a) $\mathbf{2^{100}}$ is greater. $2^{80} + 2^{80} = 2^{80}(1 + 1) = 2^{80} \cdot 2 = 2^{81} < 2^{100}$.

(b) $\mathbf{2^{102}}$ is greatest.
$2^{102} = 2^2 \cdot 2^{100} > 3 \cdot 2^{100} > 2^{101} = 2 \cdot 2^{100}$.

3. We can use the repeated addition model of multiplication and make three groups of 42:



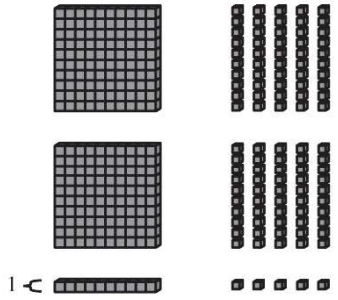
4. (a) The following partial products are obtained through the distributive property of multiplication over addition. The parenthetical computations below illustrate the corresponding area notions from the model:

$$\begin{array}{rr} & 2\ 2 \\ \times & 1\ 3 \\ \hline & 6 \quad (3 \times 2) \\ 6 & 0 \quad (3 \times 20) \\ 2 & 0 \quad (10 \times 2) \\ 2\ 0 & 0 \quad (10 \times 20) \\ \hline 2\ 8 & 6 \end{array}$$

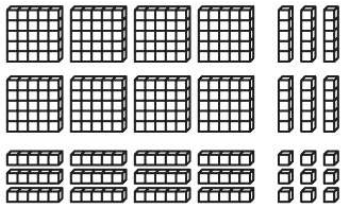
(b)

$$\begin{array}{rr} & 1\ 5 \\ \times & 2\ 1 \\ \hline & 5 \quad (1 \times 5) \\ 1 & 0 \quad (1 \times 10) \\ 1\ 0 & 0 \quad (20 \times 5) \\ 2\ 0 & 0 \quad (20 \times 10) \\ \hline 3\ 1 & 5 \end{array}$$

Which is illustrated by:



(c)



To find $43_{\,five} \cdot 23_{\,five}$ count the number of flats, longs, and ones. There are $4 \cdot 2$ flats, $2 \cdot 3 + 3 \cdot 4$ longs, and $3 \cdot 3$ units. 5 units $=$ 1 long, 5 longs $=$ 1 flat, and 5 flats $=$ 1 block, thus $43_{\,five} \cdot 23_{\,five} =$ 2 blocks, 1 flat, 4 longs, and 4 units, or $\mathbf{2144}_{\,\boldsymbol{five}}$.

5. (a) Start with multiplication of 4_6 by 3. The _ must be 2, since 1 must be regrouped from $3 \cdot 6$ and only $3 \cdot 2 + 1 = 7$. Then $3 \cdot 4 = 12$ for 2 in the _ of the first partial product. Similar reasoning gives:

$$\begin{array}{rrrr} & 4 & \underline{\mathbf{2}} & 6 \\ \times & 7 & 8 & 3 \\ \hline 1 & \underline{\mathbf{2}} & 7 & 8 \\ & 3 & 4 & 0\ 8 \\ \underline{\mathbf{2}} & 9 & 8 & 2 \\ \hline 3 & 3 & 3 & 5\ \underline{\mathbf{5}}\ 8 \end{array}$$

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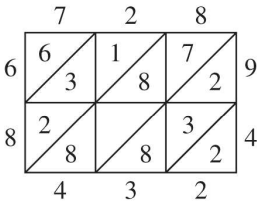
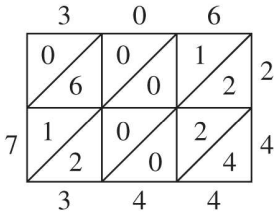
(b) The _ in the multiplier must be 4, since only $4 \cdot 7$ gives 8 in the units place of the second partial product. Similar reasoning gives:

$$\begin{array}{rrrrrr} & & 3 & 2 & 7 & \\ & \times & 9 & \underline{4} & 1 & \\ \hline & & 3 & 2 & 7 & \\ & 1 & \underline{3} & 0 & 8 & \\ \underline{2} & 9 & \underline{4} & 3 & & \\ \hline 3 & 0 & \underline{7} & \underline{7} & 0 & 7 \end{array}$$

6. (a) $728 \times 94 = 68{,}432$:



(b) $306 \times 24 = 7344$:



7. (a) Annexation can be interpreted as append. In base two, two is 10_{two}. To illustrate the property consider a three digit number in base two: $abc_{two} \cdot 10_{two} = (a \cdot 10_{two}^2 + b \cdot 10_{two} + c \cdot 1_{two}) \cdot 10_{two} = a \cdot 10_{two}^3 + b \cdot 10_{two}^2 + c \cdot 10_{two} + 0 \cdot 1_{two} = abc0_{two}$. So multiplying abc_{two} by 10_{two} "annexed" the numeral abc_{two} with a 0 in the "ones" place.

(b) In base two, 4 can be expressed as $10_{two} \cdot 10_{two}$. Thus, given what we learned in (b), multiplying by 4 in base two "annexes" a base two numeral by 00 by annexing the original number by 0 twice.

(c)
$$\begin{aligned} 110_{two} \cdot 11_{two} &= 110_{two}(10_{two} + 1_{two}) \\ &= 110_{two} \cdot 10_{two} + 110_{two} \cdot 1_{two} \\ &= 1100_{two} + 110_{two} \\ &= \mathbf{10010_{two}}. \end{aligned}$$

8. (a) Annexation can be interpreted as append. In base two, two is 10_{two}. To illustrate the property consider a three digit number in base two: $abc_{two} \cdot 10_{two} = (a \cdot 10_{two}^2 + b \cdot 10_{two} + c \cdot 1_{two}) \cdot 10_{two} = a \cdot 10_{two}^3 + b \cdot 10_{two}^2 + c \cdot 10_{two} + 0 \cdot 1_{two} = abc0_{two}$. So multiplying abc_{two} by 10_{two} "annexed" the numeral abc_{two} with a 0 in the "ones" place.

(b) In base two, 4 can be expressed as $10_{two} \cdot 10_{two}$. Thus, given what we learned in (b), multiplying by 4 in base two "annexes" a base two numeral by 00 by annexing the original number by 0 twice.

(c)
$$\begin{aligned} 110_{two} \cdot 11_{two} &= 110_{two}(10_{two} + 1_{two}) \\ &= 110_{two} \cdot 10_{two} + 110_{two} \cdot 1_{two} \\ &= 1100_{two} + 110_{two} \\ &= \mathbf{10010_{two}}. \end{aligned}$$

9.

		Halves		Doubles	
	$\rightarrow$	17	$\times$	$\boxed{63}$	
Halve	17	8		126	Double 63
Halve	8	4		252	Double 126
Halve	4	2		504	Double 252
Halve	2 $\rightarrow$	1		$\boxed{1008}$	Double 504

The numbers in the doubles column paired with odd numbers in the halves column are 63 and 1008; and $63 + 1008 = 17 \times 63 = \mathbf{1071}$.

10. (a) 3 hrs skiing $\times$ 444 calories/hr $=$ **1332** calories.

(b) Jane: 2 hrs $\times$ 462 calories/hr $=$ 924 calories.

Carolyn: 3 hrs $\times$ 198 calories/hr $=$ 594 calories.

Thus Jane burned $924 - 594 = \mathbf{330}$ more.

(c) Lyle: 3 hrs $\times$ 708 calories/hr $=$ 2124 calories.

Maurice: 5 hrs $\times$ 444 calories/hr $=$ 2220 calories.

Thus Maurice burned $2220 - 2124 =$ **96** more.

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11. $2 \text{ hrs} \times 666 \text{ calories/hr} = 1332 \text{ calories/day}$ swimming.

$1500 - 1332 = 168$ calories/day increased intake.

$168 \times 14 = 2352$ excess calories consumed.

$2352 < 3500$, so he gained **less** than 1 pound.

12. Assuming the price is \$30 per \$1000 per year, there are $\$50,000 \div \$1,000 = 50$ installments of \$30. Each quarter, the cost is $50 \cdot \$30 \div 4 = \mathbf{\$375}$.

13. **(a)** Repeated subtraction

$$\begin{array}{r} 8\overline{)\,6\;2\;3} \\ 5\;6\;0 \quad 70 \text{ eights} \\ \hline 6\;3 \\ 5\;6 \quad 7 \quad \text{eights} \\ \hline 7 \quad 77 \quad \text{reminder } 7 \end{array}$$

Standard algorithm

$$\begin{array}{r} \quad\;\; 7\;7 \quad \text{remainder } 7 \\ 8\overline{)\,6\;2\;3} \\ 5\;6\;0 \\ \hline 6\;3 \\ 5\;6 \\ \hline 7 \end{array}$$

(b) Repeated subtraction

$$\begin{array}{r} 3\;6\overline{)\,2\;9\;8} \\ 2\;8\;8 \quad 8 \quad 36\text{'s} \\ \hline 1\;0 \quad 8 \quad \text{remainder } 10 \end{array}$$

Standard algorithm

$$\begin{array}{r} \quad\;\; 8 \quad \text{remainder } 10 \\ 3\;6\overline{)\,2\;9\;8} \\ 2\;8\;8 \\ \hline 1\;0 \end{array}$$

(c) Repeated subtraction

$$\begin{array}{r} 3\;9\;1\overline{)\,4\;0\;0\;1} \\ 3\;9\;1\;0 \quad 10 \; 391\text{'s} \\ \hline 9\;1 \quad 10 \quad \text{remainder } 91 \end{array}$$

Standard algorithm

$$\begin{array}{r} \quad\;\; 1\;0 \quad \text{reminder } 91 \\ 3\;9\;1\overline{)\,4\;0\;0\;1} \\ 3\;9\;1 \\ \hline 9\;1 \end{array}$$

14. **(a)** The divisor will be the least digit value and the dividend will be arranged so that the greatest is placed for the greatest place value. $\mathbf{3\overline{)754}}$

(b) The thinking in (a) is reversed. $\mathbf{7\overline{)345}}$

15. Reverse the operation: $300 \div 10 = 30$ and $30 \div 10 = \mathbf{3}$.

16. **(a)** Answers may vary. One such is $36 \cdot 84 = 3024$ and $63 \cdot 48 = 3024$.

(b) Let the digits be a, b, c and d. Then $(10a + b) \cdot (10c + d) = (10b + a) \cdot (10d + c) \Rightarrow 100ac + 10bc + 10ad + bd = 100bd + 10bc + 10ad + ac \Rightarrow 99ac = 99bd \Rightarrow ac = bd$.

So if $a \cdot c = b \cdot d$ then the products will always be the same when the digits are reversed (e.g., in part (a) above, $a \cdot c = b \cdot d \Rightarrow 3 \cdot 8 = 6 \cdot 4$).

17. Let p be the number of pennies in box 3. Then $3p$ is the number of pennies in box 1 and $2(3p) = 6p$ is the number of pennies in box 2. Thus $3p + 6p + p = 4520 \Rightarrow 10p = 4520 \Rightarrow p = 452$.

So box $1 = 3(452) = \mathbf{1356}$ pennies;

box $2 = 6(452) = \mathbf{2712}$ pennies;

box $3 = \mathbf{452}$ pennies.

18. $36 \text{ apples} \times 50 \text{ boxes} = 1800$ apples, or 600 3-apple bags. She had $18 \div 3 = 6$ bags left over, so she sold $600 - 6 = 594$ bags at \$1 per bag. Thus her profit was $\$594 - \$452 = \mathbf{\$142}$.

19. **(a)** 5 was multiplied by 6 to obtain 30. The 3 was regrouped, then 3 was multiplied by 2 to obtain 6. The regrouping was added to obtain 9 which was recorded in the tens place.

(b) When 1 was brought down the quotient of 0 was not recorded.

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20. $56 \cdot 10$

$= (5 \cdot 10 + 6) \cdot 10$	expanded form.
$= (5 \cdot 10) \cdot 10 + 6 \cdot 10$	distributive property.
$= 5(10 \cdot 10) + 6 \cdot 10$	associative property.
$= 5 \cdot 10^2 + 6 \cdot 10$	definition of a^n.
$= 5 \cdot 10^2 + 6 \cdot 10 + 0 \cdot 1$	additive identity.
$=$ **560**	place value.

21. **(a)** Finding a number that leaves a remainder of 3 upon division by 4 is equivalent to thinking of a number multiplied by four and then add 3. For example 15 is a number that leaves a remainder of 3 upon division by 4:
$15 \div 4 = 3$ with a remainder of 3 because
$15 = 4 \bullet 3 + 3 \Rightarrow 15 = 12 + 3 \Rightarrow 15 = 15$.
In general, $4n + 3$ is all whole numbers that leave a remainder of 3 when divided by 4. In set builder notation $\{4n+3 \mid n \in W\}$.

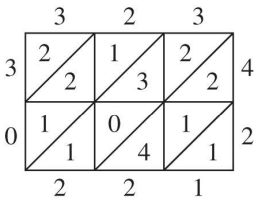
(b) $4n+3$
$4(0)+3=3$
$4(1)+3=7$
$4(2)+3=11$
$4(3)+3=15$
$4(4)+3=19$
etc.
3, 7, 11, 15, 19, …

(c) Arithmetic because each subsequent term is the result of adding 4 to the previous term.

22. **(a)** **Base nine**. In the units column $3 + 8 = 12$ with the two brought down and the 1 regrouped, so $(3 + 8)_{nine} = 12_{nine}$.

(b) **Base six**. $2 \cdot 3$ produced 0 in the units column, so $(2 \cdot 3)_{six} = 10_{six}$.

23. $323_{five} \cdot 42_{five} = \mathbf{30221}_{\boldsymbol{five}}$:



24. $32_a = 23_b \Rightarrow 3a + 2 = 2b + 3 \Rightarrow$
$b = \frac{3a-1}{2}$. The smallest value of $a (a > 1)$ for b to be whole is $a = 3 \Rightarrow a = 3$ and $b = 4$. But 32_{three} is not possible because there is no numeral 3 in base three. The smallest possible solution is thus $a = 5$ and $b = 7$, or $32_{five} = 23_{seven}$.

25. **(a)** The greatest product requires the largest multiplicands which can be formed using the four numbers: $8 \times 763 > 7 \times 863$ because $8 \times 700 = 7 \times 800$ but $8 \times 63 > 7 \times 63$. Thus:

$$\begin{array}{cccc} \boxed{7} & \boxed{6} & \boxed{3} & \\ & \times & & \boxed{8} \\ \hline 6 & 1 & 0 & 4 \end{array}$$

(b) The least product requires the smallest multiplicands which can be formed using the four numbers; $3 \times 678 < 6 \times 378$ because $3 \times 600 = 6 \times 300$ but $3 \times 78 < 6 \times 78$. Thus:

$$\begin{array}{cccc} \boxed{6} & \boxed{7} & \boxed{8} & \\ & \times & & \boxed{3} \\ \hline 2 & 0 & 3 & 4 \end{array}$$

26. In each case it may be helpful to generate a multiplication table in the appropriate base; e.g. in base 5:

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

(a)
$$\begin{array}{cccc} & {\scriptstyle 1} & & \\ & 3 & 2 & \textit{five} \\ \times & & 4 & \textit{five} \\ \hline 2 & 3 & 3 & \textit{five} \end{array}$$

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(b)
$$\begin{array}{r} 4_{five} \\ 4_{five}\,\overline{)\,3\;2_{five}} \\ \underline{3\;1} \\ 1 \end{array}$$
remainder 1_{five}

(c)
$$\begin{array}{r} 4\;3_{six} \\ \times\;2\;3_{six} \\ \hline 2\;1\;3 \\ 1\;3\;0 \\ \hline 1\;5\;1\;3_{six} \end{array}$$

(d)
$$\begin{array}{r} 3\;1_{five} \\ 3_{five}\,\overline{)\,1\;4\;3_{five}} \\ \underline{1\;4} \\ 0\;3 \\ \underline{0\;3} \\ 0 \end{array}$$

(e)
$$\begin{array}{r} 1\;1\;0_{two} \\ 11_{two}\,\overline{)\,1\;0\;0\;1\;0_{two}} \\ \underline{1\;1} \\ 1\;1\;0 \\ \underline{1\;1\;0} \\ 0 \end{array}$$

(f)
$$\begin{array}{r} 1\;0\;1\;1\;0_{two} \\ \times\qquad 1\;0\;1_{two} \\ \hline 1\;0\;1\;1\;0 \\ 1\;0\;1\;1\;0 \\ \hline 1\;1\;0\;1\;1\;1\;0_{two} \end{array}$$

27. 8 hours $\times$ 62 mph $= (8 \cdot 60) + (8 \cdot 2) =$ $480 + 16 =$ **496 miles** (i.e., the front-end multiplying method).

28. Estimates may vary. The range would be from $30 \cdot 20 = 600$ to $40 \cdot 30 = 1200$. Rounding, $38 \approx 35$ or 40 and $23 \approx 20$ or 25, thus about $35 \cdot 20 = 700$ seats, or $40 \cdot 25 = 1000$ seats. 700 would be low (rounded down) and 1000 would be high (rounded up).

29. Answers may vary:

(a) **Different**. One factor is the same in each and the other is 4 times larger.

(b) **Same**. 22 was divided by 2 to obtain 11 while 32 was multiplied by 2 to obtain 64. The result is to multiply the original computation by $\frac{2}{2} = 1$ which does not change it.

(c) **Same**. 13 was multiplied by 3 and 33 was divided by 3, thus the original computation was multiplied by 1.

30. Note that in parts (b) through (d), a larger divisor produces a lower quotient when the dividend stays the same and a larger dividend produces a higher quotient when the divisor stays the same.

(a) **High**. $299 \cdot 300 < 300 \cdot 300$.

(b) **Low**. $6001 \div 299 > 6000 \div 300$.

(c) **Low**. $6000 \div 299 > 6000 \div 300$.

(d) **Low**.
$10 \cdot 99 = 990 < 999 \Rightarrow 999 \div 99 > 10$.

31. Answers may vary. One strategy to find $(n5)^2$ would be to write $n(n+1)$ and append 25 (because 5^2 is always 25); e.g.,
$65^2 = (6 \cdot 7 = 42)$ and append $25 = 4225$ and $75^2 = (7 \cdot 8 = 56)$ and append $25 = 5625$.

32. Answers vary. If the estimated product is 42,000 and 42 = 6 • 7 so 42,000 = 600 • 70, then an estimate of $42{,}000 \approx 612 \bullet 73$.

Assessment 3-5B

1. **(a)** There are $8 + 4 = 12$ factors of 3.
Thus $3^8 \cdot 3^4 = 3^{8+4} = \mathbf{3^{12}}$.

(b) $5^2 \cdot 5^4 \cdot 5^2 = 5^{2+4+2} = \mathbf{5^8}$.

(c) $6^2 \cdot 2^2 \cdot 3^2 = 6^2 \cdot (2 \cdot 3)^2 =$
$6^2 \cdot 6^2 = \mathbf{6^4} = (2 \cdot 3)^4$.

(c) $4^8 \cdot 8^4 \cdot 32^5 = 2^{16} \cdot 2^{12} \cdot 2^{25} =$
$\mathbf{2^{53}}$.

2. **(a)** $2^{20} + 2^{20} = 2^{20}(1+1) = 2^{20} \cdot 2 = 2^{21}$,
so **they are equal**.

(b) $9 \cdot 3^{30} = 3^2 \cdot 3^{30} = 3^{32}$,
so $\mathbf{3^{31} < 9 \cdot 3^{30} < 3^{33}}$.

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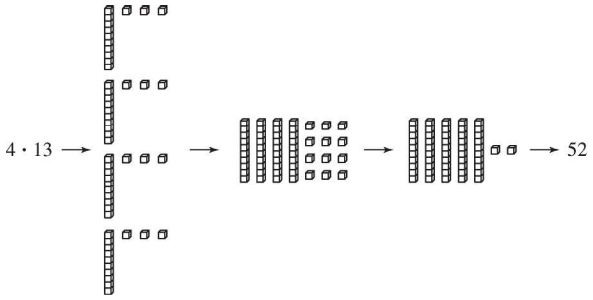
3. We can use the repeated addition model of multiplication and make four groups of 13:



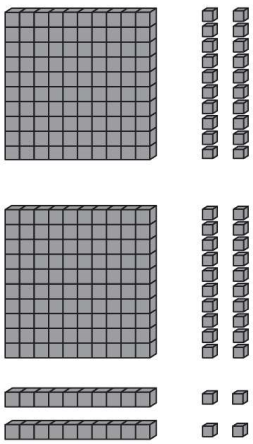
4. **(a)** The following partial products are obtained through the distributive property of multiplication over addition, as shown below:

$$\begin{array}{rr} & 1\ 3 \\ \times & 1\ 2 \\ \hline & 6 \quad (2 \times 3) \\ 2 & 0 \quad (2 \times 10) \\ 3 & 0 \quad (10 \times 3) \\ 1\ 0 & 0 \quad (10 \times 10) \\ \hline 1\ 5 & 6 \end{array}$$

(b)

$$\begin{array}{rr} & 1\ 2 \\ \times & 2\ 2 \\ \hline & 4 \quad (2 \times 2) \\ 2 & 0 \quad (2 \times 10) \\ 4 & 0 \quad (20 \times 2) \\ 2\ 0 & 0 \quad (20 \times 10) \\ \hline 2\ 6 & 4 \end{array}$$

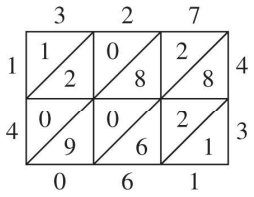
Which is illustrated by:



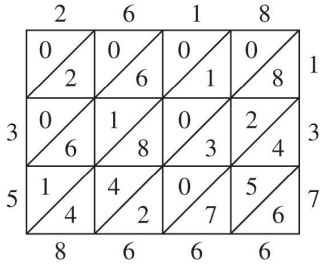
5. Start by multiplication of 4_4 by 7. The _ must be 8 since the 2 must be regrouped from $7 \cdot 4$ and only $7 \cdot 8 + 2$ yields an 8 in the tens column. Now that the factors are 484 and 327, compute the partial products in the usual way to solve for the other missing terms.

$$\begin{array}{rrrr} & 4 & \underline{\mathbf{8}} & 4 \\ \times & 3 & 2 & 7 \\ \hline 3 & \underline{\mathbf{3}} & 8 & 8 \\ 9 & 6 & 8 & \\ \underline{\mathbf{1}} & 4 & 5 & 2 \\ \hline 1 & 5 & 8 & 2\ \underline{\mathbf{6}}\ 8 \end{array}$$

6. **(a)** $327 \times 43 = 14061$:



(b) $2618 \times 137 = 358{,}666$:



7. **(a)** Answers may vary. Assume the daily averages for taking a shower, toilet flushing, washing of hands and face, drinking, and brushing teeth, and assume three dish washes and two cookings (have cold cereal for breakfast). Then $75 + 22 + 7 + 1 + 1 + (3 \cdot 30) + (2 \cdot 18) =$ 232 liters.

(b) About 310,000,000 people $\times$ 200 liters per day $=$ **about 62,000,000,000 liters** per day.

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8. (a) Use a three-digit number in base five, abc_{five}.

$abc_{five} \cdot 10_{five} = (a \cdot 10^2 + b \cdot 10^1 + c \cdot 10^0)_{five} \cdot 10_{five} = (a \cdot 10^3 + b \cdot 10^2 + c \cdot 10^1)_{five} = \boldsymbol{abc0}_{\boldsymbol{five}}$.

(b) Multiplying by 100_{five} is the same as multiplying twice by 10_{five} (i.e., $10_{five} \cdot 10_{five} = 100_{five}$). Since a zero is annexed each time a number is multiplied by 10, multiplication by 100 will result in annexation of two zeros (this is true for multiplying by 100 in any base, not just base five).

(c) $14_{five} \cdot 23_{five} = (10 + 4)_{five} \cdot 23_{five} = (10 \cdot 23 + 4 \cdot 23)_{five} = (230 + 202)_{five} = \boldsymbol{432}_{\boldsymbol{five}}$.

Observe that in base five notation
$4_{five} \cdot 23_{five} = [4 \cdot (2 \cdot 10 + 3)]_{five}$
$= (130 + 22)_{five} = 202_{five}$.

9.

		Halves		Doubles	
	→	31	×	69	
Halve 31	→	15		138	Double 69
Halve 15	→	7		276	Double 138
Halve 7	→	3		552	Double 276
Halve 3	→	1		1104	Double 552

69
138
276
552
1104
2139 which is the product of 31×69.

10. (a) 444 calories per hour $\cdot$ 4 hours = **1776 calories**.

(b) Jane burned $462 \cdot 3 = 1386$ calories and Carolyn burned $198 \cdot 4 = 792$ calories. **Jane** burned 594 more calories.

(c) Lyle burned $708 \times 4 = 2832$ calories and Maurice burned $444 \times 5 = 2220$ calories. **Lyle** burned 612 more calories.

11. Each day Glenn burned $666 \times 3 = 1998$ calories. He burned more calories working out than he consumed in additional calories. Assuming that the three hour swims were not part of his usual daily routine, he **did not gain weight**.

12. \$24/thousand $\times$ 30 thousands = \$720 annual premiums. \$720 $\div$ 12 months = **\$60**/month in installment payments.

13. (a) Repeated subtraction

7	3	9	2		
	3	5	0	50	sevens
		4	2		
		4	2	6	sevens
			0	56	reminder 0

Standard algorithm

		5	6	remainder 0
7	3	9	2	
	3	5		
		4	2	
		4	2	
			0	

(b) Repeated subtraction

3	7	9	2	5		
		7	4	0	20	37's
		1	8	5		
		1	8	5	5	37's
				0	25	remainder 0

Standard algorithm

		2	5	remainder 0
3	7	9	2	5
		7	4	
		1	8	5
		1	8	5
				0

(c) Repeated subtraction

4	2	3	5	0	0	2	
			4	6	5	3	11 423's
				3	4	9	11 remainder 349

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Standard algorithm

$$\begin{array}{r} \quad\;\; 1\;\;1 \\ 4\;\;2\;\;3\;\overline{)\;5\;\;0\;\;0\;\;2} \\ \quad\;\; 4\;\;2\;\;3 \\ \hline \quad\;\; 7\;\;7\;\;2 \\ \quad\;\; 4\;\;2\;\;3 \\ \hline \quad\;\; 3\;\;4\;\;9 \end{array}$$

remainder 349

14. **(a)** The greatest quotient follows from dividing the smallest number possible into the largest number possible $(876 \div 3 = 292)$:

$$\boxed{3}\;\overline{)\;\boxed{8}\;\boxed{7}\;\boxed{6}}$$

(b) The least quotient follows from dividing the largest number possible into the smallest number possible
$(367 \div 8 = 45$ remainder $7)$:

$$\boxed{8}\;\overline{)\;\boxed{3}\;\boxed{6}\;\boxed{7}}$$

15. $5 \cdot x = 250 \Rightarrow x = 50,$ the original number. The answer should have been $50 \div 5 = \mathbf{10}$.

16. Because sum of two numbers less than 10,000 is less than 20,000, ***M* must be 1**. To produce an exchange (carry), S must be 8 or 9. Since $S + M$ is 9 or 10, O must be 0 or 1. But M is 1, so ***O* is 0**. At this point we have

$$\begin{array}{rcccc} & S & E & N & D \\ + & & 1 & 0 & R & E \\ \hline & 1 & 0 & N & E & Y \end{array}$$

and we know that S is 8 or 9.

If there is an exchange from column three to four, then $E = 9$ (since $E + 0 = E$) and then $N = 0$. But $O = 0$. So there is no exchange from column three to four and thus, $\boldsymbol{S = 9}$.

If there is no exchange from column two to column three, then $E = N,$ which is not possible. There is a carry and thus $N = E + 1$.

We have

$$\begin{array}{rcccc} & 9 & E & E+1 & D \\ + & 1 & 0 & R & E \\ \hline 1 & 0 & E+1 & E & Y \end{array}$$

If there were no exchange from column 1, then R is 9. But S is 9. So $\boldsymbol{R = 8}$.

Similar reasoning results in
$\boldsymbol{E = 5, N = 6, D = 7,\ \&\ Y = 2}$

$$\begin{array}{r} \quad\;\; 1\;\;1 \\ \quad\;\; 9\;\;5\;\;6\;\;7 \\ + \quad 1\;\;0\;\;8\;\;5 \\ \hline 1\;\;0\;\;6\;\;5\;\;2 \end{array}$$

17. Let x represent the number of dimes in the second box. Then $4x + x + 3(4x) = 340$. Solving we find that $x = 20$ dimes. Converting to a dollar value, the second box contains $20 \times \$0.10 = \$2$. Thus the boxes contain **8, 2, and 24 dollars** respectively.

18. $\$5340 - (12$ months $\times\ \$95$ per month$) = \$4200$ saved in the final two years.
$\$4200 \div 24$ months $=$ **\$175** per month.

19. **(a)** Regrouping was not used. Place value was not observed.

(b) 4 was multiplied by 6 to obtain 24. The 2 was regrouped; then 6 and 3 were added instead of multiplied and the 2 was added to obtain 11.

20.
$$\begin{aligned} 35 \cdot 100 &= (3 \cdot 10 + 5) \cdot 100 && \text{expanded form} \\ &= (3 \cdot 10 + 5) \cdot 10^2 && \text{definition of } a^n \\ &= (3 \cdot 10) \cdot 10^2 + && \\ &\qquad 5 \cdot 10^2 && \text{distributive property} \\ &= 3(10 \cdot 10^2) + && \text{associative property} \\ &\qquad 5 \cdot 10^2 && \text{of multiplication} \\ &= 3 \cdot 10^3 + 5 \cdot 10^2 && \text{rules of exponents} \\ &\quad + 0 \cdot 10 + 0 \cdot 1 && \text{multiplication} \\ &&& \text{by zero property} \\ &= 3500 && \text{place value} \end{aligned}$$

21. **(a)** Let n be a whole number. Then $\boldsymbol{n = 4q + 1}$, $q \in W,$ will produce all the whole numbers with remainder of 1 when divided by 4.

(b)

q	n
0	1
1	5
2	9

etc

(c) This is an **arithmetic sequence** with $a_1 = 1$ and $d = 4$.

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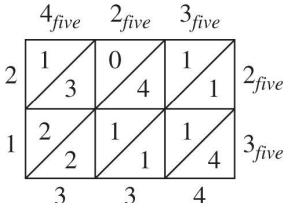
22. **(a)** **Base four**. In the units column it was necessary to regroup to subtract; since $6 - 3 = 3$, a 4 must have been regrouped. Then $(4 + 2)_{ten} = 12_{four}$ and $(12 - 3)_{four} = 3_{four}$.

(b) **Any base $\geq$ two** would yield this quotient.

23. $423_{five} \cdot 23_{five} = 21334_{five}$:



24. Form the equation $4a + 1 = b + 4 \Rightarrow 4a - 3 = b$. When $a = 2, b = 5$, which is a solution to the equation but not a solution to the problem since $a = 2$ cannot be the base for 41_a. Continuing, the first useable value for a is 5, which leads to a base of 17 for b. Thus the solution is $\boldsymbol{a = 5, b = 17}$, or $41_{five} = 14_{seventeen}$.

25. **(a)** The greatest product requires the largest multiplicands which can be formed using the five numbers; $83 \times 762 > 73 \times 862$ because $80 \times 700 = 70 \times 800$ but $83 \times 62 > 73 \times 62$:

$$\begin{array}{r} \boxed{7}\ \boxed{6}\ \boxed{2} \\ \times\ \boxed{8}\ \boxed{3} \\ \hline 6\ 3\ 2\ 4\ 6 \end{array}$$

(b) The least product requires the smallest multiplicands which can be formed using the five numbers; $26 \times 378 < 36 \times 278$ because $20 \times 300 = 30 \times 200$: but $26 \times 78 < 36 \times 78$

$$\begin{array}{r} \boxed{3}\ \boxed{7}\ \boxed{8} \\ \times\ \boxed{2}\ \boxed{6} \\ \hline 9\ 8\ 2\ 8 \end{array}$$

26. In each case it may be helpful to generate a multiplication table in the appropriate base; e.g. in base 5:

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

(a)
$$\begin{array}{rr} & {\scriptstyle 1} \\ & 4\ \ 2_{\,five} \\ \times & \ \ 3_{\,five} \\ \hline 2\ \ 3\ \ 1_{\,five} \end{array}$$

(b)
$$\begin{array}{r} \ \ 3_{\,five} \\ 4_{\,five}\ \overline{)\ 2\ \ 2_{\,five}} \\ \underline{2\ \ 2} \\ \ \ 0 \end{array}$$

(c)
$$\begin{array}{r} \ \ 3\ \ 2_{\,five} \\ \times\ 4\ \ 2_{\,five} \\ \hline 1\ \ 1\ \ 4 \\ \hline 2\ \ 3\ \ 3 \\ \hline 2\ \ 4\ \ 4\ \ 4_{\,five} \end{array}$$

(d)
$$\begin{array}{r} \ \ 3\ \ 1_{\,five} \\ 2\ \ 3_{five}\overline{)\ 1\ \ 3\ \ 1\ \ 3_{\,five}} \\ \underline{1\ \ 2\ \ 4} \\ \ \ 2\ \ 3 \\ \underline{2\ \ 3} \\ \ \ \ \ 0 \end{array}$$

(e)
$$\begin{array}{r} \ \ 1\ \ 0\ \ 1_{\,two} \\ \times\ 1\ \ 0\ \ 1_{\,two} \\ \hline \ \ 1\ \ 0\ \ 1 \\ \hline 1\ \ 0\ \ 1 \\ \hline 1\ \ 1\ \ 0\ \ 0\ \ 1_{\,two} \end{array}$$

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(f)
$$\begin{array}{r} 1\ 1\ _{two} \\ 11_{two}\overline{)\ 1\ 0\ 0\ 1\ _{two}} \\ \underline{0\ 1\ 1} \\ 1\ 1 \\ \underline{1\ 1} \\ 0 \end{array}$$

27. 6 hours $\times$ 65 mph $= (6 \cdot 60) + (6 \cdot 5) =$ $360 + 30 = 390$ **miles** (i.e., the front-end multiplying method).

28. About **21,000** calories. 3540 calories per pound is about 3500 calories, and $6 \cdot 3500 = 3 \cdot (2 \cdot 3500) = 3 \cdot (7000) = 21{,}000$ calories. (Actual is 21,240.)

29. Answers may vary:

(a) **Different**. The estimates are quite different. The second number (22) is $\frac{1}{4}$ the first number (88).

(b) **Same**. 93 was divided by 3 to obtain 31 and 15 was multiplied by 3 to obtain 45.

(c) **Different**. $20 \cdot 17 = 17 \cdot 20$; $12 < 17$ and $18 < 20$.

30. Note that in parts (a) through (d), a larger divisor produces a lower quotient when the dividend stays the same and a larger dividend produces a higher quotient when the divisor stays the same.

(a) **High**. $398 \cdot 500 < 400 \cdot 500$.

(b) **Low**. $8001 \div 398 > 8001 \div 400 > 8000 \div 400$.

(c) **Low**. $10{,}000 \div 999 > 10{,}000 \div 1000$.

(d) **High**. $1999 \div 201 < 2000 \div 200$.

31. Answers may vary. One strategy to mentally find $ab \cdot 99$, where ab represents the number $10a + b$, would be to write $(ab - 1)$ as the first two digits and then append the digits obtained from $100 - ab$. E.g., to find $12 \cdot 99$ the first two digits are $12 - 1 = 11$ and the second two digits are $100 - 12 = 88$, or $12 \cdot 99 = 1188$.

Another method is to show $ab \cdot 99$ as $ab(100 - 1) = 100ab - ab$.

32. Answers vary. If the estimated product is 2400 and 24 = 3 • 8 so 2400 = 30 • 80, then an estimate of $2400 \approx 34 \cdot 82$.

Mathematical Connections 3-5: Review Problems

22. Answers may vary; for example, $3 + 0 = 3 = 0 + 3$.

23. **(a)** $ax + bx + 2x = \mathbf{(a + b + 2)x}$.

(b) $3(a + b) + x(a + b) = \mathbf{(3 + x)(a + b)}$.

24. $59{,}260$ miles $- 52{,}281 =$ **6979 miles** traveled.

25. **(a)** $36 \div 4 = 9 \Rightarrow \mathbf{36 = 4 \cdot 9}$.

(b) $112 \div 2 = x \Rightarrow \mathbf{112 = 2x}$.

(c) $48 \div x = 6 \Rightarrow \mathbf{48 = x6}$.

(d) $x \div 7 = 17 \Rightarrow \mathbf{x = 7 \cdot 17}$.

Chapter 3 Review

1. **(a)** Tens.

(b) Thousands.

(c) Hundreds.

2. **(a)** $\overline{\text{CD}}\text{XLIV} = 1000 \cdot \text{CD} + \text{XLIV} = 1000 \cdot 400 + 40 + 4 =$ **400,044**.

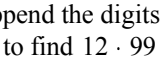
(b) $432_{five} = 4 \cdot 5^2 + 3 \cdot 5 + 2 = 100 + 15 + 2 =$ **117**.

(c) $\text{ET0}_{twelve} = 11 \cdot 12^2 + 10 \cdot 12 + 0 = 1584 + 120 =$ **1704**.

(d) $1011_{two} = 1 \cdot 2^3 + 1 \cdot 2 + 1 = 8 + 2 + 1 =$ **11**.

(e) $4136_{seven} = 4 \cdot 7^3 + 1 \cdot 7^2 + 3 \cdot 7 + 6 = 1372 + 49 + 21 + 6 =$ **1448**.

3. **(a)** CM $\rightarrow$ 900; XC $\rightarrow$ 90; IX $\rightarrow$ 9, or $999 =$ **CMXCIX**.

(b) Eight groups of 10 and 6 units of one, or  .

(c) Six units of 20 and three units of 1, or .

(d) 2 groups of $125 + 3$ groups of $25 + 4$ groups of $5 + 1$, or $\mathbf{2341}_{\textbf{\textit{five}}}$.

(e) 1 group of $16 + 1$ group of $8 + 0$ groups of $4 + 1$ group of $2 + 1$, or $\mathbf{11011}_{\textbf{\textit{two}}}$.

4. **(a)** $3^{4+7+6} = \mathbf{3^{17}}$.

(b) $2^{10+11} = \mathbf{2^{21}}$.

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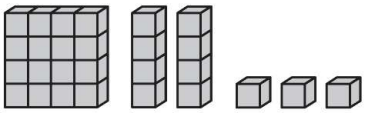
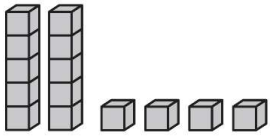
5. $1000_{three} + 200_{three} + 100_{three} + 20_{three} = 1000_{three} + 1000_{three} + 20_{three} = \mathbf{2020_{three}}$.

6. $51_{ten} = (1 \cdot 10^3)_{three} + (2 \cdot 10^2)_{three} + (2 \cdot 10)_{three} + (0 \cdot 1)_{three} \Rightarrow$ **1 block, 2 flats, 2 longs, 0 units**. So the fewest blocks needed is **5.**

7. (a) 123_{four}:



(b) 24_{five}:



8. (a) $40,000,000,000 = 4 \cdot 10^{10}$, so the place value of 4 is $\mathbf{10^{10}}$.

(b) A number in base five having ten digits is $m \cdot 5^9 + n \cdot 5^8 + \cdots$. The place value of the second digit is therefore $\mathbf{5^8}$.

(c) 30 zeros and a 1 in base two represents a number $n = 1 \cdot 2^{31} + 0 \cdot 2^{30} + \cdots + 1$. The place value of the lead digit is therefore $\mathbf{2^{31}}$.

9. (a) $10^{10} + 23 = 1 \cdot 10^{10} + 0 \cdot 10^9 + \cdots + 2 \cdot 10 + 3$, or $\mathbf{10,000,000,023}$.

(b) $2^{10} + 1 = 1 \cdot 2^{10} + 0 \cdot 2^9 + \cdots + 1$, or $\mathbf{10,000,000,001_{two}}$.

(c) $5^{10} + 1 = 1 \cdot 5^{10} + 0 \cdot 5^9 + \cdots + 1$, or $\mathbf{10,000,000,001_{five}}$.

(d)

$$\begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ -\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 1 \\ \hline 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9\ 9 \end{array}$$

(e) 10 must be borrowed from each 0 digit in the subtrahend, starting with the rightmost, just as with base ten.

$$\begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0_{two} \\ -\qquad\qquad\qquad\qquad\qquad\qquad\quad 1 \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1_{two} \end{array}$$

(f)

$$\begin{array}{r} 1\ 0\ 0\ 0\ 0\ 0_{twelve} \\ -\qquad\qquad\quad 1 \\ \hline E\ E\ E\ E\ E_{twelve} \end{array}$$

(g) $7 \cdot 5^6 + 3 \cdot 5^4 + 11$

$= (5+2) \cdot 5^6 + 0 \cdot 5^5 + 3 \cdot 5^4 + 0 \cdot 5^3 + 0 \cdot 5^2 + 10 + 1$

$= 5 \cdot 5^6 + 2 \cdot 5^6 + 0 \cdot 5^5 + 3 \cdot 5^4 + 0 \cdot 5^3 + 0 \cdot 5^2 + 2 \cdot 5^1 + 1 \cdot 5^0$

$= 1 \cdot 5^7 + 2 \cdot 5^6 + 0 \cdot 5^5 + 3 \cdot 5^4 + 0 \cdot 5^3 + 0 \cdot 5^2 + 2 \cdot 5^1 + 1 \cdot 5^0$

$= 12030021$

10. Answers may vary. One example would be selling pencils by units, dozens, and gross; i.e., base twelve.

11. (a) The Egyptian system had seven symbols. It was a tally system and a grouping system, and it used the additive property. It did not have a symbol for zero which was not important inasmuch as they did not use place value.

(b) The Babylonian system used only two symbols. It was a place value system (base 60) and was additive within the positions. It lacked a symbol for zero until around 300 B.C.

(c) The Roman system used seven symbols. It was additive, subtractive, and multiplicative. It did not have a symbol for zero.

(d) The Hindu-Arabic system uses ten symbols. It uses place value involving base ten and has a symbol for zero.

12. (a) There is 1 group of $125(5^3)$ in 128, with remainder 3.

There are 0 groups of 25 in 3, with remainder 0.

There are 0 groups of 5 in 3, with remainder 0.

There are 3 groups of 1 in 3, with remainder 0.

Thus $128 = \mathbf{1003_{five}}$.

(b) There is 1 group of $128(2^7)$ in 128, with remainder 0.

There are 0 groups of 64 in 0, with remainder 0.

There are 0 groups of 32 in 0, with remainder 0.

There are 0 groups of 16 in 0, with remainder 0.

There are 0 groups of 8 in 0, with remainder 0.

There are 0 groups of 4 in 0, with remainder 0.

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There are 0 groups of 2 in 0, with remainder 0.
There are 0 groups of 1 in 0, with remainder 0.

Thus $128 = \mathbf{10000000}_{\textit{two}}$.

(c) There are $10(T)$ groups of 12 in 128, with remainder 8. There are 8 groups of 1 in 8, with remainder 0.

Thus $128 = \boldsymbol{T8}_{\textit{twelve}}$.

13. Place value is determined by powers of each base.

(a) $2^{10} + 2^3 = \mathbf{10000001000}_{\textit{two}}$.

(b) $11 \cdot 12^5 + 10 \cdot 12^3 + 20 = 11 \cdot 12^5 + 10 \cdot 12^3 + 1 \cdot 12 + 8 = \boldsymbol{E0T018}_{\textit{twelve}}$.

14. $1 \cdot b^2 + 2b + 3 = 83 \Rightarrow b^2 + 2b - 80 = 0 \Rightarrow (b - 8)(b + 10) = 0 \Rightarrow b = 8$ or $b = -10$. Since the base must be positive, $\boldsymbol{b = 8}$.

15. **(a)** **Distributive** property of multiplication over addition.

(b) **Commutative** property of addition.

(c) **Identity** property of multiplication for whole numbers.

(d) **Distributive** property of multiplication over addition.

(e) **Commutative** property of multiplication.

(f) **Associative** property of multiplication.

16. **(a)** $13 = 3 + 10$. Since 10 is a natural number, $3 < 13$.

(b) $12 = 3 + 9$. Since 3 is a natural number, $12 > 9$.

17. **(a)** $4 \cdot \boxed{\mathbf{10 \le \textit{whole number} \le 15}} - 37 < 27$.

(b) $398 = \boxed{\mathbf{10}} \cdot 37 + 28$.

(c) $\boxed{\boldsymbol{n}} \cdot (3 + 4) = \boxed{\boldsymbol{n}} \cdot 3 + \boxed{\boldsymbol{n}} \cdot 4$, where $n \in W$ is any whole number.

(d) $42 - \boxed{\textit{Any whole number } \mathbf{\le 26}} \ge 16$.

18. **(a)** $3a + 7a + 5a = (3 + 7 + 5)a = \mathbf{15}\boldsymbol{a}$.

(b) $3x^2 + 7x^2 - 5x^2 = (3 + 7 - 5)x^2 = \mathbf{5}\boldsymbol{x^2}$.

(c) $x(a + b + y) = \boldsymbol{xa + xb + xy}$.

(d) $(x + 5)3 + (x + 5)y = \boldsymbol{3x + 15 + xy + 5y}$

or

$(x + 5)3 + (x + 5)y = \boldsymbol{(x + 5)(3 + y)}$.

(e) $3x^2 + x = x(3x + 1)$.

(f) $2x^5 + x^3 = x^3(2x^2 + 1)$.

19. 60 people $\times$ 8 ounces $=$ 480 ounces required.
480 $\div$ 12 ounces per can $=$ **40** twelve-ounce cans.

20. 2 slacks $\times$ 3 blouses $\times$ 2 sweaters $=$ **12** outfits.

21. Work backward from 93 using inverse operations:
Subtract 89, giving 4;
Add 20, giving 24;
Divide by 12, giving 2; then
Multiply by 13, giving **26** as the original number.

22. \$80 per person $\times$ 80 people $=$ \$6400. The **\$6000** package is less expensive.

23. 30 hours per week $\times$ \$5 per hour $+$ 8 hours overtime $\times$ \$8 per overtime hour $=$ **\$214**.

24. Let q be the amount from the first question. Then winnings are $q + 2q + 4q + \cdots$ which is a geometric sequence with $a_1 = q, r = 2,$ and $n = 5$. $6400 = q(2)^{5-1} \Rightarrow 16q = 6400 \Rightarrow q = \mathbf{\$400}$.

25. **(a)** Let n be the original number. Then

$$\frac{2[2(n + 17) - 4] + 20}{4} - 20 =$$

$$\frac{4(n + 17) - 8 + 20}{4} - 20 =$$

$$\frac{4n + 80}{4} - 20 = n + 20 - 20 = \boldsymbol{n}.$$

(b) Answers may vary. For example, if n is the original number:

$4(n + 18) - 7 = 4n + 65$.

Then two more steps might be:
$4n + 65 - 65$ (subtract 65);

$\frac{4n}{4}$ (divide by 4).

(c) Answers may vary; use the techniques of parts (a) and (b).

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26. Scratch:

$$\begin{array}{r} {\scriptstyle 1} \\ 3\ 1\ 6 \\ 7\!\!\not{1}_{1}\ 1\ 2 \\ +\quad 9\!\!\not{1}_{1}\ 1 \\ \hline 1\ 1\ 1\ 9 \end{array}$$

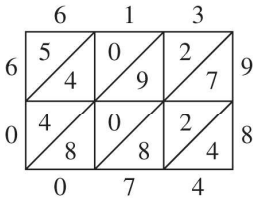
Traditional:

$$\begin{array}{r} {\scriptstyle 1} \\ 3\ 1\ 6 \\ 7\ 1\ 2 \\ +\quad 9\ 1 \\ \hline 1\ 1\ 1\ 9 \end{array}$$

27. Traditional:

$$\begin{array}{r} 6\ 1\ 3 \\ \times\quad 9\ 8 \\ \hline 4\ 9\ 0\ 4 \\ 5\ 5\ 1\ 7 \\ \hline 6\ 0\ 0\ 7\ 4 \end{array}$$

Lattice:



28. **(a)** Repeated subtraction:

$$\begin{array}{r|l} 9\ 1\ 2 & 4\ 8\ 0\ 3 \\ & \underline{4\ 5\ 6\ 0} \quad 5\text{-}912\text{'s} \\ & 2\ 4\ 3 \quad 5\text{-}912\text{'s} \end{array} \Rightarrow 5 \text{ r } 243$$

Traditional:

$$\begin{array}{r|l} & \ 5 \\ 9\ 1\ 2 & 4\ 8\ 0\ 3 \\ & \underline{4\ 5\ 6\ 0} \\ & 2\ 4\ 3 \end{array} \Rightarrow 5 \text{ r } 243$$

(b) Repeated subtraction:

$$\begin{array}{r|l} 1\ 1 & 1\ 0\ 1\ 1 \\ & \underline{9\ 9\ 0} \quad 90\text{-}1\,1\text{'s} \\ & 2\ 1 \\ & \underline{1\ 1} \quad 1 - 11 \\ & 1\ 0 \quad 91\text{-}1\,1\text{'s} \end{array} \Rightarrow 91 \text{ r } 10$$

Traditional:

$$\begin{array}{r|l} & \ 9\ 1 \\ 1\ 1 & 1\ 0\ 1\ 1 \\ & \underline{9\ 9\ 0} \\ & 2\ 1 \\ & \underline{1\ 1} \\ & 1\ 0 \end{array} \Rightarrow 91 \text{ r } 10$$

(c) Repeated subtraction:

$$\begin{array}{r|l} 2\ 3_{five} & 3\ 3\ 1\ 2_{five} \\ & \underline{2\ 3\ 0\ 0} \quad (100\text{-}23\text{'s})_{five} \\ & 1\ 0\ 1\ 2 \\ & \underline{1\ 0\ 1\ 0} \quad (20\text{-}23\text{'s})_{five} \\ & 2_{five} \quad (120\text{-}23\text{'s})_{five} \end{array}$$

$\Rightarrow 120_{five}$ remainder 2_{five}

Traditional:

$$\begin{array}{r|l} & \phantom{3\ 3\ 1\ 2_{five}}\ 1\ 2\ 0_{five} \\ 2\ 3_{five} & 3\ 3\ 1\ 2_{five} \\ & \underline{2\ 3} \\ & 1\ 0\ 1 \\ & \underline{1\ 0\ 1} \\ & 0\ 2_{five} \end{array}$$

$\Rightarrow 120_{five}$ remainder 2_{five}

(d) Repeated subtraction:

$$\begin{array}{r|l} 1\ 1_{two} & 1\ 0\ 1\ 1_{two} \\ & \underline{1\ 1\ 0} \quad (10\text{-}11\text{'s})_{two} \\ & 1\ 0\ 1 \\ & \underline{1\ 1} \quad (1\text{-}11)_{two} \\ & 1\ 0_{two} \quad (11\text{-}11\text{'s})_{two} \end{array}$$

$\Rightarrow 11_{two}$ remainder 10_{two}

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(d) Traditional:

$$\begin{array}{r} \quad\;\; 1\;\; 1_{two} \\ 1\;\;1_{two}\;\overline{)\,1\;\;0\;\;1\;\;1_{two}} \\ \quad\;\; 1\;\;1 \\ \hline \quad\;\; 1\;\;0\;\;\;1 \\ \quad\;\;\;\; 1\;\;1 \\ \hline \quad\;\; 1\;\;\;0\;\; two \end{array}$$

$\Rightarrow 11_{two}$ remainder 10_{two}

29. **(a)** If $4803 \div 912 = 5$ remainder 243
Then $912 \cdot 5 + 243 = 4803$.

(b) If $1011 \div 11 = 91$ remainder 10 Then $11 \cdot 91 + 10 = 1011$.

(c) If $(3312 \div 23)_{five} = (120 \text{ remainder } 2)_{five}$
Then $(23 \cdot 120)_{five} + 2_{five} = 3312_{five}$.

(d) If $(1011 \div 11)_{two} = (11 \text{ remainder } 10)_{two}$
Then $(11 \cdot 11)_{two} + 10_{two} = 1011_{two}$.

30. **(a)** $19 \cdot 5 \cdot 194 \cdot 2 = (19 \cdot 194) \cdot 10 = \mathbf{36{,}860}$.

(b) $379 \cdot 4 \cdot 193 \cdot 25 = (379 \cdot 193) \cdot 100 = \mathbf{7{,}314{,}700}$.

(c) $8 \cdot 481 \cdot 73 \cdot 125 = (481 \cdot 73) \cdot 1000 = \mathbf{35{,}113{,}000}$.

(d) $374 \cdot 200 \cdot 893 \cdot 50 = (374 \cdot 893) \cdot 10{,}000 = \mathbf{3{,}339{,}820{,}000}$.

31. (320×6 mos) + (410×6 mos) = **$4380**.

32. 15,600 cans per hour $\div$ 24 cans per case = 650 cases per hour. Then 650×4 hours = **2600 cases**.

33. There are 8 groups of 3 in 24 (or two dozen) apples. $8 \times 69¢$ per group $= 552¢$ on sale;
$32¢$ each $\times$ 24 apples $= 768¢$ regular price.
$768¢ - 552¢ =$ **216¢ saved** (or $2.16).

34. Let b be the number of bicycles and t be the number of tricycles. Then

$2b + 3t = 126$ wheels $\quad (i)$
$2b + 2t = 108$ pedals $\quad (ii)$

Subtracting (ii) from $(i) \Rightarrow t =$ **18 tricycles**.

Substituting $t = 18$ into $(i) \Rightarrow b =$ **36 bicycles**.

35. **(a)**
$$\begin{array}{r} \;\;1\;\;1 \\ 1\;\;2\;\;3_{five} \\ +\;\;3\;\;4_{five} \\ \hline \mathbf{2\;\;1\;\;2}_{five} \end{array}$$

(b)
$$\begin{array}{r} \quad 10\;\;\;0\;\;\;10 \\ \not{1}\;\;0\;\;\not{1}\;\;0_{two} \\ -\;\;1\;\;0\;\;1_{two} \\ \hline \quad\mathbf{1\;\;0\;\;1}_{two} \end{array}$$

(c)
$$\begin{array}{r} \quad\;\;2\;\;3_{five} \\ \times\;\;3\;\;4_{five} \\ \hline \quad 2\;\;0\;\;2 \\ 1\;\;2\;\;4 \\ \hline \mathbf{1\;\;4\;\;4\;\;2}_{five} \end{array}$$

(d)
$$\begin{array}{r} \quad\;\;1\;\;0\;\;0\;\;1_{two} \\ \times\;\;1\;\;0\;\;1_{two} \\ \hline \quad 1\;\;0\;\;0\;\;1 \\ 0\;\;0\;\;0\;\;0 \\ 1\;\;0\;\;0\;\;1 \\ \hline \mathbf{1\;\;0\;\;1\;\;1\;\;0\;\;1}_{two} \end{array}$$

36. $44_{five} \cdot 34_{five} = (4 \cdot 10_{five} + 4) \cdot 34_{five}$
$= 4 \cdot 34_{five} \cdot 10_{five} + 4 \cdot 34_{five}$
$= 3010_{five} + 301_{five}$
$= \mathbf{3311}_{five}$

37.
$$\begin{array}{l} 4_{five}\overline{)\;434_{five}} \\ \underline{-\;400_{five}} \quad (100 \cdot 4)_{five} \\ \quad\;\; 34_{five} \\ \underline{-\;\;31_{five}} \quad (4 \cdot 4)_{five} \\ \quad\;\;\; 3_{five} \end{array}$$

Thus, $\mathbf{434}_{five} = \mathbf{104}_{five} \cdot \mathbf{4}_{five} + \mathbf{3}_{five}$

38. Answers may vary; for example:

(a) $(26 + 24) + (37 - 7) = 50 + 30 =$ **80**.

(b) $(7 \cdot 9) \cdot (4 \cdot 25) = 63 \cdot 100 =$ **6300**.

39. Methods may vary; for example:

(a) $63 \cdot 7 = (7 \cdot 60) + (7 \cdot 3) = 420 + 21 =$ **441** (front-end multiplying).

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(b) $85 - 49 = (85 + 1) - (49 + 1) = 86 - 50 =$ **36** (trading off).

(c) $(18 \cdot 5) \cdot 2 = 18 \cdot (5 \cdot 2) = 18 \cdot 10 =$ **180** (using compatible numbers).

(d) $2436 \div 6 = (2400 \div 6) + (36 \div 6) =$ $400 + 6 =$ **406** (breaking up the dividend).

40. Answers may vary; for example:

(a) Front-end: $5 + 3 + 2 + 4 + 9 = 23$; place value 2300; adjustments $40 + 100 + 60 +$ $0 + 100 = 300$; adjusted sum $= 2300 +$ $300 = 2600$.

(b) Rounding: $500 + 400 + 300 + 400 +$ $1000 = 2600$.

In this case, both estimates give the same result (the actual sum is 2602, so both are reasonable).

41. The addends cluster around 2400, so one would estimate the sum to be $4 \cdot 2400 =$ **9600**.

42. **(a)** $999 \cdot 47 + 47 = 47(999 + 1) =$ $47 \cdot 1000 =$ **47,000**.

(b) $43 \cdot 59 + 41 \cdot 43 = 43(59 + 41) =$ $43 \cdot 100 =$ **4300**.

(c) $1003 \cdot 79 - 3 \cdot 79 = 79(1003 - 3) =$ $79 \cdot 1000 =$ **79,000**.

(d) $1001 \cdot 113 - 113 = 113(1001 - 1) =$ $113 \cdot 1000 =$ **113,000**.

(e) $101 \cdot 35 = (100 + 1) \cdot 35 = 35 \cdot 100 +$ $35 \cdot 1 = 3500 + 35 =$ **3535**.

(f) $98 \cdot 35 = (100 - 2) \cdot 35 = 35 \cdot 100 -$ $35 \cdot 2 = 3500 - 70 =$ **3430**.

43. **(a)**

$$\begin{array}{rccccccc} & 3x^3 & + & 4x^2 & + & 7x & + & 8 \\ + & & & 5x^2 & + & 2x & + & 1 \\ \hline & 3x^3 & + & 9x^2 & + & 9x & + & 9 \end{array}$$

(b) Answers may vary. For example:

$$\begin{array}{rccccccc} & 3\cdot10^3 & + & 5\cdot10^2 & + & 7\cdot10 & + & 8 \\ - & & & 4\cdot10^2 & + & 2\cdot10 & + & 1 \\ \hline & 3\cdot10^3 & + & 1\cdot10^2 & + & 5\cdot10 & + & 7 \end{array}$$

Is equivalent (when $x = 10$) to:

$$\begin{array}{rccccccc} & 3x^3 & + & 5x^2 & + & 7x & + & 8 \\ - & & & 4x^2 & + & 2x & + & 1 \\ \hline & 3x^3 & + & x^2 & + & 5x & + & 7 \end{array}$$

(c) Answers may vary. For example:

$25 \cdot 10^2 = (2 \cdot 10 + 5) \cdot 10^2 = 2 \cdot 10^3 +$ $5 \cdot 10^2 + 0 \cdot 10 + 0 = 2500$.

Is equivalent to:

$(2x + 5)x^2 = 2x^3 + 5x^2$.

44.

Suppose $a, b \in B$. Then $a = 5j$ and $b = 5k$, where $j, k \in W$. Therefore $a + b = 5j + 5k = 5(j + k)$, where $(j + k) \in W$. Therefore $a + b \in B$ and B is closed under addition.

45. After 0 seconds 1 person knew (the principal).

After 30 seconds the principal had called 1 person (2 members notified).

After 60 seconds the principal and one member called 2 other members (4 members notified).

After 90 seconds the one member and the 2 new members called another 3 members (7 members notified).

Continuing this pattern:

After 120 seconds 12 members notified.

After 150 seconds 20 members notified.

After 180 seconds 33 members notified.

After 210 seconds 54 members notified.

After 240 seconds 88 members notified.

It took 240 seconds = 4 minutes to notify all 85 members.

46. She started with 124 cookies and there were 7 left so the children took 124 – 7= 117. A number between 10 and 30 that evenly divides into 117 is 13. $117 \div 13 = 9$. There were 13 children who each took 9 cookies.

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47. In the list of digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 there are three digits that read the same upside down: 0, 1, and 8. A 6 reads as 9 upside down and 9 reads as 6 upside down. So the five digits that are possible are 0, 1, 6, 8, and 9.

The difference of the first digit in the upside down number (call this number B) and the first number in the real license plate number (call this number A) is 7. Using the list of possible digits only

$8 - 1 = 7$ works.

This gives us:

$$\begin{array}{r} 8\_,\_\_1\,(\text{B}) \\ -\underline{1\_,\_\_8}\,(\text{A}) \\ = 78,633 \end{array}$$

In order for the last digit in B (1) minus the last digit in A (8) to be 1, a ten was added to the 1 to make 11 units to subtract 8 and get 3. In order for the difference between the second to last digit in B and the second last digit in A to be 3 and using the 3 left over digits 0, 6, 9 one can only use 0 and 6 because using 9-6 would not allow for the ten to be added to the 1in B.

This gives us:

$$\begin{array}{r} 89,\_01\ (\text{B}) \\ -\underline{10,\_68}\ (\text{A}) \\ = 78,633 \end{array}$$

The only digit not used in the top is 6. Placing the numbers into the five digit numbers:

$$\begin{array}{r} 89,601\ (\text{B}) \\ -\underline{10,968}\ (\text{A})\ . \\ = 78,633 \end{array}$$

The real license plate is 10,968.

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CHAPTER 4

NUMBER THEORY

Assessment 4-1A: Divisibility

1. Odd. For example: 2 times 2 plus 1 is 5 which is odd.

2. (a) **False; there is no value $c \in W$ such that $24c = 5$.**

(b) True; $30 \div 10 = 3$.

(c) False; there is no value $c \in W$ such that $8c = 324$.

(d) False; there is no value $c \in W$ such that $0c = 24$.

3. Use these tests for each number:

(*i*) $2|n$ if the units digit is divisible by 2.

(*ii*) $3|n$ if the sum of the digits is divisible by 3.

(*iii*) $4|n$ if the last two digits are divisible by 4.

(*iv*) $5|n$ if the units digit is 0 or 5.

(*v*) $6|n$ if $2|n$ and $3|n$.

(*vi*) $8|n$ if the last three digits are divisible by 8.

(*vii*) $9|n$ if the sum of the digits is divisible by 9.

(*viii*) $10|n$ if the units digit is 0.

(*ix*) $11|n$ if the sum of the digits in places that are even powers of 10 minus the sum of the digits in the places that are odd powers of 10 is divisible by 11.

		2	3	4	5	6	8	9	10	11
(a)	4,201,012	Y	N	Y	N	N	N	N	N	N
(b)	1573	N	N	N	N	N	N	N	N	Y
(c)	15,810	Y	Y	N	Y	Y	N	N	Y	N

4. (a) **Yes**. The question is really, "does $9|1379$?" Using the divisibility test for 9, $9 \nmid (1 + 3 + 7 + 9)$ so $9 \nmid 1379$ and there will be a remainder; i.e., a group of less than 9 players.

(b) **No**. The question is really, "does $11|1354$?" Using the divisibility test for 11, $11 \nmid [(1 + 5) - (3 + 4)]$ so $11 \nmid 1354$ and there will be a remainder; i.e. trees left over.

(c) **Yes**. The question is really, "does $8|1216$?" Using the divisibility test for 8, $8 \mid 216$ so there will be no remainder.

5. (a) **Any digit from 0 to 9**. The units digit of $1,427,4\square 2$ is divisible by 2.

(b) **1, 4, or 7**. $3|(1 + 4 + 2 + 7 + 4 + \square + 2) \Rightarrow 3|(20 + \square) \Rightarrow \square = 1, 4, \text{or } 7$.

(c) **1, 3, 5, 7, or 9**. $4|\square 2 \Rightarrow \square = 1, 3, 5, 7, \text{or } 9$.

(d) **7**. $9|(1 + 4 + 2 + 7 + 4 + \square + 2) \Rightarrow 9|(20 + \square) \Rightarrow \square = 7$.

6. (a) $3|747$. $3|(7 + 4 + 7 = 18)$. The _ could be filled by 1, 4, or 7, but 7 is the greatest.

(b) $9|83745$. $9|(8 + 3 + 7 + 4 + 5 = 27)$. Only 7 makes them viable.

(c) $11|6655$. $11|(6 + 5) - (6 + 5)$. 6_55 in this case must result in the sum of the digits in places that are even powers of 10 being equal to the sum of the digits in places that are odd powers of 10.

(d) $5|1365$. The _ could be filled in with 0 or 5, but 5 is the greatest.

(e) $6|2844$. $2|2\_44$ and $3|(2+\_+4+4) \Rightarrow 3|(10+\_)$ so _ could be 2,5,8, but 8 is the greatest.

(f) $8|3864$. The _ could be filled in with 2 or 6, but 6 is the greatest.

7. (a) $7|280$ because $280 = 7 \cdot 40$ (definition of division).

(b) $19|(3800 + 19)|$ because $19|3800$ and $19\ |19$. [Theorem 4-2(a)].

(c) $15|(2^4 \cdot 3^5 \cdot 5)$ because $2^4 \cdot 3^5 \cdot 5 = (3 \cdot 5) \cdot 2^4 \cdot 3^4$ (definition of division).

(d) $19 \nmid (3800 + 37)$ because $19|3800$ but $19 \nmid 37$ [Theorem 4-2(b)].

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8. **(a)** **True**. Break into parts of compatible number: $390026 = 390000 + 26 = 13(30000) + 13(2) = 13(30002)$.

(b) **True**. By Theorem 4-2(b), 13 | 260000 and $13 \nmid 33$. Thus $13 \nmid (260000 + 33)$.

(c) **False.** Because 17 | 34,000 and $17 \nmid 15$; 17 does not divide the sum.

(d) **True**. Because 17 | 34,000 and 17 | 15; 17 divides the sum.

(e) **False.** Because 19 | 19,000 and $19 \nmid 31$; 19 does not divide the sum.

(f) **True**. $93^{11} = 93 \cdot 93^{10} = 31 \cdot 3 \cdot 93^{10} = 31(3 \cdot 93^{10})$. Therefore, since $3 \cdot 93^{10} \in W$, there exist a $g \in W$ such that $93^{11} = 31(g)$.

9. **(a)** **True** by Theorem 4-1. For any whole numbers a and d, if $d \mid a$ and n is any whole number, then $d \mid na.$. So if 7 | 21, then 7 | (21•21) or $7 \mid 21^2$.

(b) **False**. Let $a = 4, b = 2,$ and $c = 3$. Two divides 4 $(b \mid a)$ and $(2 + 3)$ does not divide $(4 + 3), (b + c) \nmid (a + c)$.

(c) **True**. Because $b \mid a,$ there is a whole number c such that $a = bc$. Then, $a^3 = b^3c^3 = (bc^3)b^2,$ therefore $b^2 \mid a^3$.

(d) **True**. Because $b \mid a,$ there exist $c \in W$ such that $a = bc$. Thus, $a + b = bc + b = b(c + 1)$. Since $c + 1 \in W,$ b divides $a + b,$ written $b \mid (a + b)$.

10. **(a)** A number divisible by 2 and 3 but not by 5 has to end on 2, 4, 6, or 8 and the sum of its digits has to be divisible by 3. For example 6.

(b) A number divisible by 2 and 4 but not by 8 has to end on 0, 2, 4, 6, or 8 and the last two digits are divisible by 4. However, the last three digits cannot be divisible by 8. For example 12.

(c) A number divisible by 5 and t10 has to end in 0 but if it ends in 0, then it is divisible by 2. Such a number does not exist.

11. **19¢.** $209 = 11 \cdot 19$ (both prime numbers). For whole cent pricing $1¢, 11¢, 19¢,$ and $209¢$ are possibilities, but the problem statement is for pencils (plural) and the pencils must cost more than $12¢$.

12. **(a)** **True**. For example $3 \mid (1 \cdot 2 \cdot 3)$. In general for three consecutive numbers n, $n + 1$, $n + 2$ every other number is even, at least one (or two) of the three numbers n, $n + 1$, $n + 2$ will be even. Since every third number is divisible by three, exactly one of n, $n + 1$, $n + 2$ will have a factor of three. So in $n \cdot (n+1) \cdot (n+2)$ there will be at least one factor of two and exactly one factor of three.

(b) **False.** For example $4 \nmid (5 \cdot 6)$ the product of two consecutive numbers is divisible by four if one of the numbers is divisible by 4.

13. There are 11 ways: 99911, 99731, 99551, 99533, 97751, 97733, 97553, 95555, 77771, 77753, 77555.

14. **(a)** **True**. Let $n = 3(a \cdot 10^k + b \cdot 10^{k-1} + \cdots + x \cdot 10^0)$, where $3a,$ $3b,$ $\ldots 3x$ are all single digits. Then every digit is divisible by 3, and the number itself is divisible by 3.

(b) **False**. Twelve is divisible by 3 but its digits $\{1, 2\}$ are not divisible by 3.

(c) **False**. This is an "if and only if" statement. For it to be true, both (a) and (b) must be true. Since (b) is false, this statement is false.

15. (a)

$$
\begin{array}{r|l}
2414271 & 7 + 2 + 4 + 2 + 8 + 1 + 5 = 29 \\
3\overline{)7242815} & \\
-6000000 & \\
\hline
1242815 & \\
-1200000 & \\
\hline
42815 & \\
-30000 & \\
\hline
12815 & \\
-12000 & \\
\hline
815 & \\
-600 & \\
\hline
215 & \\
-210 & \\
\hline
5 & \\
-3 & \\
\hline
\mathbf{2} &
\end{array}
\qquad
\begin{array}{r}
9 \\
3\overline{)\ 29} \\
-27 \\
\hline
\mathbf{2}
\end{array}
$$

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(b) $7242815 = 7 \cdot 1{,}000{,}000 + 2 \cdot 100{,}000 + 4 \cdot 10{,}000 + 2 \cdot 1{,}000 + 8 \cdot 100 + 1 \cdot 10 + 5$
$= 7(999{,}999 + 1) + 2(99{,}999 + 1) + 4(9{,}999 + 1) + 2(999 + 1) + 8(99 + 1) + 1(9 + 1) + 5$
$= 7 \cdot 999{,}999 + 2 \cdot 99{,}999 + 4 \cdot 9{,}999 + 2 \cdot 999 + 8 \cdot 99 + 1 \cdot 9 + 7 + 2 + 4 + 2 + 8 + 1 + 5$

Because a 3 can be factored out of the first six terms, which is equivalent to dividing by 3, applying the division algorithm to the sum of the digits at the end will yield the reminders.

(c) **Yes.** The key idea in (b) was that each place value greater than 1(10°) can be written as the digit times 9…9 plus 1, for example $d \cdot (9{,}999 + 1) = d \cdot 10^4$. Thus, any number can be rewritten as 9 times some whole number plus the sum of the digits.

16. 1, 3, 5, 9, and 15 divide n. 1 divides every number.

Because $n = 45 \cdot d$, $d \in W$, then $n = (3 \cdot 15)\, d$ which implies n is divisible by 3 and 15. And, $n = (5 \cdot 9)d$ which implies n is divisible by 5 and 9.

17. **16|*n* if 16|(last four digits of *n*)**. This continues the pattern of divisibility by 2, 4, and 8.

18. The term "casting out nines" implies that any 9 or sum of digits equaling 9 in n may be "cast out." The remaining digit is the remainder when n is divided by 9. Then the remainder when n is divided by 9 is the same as the remainder when the sum of the digits of n is divided by 9 ("casting out" as needed).

(a) ***(i)*** $12{,}343 + 4546 + 56 = 16{,}945.$

(ii) $12343 = 9 \cdot 1371 + 4$
$4546 = 9 \cdot 505 + 1$
$56 = 9 \cdot 6 + 2$
and $4 + 1 + 2 = 7.$

(iii) $16945 = 9 \cdot 1882 + 7.$

(b) ***(i)*** $987 + 456 + 8765 = 10{,}208.$

(ii) $987 = 9 \cdot 109 + 6$
$456 = 9 \cdot 50 + 6$
$8765 = 9 \cdot 973 + 8$
and $6 + 6 + 8 = 20$; $20 = 9 \cdot 2 + 2.$

(iii) $10208 = 9 \cdot 1134 + 2.$

(c) $1003 - 46 = 957.1003$ has a remainder of 4 when divided by 9; 46 has a remainder of 1 when divided by 9; $4 - 1 = 3$ has a remainder of 3 when divided by 9. $9 + 5 + 7 = 21$ with a remainder of 3 when divided by 9.

(d) $345 \cdot 56 = 19{,}320.$ 345 has a remainder of 3 when divided by 9; 56 has a remainder of 2 when divided by 9; $3 \cdot 2 = 6$ has a remainder of 6 when divided by 9. $1 + 9 + 3 + 2 + 0 = 15$ has a remainder of 6 when divided by 9.

(e) Answers may vary. The division may not have a whole number quotient, in which case the test fails.

19. **(a)** **False**. The palindrome 12121 is not divisible by 11.

(b) **True.** Let a, b, and c be digits. All six digit palindromes can be written in the form *abccba*. The sum of the digits in places with even power is $b + c + a$. The sum of the digits in places with odd powers is $a + c + b$. This difference is zero, which is divisible by 11.

Alternative: *abccba*

$= a \cdot 10^5 + b \cdot 10^4 + c \cdot 10^3 + c \cdot 10^2 + b \cdot 10 + a \cdot 1$
$= (a \cdot 10^5 + a) + (b \cdot 10^4 + b \cdot 10) + (c \cdot 10^3 + c \cdot 10^2)$
$= a \cdot 100{,}001 + b \cdot 10{,}010 + c \cdot 1{,}100$
$= 11(a \cdot 9091 + b \cdot 910 + c \cdot 100).$

20. Any five-digit number may be written as
$a \cdot 10^4 + b \cdot 10^3 + c \cdot 10^2 + d \cdot 10 + e =$
$a(9999 + 1) + b(999 + 1) + c(99 + 1)$
$+ d(9 + 1) + e = (9999a + 999b + 99c + 9d)$
$+ (a + b + c + d + e).$

The first group is divisible by 9 [Theorem 4-2(a)]. Then the five-digit number is divisible by 9 if and only if the second group is divisible by 9 [Theorem 4-2(b)]; i.e., if the sum of the digits is divisible by 9.

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Assessment 4-1B

1. Odd. For example: 3 times 2 plus 1 is 7 which is odd.

2. **(a)** **True**. 5 is a factor of 20.

(b) **False**. There is no value $c \in W$ such that $20c = 10$.

(c) **True**. $20 \div 5 = 4$.

(d) **False**. There is no value $c \in W$ such that $30c = 6$.

3. Use these tests for each number:

(i) $2|n$ if the units digit is divisible by 2.

(ii) $3|n$ if the sum of the digits is divisible by 3.

(iii) $4|n$ if the last two digits are divisible by 4.

(iv) $5|n$ if the units digit is 0 or 5.

(v) $6|n$ if $2|n$ and $3|n$.

(vi) $8|n$ if the last three digits are divisible by 8.

(vii) $9|n$ if the sum of the digits is divisible by 9.

(viii) $10|n$ if the units digit is 0.

(ix) $11|n$ if the sum of the digits in places that are even powers of 10 minus the sum of the digits in the places that are odd powers of 10 is divisible by 11.

		2	3	4	5	6	8	9	10	11
(a)	7163	N	N	N	N	N	N	N	N	N
(b)	34,200	Y	Y	Y	Y	Y	Y	Y	Y	N
(c)	199,990	Y	N	N	Y	N	N	N	Y	N

4. **(a)** **Yes**. 9 | 1,111,500 because 9 | (1 + 1 + 1 + 1 + 5 + 0 + 0), 9 | (9).

(b) **Yes**. 4 | 548 because 4 | 48.

(c) **No**. $12 \nmid 26{,}600$.

5. **(a)** Since $1{,}182{,}1\square 6$ will always be even, **any digit** 0-9 can be placed in the square.

(b) Represent the digit to be place in the square by x.
$1 + 1 + 8 + 2 + 1 + x + 6 = 19 + x$.
$19 + x$ is divisible by 3 if and only if $x =$ **2, 5, or 8.**

(c) For $1{,}182{,}1\square 6$ to be divisible by 4, $\square 6$ must be divisible by 4. This happens if and only if **1, 3, 5, 7, or 9** are placed in the square.

(d) Considering 5(b), $19 + x$ must be divisible by 9. Therefore, $x =$ **8**.

6. **(a)** **True.** Because $5 \mid 2 \cdot 3 \cdot 5 \cdot 7$ because $5 \mid 5$.

(b) False. $5 \mid 2 \cdot 3 \cdot 5 \cdot 7$, but $5 \nmid 1$, so $5 \nmid [(2 \cdot 3 \cdot 5 \cdot 7) + 1]$.

(c) **True**. $6 \mid (2 \cdot 3) \cdot 2^2 \cdot 3^2 \cdot 17^4$ because $6 \mid (2 \cdot 3)$.

(d) True. $7 \mid 4200$ but $7 \nmid 22$ so $7 \nmid 4222$.

7. **(a)** $26|(13^4 \cdot 100)$ because $13^4 \cdot 100 = 13 \cdot 2 \cdot 13^3 \cdot 50 =$ **$26(13^3 \cdot 50)$** [Theorem 4-1].

(b) $13 \nmid (2^4 \cdot 5^3 \cdot 26 + 1)$ because **$13|(2^4 \cdot 5^3 \cdot 26)$** but **$13 \nmid 1$** [Theorem 4-2 (b)].

(c) $2^4 \nmid (2 \cdot 4 \cdot 6 \cdot 8 \cdot 17^{10} + 1)$ because $2^4 | (1 \cdot 2 \cdot 3 \cdot 4) \cdot 2^4 \cdot 17^{10}$ but **$2^4 \nmid 1$** [Theorem 4-2(b)].

(d) $2^4 | (10^4 + 6^4)$ because **$2^4 | 10^4 = 2^4 \cdot 5^4$** and **$2^4 | 6^4 = 2^4 \cdot 3^4$** [Theorem 4-2 (a)].

8. **(a)** **False.** Because 12 | 24,000 and $12 \nmid 13$; 12 does not divide the sum.

(b) **True**. Because 12 | 24,000 and 12 | 36; 12 divides the sum.

(c) **False**. Because 19 | 38,000 and 19 37; 19 does not divide the sum.

(d) **True**. Because 19 | 3,800,000 and $19 \nmid 18$; 19 does not divide the sum.

(e) **False.** Because 23 | 23,000 and 23 | 23; 23 divides the sum.

(f) **True**. Because 23 | 46; $23 \mid 46 \cdot 46^{10}$.

9. **(a)** **True** by Theorem 4-2(b). If $d \mid a$ and $d \nmid b$, then $d \nmid (a + b)$.

(b) **False.** 3 | (50 • 30) because 3 | 30 so 3 will divide any multiple of 30 by Theorem 4-1. if $d \mid a$ and n is any whole number, then $d \mid na$..

(c) **True** by Theorem 4-2(b). Consider $a + b$ a single whole number.

(d) **True** by Theorem 4-1.

10. **(a)** **No.** 12 | 24,000 and $12 \nmid 13$, so $12 \nmid 24{,}013$.

(b) **Yes**. 12 | 24,000 and 12 | 36, so 12 | 24,036.

(c) **Yes**. 17 | 17,000 and 17 | 34, so 17 | 17,034.

(d) **Yes**. 3 is a factor of $2 \cdot 3 \cdot 5 \cdot 7$.

(e) **No**. $6 \mid (2 \cdot 3 \cdot 5 \cdot 7)$ and $6 \nmid 1$, so $6 \nmid (2 \cdot 3 \cdot 5 \cdot 7) + 1$.

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11. The number of notepads sold must be between 16 (if the price was close to \$2) and 31 (if the price was close to \$l) and 17|3145. **17 notepads** at \$1.85 each totals \$31.45.

12. **(a)** True. For example 3 | (1+2+3). In general $3 \mid (x + (x+1) + (x+2)) \Rightarrow 3 \mid (3x + 3)$ because $3 \mid 3x$ and $3 \mid 3$.

(b) False. For example $4 \nmid (3 + 4 + 5 + 6)$. In general $4 \nmid (x + (x+1) + (x+2) + (x+3)) \Rightarrow 4 \Rightarrow (4x + 6)$ because $4 \nmid 6$.

13. **(a)** **1, 2, 4, 5, 8, and 11**. None are the result of combinations divisible by 3 or 7.

(b) **1 touchdown and 11 field goals or 4 touchdowns and 4 field goals**. Consider 0 touchdowns: $3 \nmid 40$; impossible. Consider 2 touchdowns: $3 \nmid (40 - 14)$; impossible. Other situations follow.

(c) **5 Field goals**. 6 touchdowns $\cdot 7$ points each $= 42$ points; $57 - 42 = 15$; and $15 \div 3 = 5$ field goals.

14. **(a)** **True**. Any number is divisible by 6 if and only if it is divisible by 2 and by 3.

(b) **True**. $6 = 2 \cdot 3$.

(c) **False** E.g., 20. The difference between (c) and (b) is that 2 and 3 have no common factors, while 2 and 4 do.

(d) **True**. $8 = 2 \cdot 4$.

(e) **False.** Consider the number 4. 2|4 and 4|4, but $8 \nmid 4$. Alternative: 2|12 and 4|12, but $8 \nmid 12$.

15.

	n	Remainder when n is divided by 9	Sum of the digits of n	Remainder when the sum of the digits of n is divided by 9
(a)	31	4	4	4
(b)	143	8	8	8
(c)	345	3	12	3
(d)	2987	8	26	8
(e)	7652	2	20	2

(f) The remainder when n is divided by 9 seems to be equal to the remainder when the sum of the digits of n is divided by 9.

16. If $28|n$, then $n = 28k$, where k is a whole number. Thus, $n = 2^2 \cdot 7 \cdot k$. This shows that 1, 2, 2^2(=4), $2 \cdot 7$ (=14), and 7 also divide n.

17. **25|*n* if 25|(last two digits of *n*)**; i.e., if n ends in 00, 25, 50, or 75.

18. **(a)** The sum of the digits in $99 + 28$ is $9 + 9 + 2 + 8$, which leaves remainder 1 upon division by 9. However, 227 leaves the same remainder as $2 + 2 + 7$, i.e., 2 upon division by 9.

(b) 11,199 – 21 leaves remainder 3 – 3 or 0 upon division by 9 (using the casting out technique). However, 11,168 has remainder 8, when divided by 9.

(c) $99 \cdot 26$ divided by 9 has remainder 0, while 2575 divided by 9 has remainder 1.

19. **(a)** ***(i)*** $11|[(4 + 5) - (5 + 4)] \Rightarrow$ **11|4554**.

(ii) $11|[(9 + 3) - (3 + 9)] \Rightarrow$ **11|9339**.

(iii) $11|[(2 + 0) - (0 + 2)] \Rightarrow$ **11|2002**.

(iv) $11|[(2 + 2) - (2 + 2)] \Rightarrow$ **11|2222**.

(b) **Yes**. By definition, all four-digit palindromes are of the form $a \cdot 10^3 + b \cdot 10^2 + b \cdot 10 + a$ for whole numbers a and b (where a and b may be equal). The divisibility test for 11 shows that n is divisible by 11 if $11|[(a + b) - (b + a)]$, or $11|0$.

20. Let *abcd* represent any four-digit number.

$$\begin{aligned} abcd &= a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d \\ &= a(999 + 1) + b(99 + 1) \\ &\quad + c(9 + 1) + \mathrm{d} \\ &= a \cdot 999 + b \cdot 99 + c \cdot 9 \\ &\quad + a + b + c + d \\ &= 3(a \cdot 333 + b \cdot 33 + c \cdot 3) \\ &\quad + (a + b + c + d) \end{aligned}$$

Since 3 divides $a \cdot 999 + b \cdot 99 + c \cdot 9 + d$, if 3 divides $(a + b + c + d)$, then 3 divides *abcd*. If 3 divides *abcd*, since 3 divides $a \cdot 999 + b \cdot 99 + c \cdot 9$, then 3 must divide $(a + b + c + d)$.

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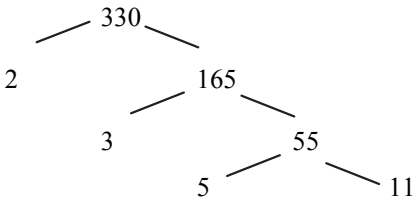
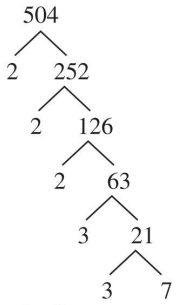
Assessment 4-2A: Prime and Composite Numbers

1. (a)



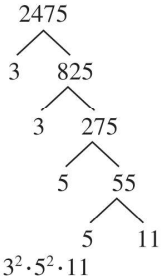
(b) $2 \cdot 3 \cdot 5 \cdot 11 = 330$

2. (a)



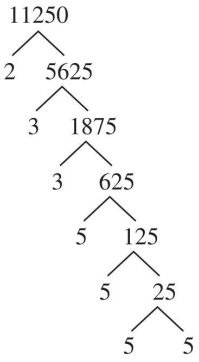
$504 = 2^3 \cdot 3^2 \cdot 7$

(b)



$2475 = 3^2 \cdot 5^2 \cdot 11$

(c)



$11250 = 2 \cdot 3^2 \cdot 5^4$

3. (a) $1 \cdot 2 \cdot 3 \cdot 2^2 \cdot 5 \cdot (2 \cdot 3) \cdot 7 \cdot 2^3 \cdot 3^2$
$= \mathbf{2^7 \cdot 3^4 \cdot 5 \cdot 7}.$

(b) $(2^2 \cdot 5^2) \cdot (13) \cdot (7^2)^{10} =$
$\mathbf{2^2 \cdot 5^2 \cdot 7^{20} \cdot 13}.$

(c) 251 is prime, so its factorization is **251**.

(d) $100^{10} = (2^2 \cdot 5^2)^{10} = 2^{20} \cdot 5^{20}$.

4. 23. $23^2 < 769$, but $29^2 > 769$ (23 is prime and 29 is the next largest prime).

5. (a) **Prime**. Fails divisibility test for primes up to 7; no need to test for primes >7 because $11^2 > 103$.

(b) **Not prime**. $7 \cdot 17 = 119$.

(c) **Prime**. Fails divisibility test for primes up to 5.

(d) **Not prime.** $101 \cdot 3 = 303$.

(e) **Prime.** Fails divisibility test for primes up to 19; no need to test for primes >19.

(f) **Prime.** Fails divisibility test for primes up to 7.

(g) **Prime.** $2 \cdot 3 \cdot 5 \cdot 7 + 1 = 211$, which fails divisibility test for primes up to 13.

(h) **Not prime**. $2 \cdot 3 \cdot 5 \cdot 7 + 11 = 221 = 13 \cdot 17$.

6. The three smallest primes are 2, 3, and 5, so the least number divisible by three different primes is $2 \cdot 3 \cdot 5 = \mathbf{30}$.

7. Since 2 is the least prime, 2^k for some k must be our answer. If $k = 4$, then $\mathbf{2^4 = 16}$ has factors 1, 2, 4, 8, and 16. But this is a two digit number. However, $5^4 = \mathbf{625}$ has five factors 1, 5, 25, 125, and 625.

8. (a) The Fundamental Theorem of Arithmetic tells us that each composite number, n, can be written as a product of primes in one and only one way. Since $2|n$ and $3|n$, we know that 2 and 3 must both occur in ns prime decomposition. Therefore, $6|n$.

(b) **True.** If $a \mid n$ and $b \mid n$ then there exist natural numbers k and l such that $n = ak$ and $n = bl$. Thus, $n^2 = ak \cdot bl = ab \cdot kl$.

9. (a) $36^{10} \cdot 49^{20} \cdot 6^{15}$
$= (2^2 \cdot 3^2)^{10} \cdot (7^2)^{20} \cdot (2 \cdot 3)^{15}$
$= 2^{20} \cdot 3^{20} \cdot 7^{40} \cdot 2^{15} \cdot 3^{15}$
$= \mathbf{2^{35} \cdot 3^{35} \cdot 7^{40}}.$

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(b) $100^{60} \cdot 300^{40} = (2^2 \cdot 5^2)^{60} \cdot (2^2 \cdot 3 \cdot 5^2)^{40}$
$= 2^{120} \cdot 5^{120} \cdot 2^{80} \cdot 3^{40} \cdot 5^{80}$
$= \mathbf{2^{200} \cdot 3^{40} \cdot 5^{200}}$.

(c) $(2 \cdot 3^4 \cdot 5^{110} \cdot 7) + (2^2 \cdot 3^4 \cdot 5^{110})$
$= 2 \cdot 3^4 \cdot 5^{110} \cdot (7 + 2)$
$= 2 \cdot 3^4 \cdot 5^{110} \cdot 3^2$
$= \mathbf{2 \cdot 3^6 \cdot 5^{110}}$.

(d) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 + 1 =$ **2311**, which is prime.

10. **(*i*)** $2^3 \cdot 3^2 \cdot 25^3$ is not a prime factorization because 25 is not prime.

(*ii*) $25^3 = (5^2)^3 = 5^6$, so **prime factorization** is $2^3 \cdot 3^2 \cdot 5^6$.

11. **No**. $8^z = (2^3)^z = 2^{3z}$. In other words, 8^z will always have a unique prime factorization that contains only 2*s*. On the other hand $3^x \cdot 5^y$ will always have a unique prime factorization that contains only 3*s* and 5*s*.

12. $32\,n = 2^6 \cdot 3^5 \cdot 5^4 \cdot 7^3 \cdot 11^7$
$= 2^5 \cdot 2 \cdot 3^5 \cdot 5^4 \cdot 7^3 \cdot 11^7$
$= 32 \cdot \mathbf{2 \cdot 3 \cdot 5 \cdot 7 \cdot 11^6} \cdot 3^4 \cdot 5^3 \cdot 7^2 \cdot 11$

The Fundamental Theorem of Arithmetic tells us that

$m = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11^6 \cdot (3^2 \cdot 5^3 \cdot 7^2 \cdot 11)$.

Since $3^2 \cdot 5^3 \cdot 7^2 \cdot 11$ is a whole number, $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11^6$ divides *m*.

13. **Yes**. $7^5 \cdot 11^3 = 7(7^4 \cdot 11^3)$.

14. **(a)** **$1 \cdot 48$; $2 \cdot 24$; $3 \cdot 16$; or $4 \cdot 12$**, all pairs of divisors of 48.

(b) One**. 47 is prime, so the only possibility would be** 1×47.

15. **(a)** **1, 2, 3, 5, 6, 10, 15, or 30** rows. The prime factorization of 30 is $2 \cdot 3 \cdot 5$, so there are $(1 + 1) \cdot (1 + 1) \cdot (1 + 1) = 8$ divisors.

(b) **1, 2, 4, 7, 14, or 28** rows. The prime factorization 28 is $2^2 \cdot 7$, so there are $(2 + 1) \cdot (1 + 1) = 6$ divisors.

(c) **1 or 23** rows. 23 is prime, so there are only two divisors.

(d) **1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, or 120** rows. The prime factorization of 120 is $2^3 \cdot 3 \cdot 5$, so there are $(3 + 1) \cdot (1 + 1) \cdot (1 + 1) = 16$ divisors.

16. There are 70 days in 10 weeks. Briah swims on day 1 and then every other day (1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69). Emma swims on day 1 and then every three days (1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70). They swim together 12 times (1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67).

17. **91 eggs.** Let *n* be the number of eggs in the basket. Then $n - 1$ is a multiple of 3 and of 5. Hence, $n - 1 = 3 \cdot 5k$ for some whole number *k*. Thus, $n = 15k + 1$. We know that $n \leq 100$ and that $7 \mid n$. We substitute $k = 1, 2, 3, \ldots$ to obtain the following values for *n* that are less than or equal to 100: 16, 31, 46, 61, 76, 91. Among these values, only 91 is divisible by 7. Hence, $n =$ **91**.

18. The least number of coins the pirates stole is a number that is a multiple of 15 and has a remainder of 3 when divided by 17 and a remainder of 10 when divided by 16. On a spreadsheet we can create a list of multiples of 15 (15, 30, 45, 60, 75, …) and a list of multiples of 16 plus 10 (26, 42, 58, 74, 90, …) and a list of multiples of 17 plus 3 (20, 37, 54, 71, 88, …) to find out that the least number of coins is **3930**.

19. Consider the two-digit primes: 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. The ones with the tens digit greater than the units digit are 31, 41, 43, 53, 61, 71, 73, 83, and 97. The sum of the two digits is a two-digit prime so we only have 83 left. The last three digits are all and different: 1, 3, 5, 7, 9. The sum of the three digits is palindromic. Only $1 + 3 + 7 = 11$ works. The sum of the first and third digit is one-half the sum of the first and second. The only order possible is 173 because $1 + 7 = \frac{(1+3)}{2}$. So the license plate is 83-173.

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20. There is no analytic method; one must work through the list of primes. The twin primes less than 200 are: 3 and 5; 5 and 7; 11 and 13; 17 and 19; 29 and 31; 41 and 43; 59 and 61; 71 and 73; 101 and 103; 107 and 109; 137 and 139; 149 and 151; 179 and **181; 191 and 193; and 197 and 199.**

21. **(a) (i)** $1 + 2 + 3 + 4 + 6 = 16$, so 12 is not perfect.

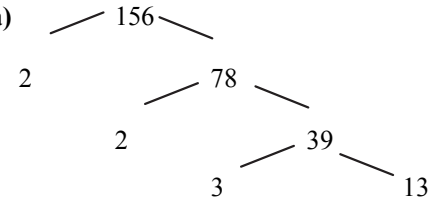
(ii) $1 + 2 + 4 + 7 + 14 = 28$, so 28 is perfect.

(iii) $1 + 5 + 7 = 13$, so 35 is not perfect.

(b) Answers vary. For example 496 is perfect.

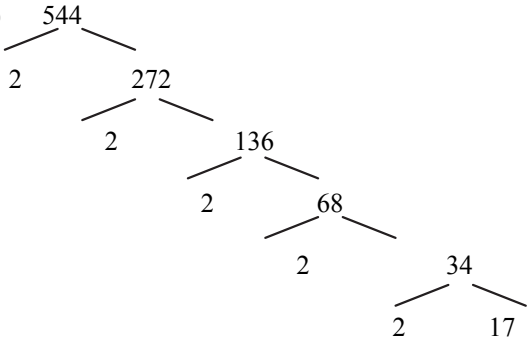
Assessment 4-2B

1. **(a)**



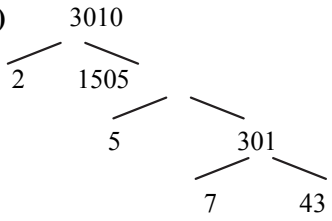
(b) $2 \cdot 2 \cdot 3 \cdot 13 = 156$

2. **(a)**



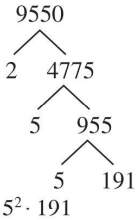
$544 = 2^5 \cdot 17$

(b)



$3010 = 2 \cdot 5 \cdot 7 \cdot 43$

(c)



$9550 = 2 \cdot 5^2 \cdot 191$

3. **(a)** $1001 = \mathbf{7 \cdot 11 \cdot 13}$.

(b) $1001^2 = (7 \cdot 11 \cdot 13)^2 = \mathbf{7^2 \cdot 11^2 \cdot 13^2}$.

(c) $999^{10} = (3^3 \cdot 37)^{10} = \mathbf{3^{30} \cdot 37^{10}}$.

(d) $111^{10} - 111^9 = 111^9(111 - 1) =$
$(3 \cdot 37)^9 \cdot (2 \cdot 5 \cdot 11) =$
$\mathbf{2 \cdot 3^9 \cdot 5 \cdot 11 \cdot 37^9}$.

4. 23. $23^2 < 671$, **but** $29^2 > 671$. **(23 is prime and 29 is the next largest prime.)**

5. **(a)** **Prime**. Fails divisibility test for primes less than 11; no need to test for primes >7 because $11^2 > 101$.

(b) **Prime**. Fails divisibility test for primes less than 11; no need to test for primes >7 because $11^2 > 113$.

(c) **Not prime**. 153=3(51) is divisible by 3.

(d) **Not prime**. $9 \cdot 21 = 189$.

(e) **Prime.** Fails divisibility test for primes less than or equal to 23; no need to test for primes >23 because $29^2 > 541$.

(f) **Prime.** Fails divisibility test for primes less than 23; no need to test for primes >23 because $29^2 > 601$.

(g) **Not prime.** 5 is a factor of both $2 \cdot 3 \cdot 5 \cdot 7$ and 5.

(h) **Not prime.** See (g).

6. $2 \cdot 3 \cdot 5 \cdot 7 = \mathbf{210}$.

7. Any number with an odd number of factors must be a perfect square; i.e., for exactly three factors the factors must be of the form: 1, p, p^2, where p is a prime. Thus find the largest two-digit prime and square it, or $97^2 = \mathbf{9409}$, which has factors 1, 97, and 9409.

8. **(a)** Since $4|n$, 2^2 must be in the prime factorization of n. Since $9|n$, 3^2 must be in the prime factorization of n. Because 2^2 and 3^2 have no common factors, $2^2 \cdot 3^2 = 36$ must divide n.

(b) Part (a) has the key idea: that a and b have no common factors. This notion is after expressed as $GCD(a, b) = 1$.

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9. **(a)** $16^4 \cdot 81^4 \cdot 6^6 = (2^4)^4 \cdot (3^4)^4 \cdot (2 \cdot 3)^6 = 2^{16} \cdot 3^{16} \cdot 2^6 \cdot 3^6 = \mathbf{2^{22} \cdot 3^{22}}$.

(b) $8^4 \cdot 32^5 = (2^3)^4 \cdot (2^5)^5 = 2^{12} \cdot 2^{25} = \mathbf{2^{37}}$.

(c) $2^2 \cdot 3^5 \cdot 7^{55} + 2^4 \cdot 3^4 \cdot 7^{55} = 2^2 \cdot 3^4 \cdot 7^{55}(1 \cdot 3 \cdot 1 + 2^2 \cdot 1 \cdot 1) = 2^2 \cdot 3^4 \cdot 7^{55}(7) = \mathbf{2^2 \cdot 3^4 \cdot 7^{56}}$.

(d) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 + 1 = 2731$, which is prime.

10. **(*i*)** $2^2 \cdot 5^3 \cdot 9^2$ is not a prime factorization because 9 is not prime.

(*ii*) The prime factorization is $2^2 \cdot 5^3 \cdot (3^2)^2 = \mathbf{2^2 \cdot 5^3 \cdot 3^4}$.

11. **No**. 5^z has factors of only 5; there are no factors of 5 in 2^x or 3^y.

12. If $2n = 2^6 \cdot 3^5 \cdot 5^4 \cdot 7^3 \cdot 11^7$ then $n = 2^5 \cdot 3^5 \cdot 5^4 \cdot 7^3 \cdot 11^7 = (2 \cdot 3 \cdot 5 \cdot 7 \cdot 11) \cdot (2^4 \cdot 3^4 \cdot 5^3 \cdot 7^2 \cdot 11^6)$. Thus $(2 \cdot 3 \cdot 5 \cdot 7 \cdot 11)$ is a factor of n.

13. **No**. No whole number times $3^2 \cdot 2^4 = 144$ will yield $3^3 \cdot 2^2 = 108$.

14. **(a)** The other arrays are 1 by 24, 2 by 12, 3 by 8, because $1 \cdot 24 = 24$, $2 \cdot 12 = 24$, and $3 \cdot 8 = 24$.

(b) There would only be 1 array: 1 by 43 because $1 \cdot 43 = 43$ and there are no other factors of 43. 43 is prime.

15. **(a)** $15 = 3 \cdot 5$. Possible lengths and widths are 1 by 15, 3 by 5.

(b) $24 = 2^3 \cdot 3$. Possible lengths and widths are 1 by 24, 2 by 12, 3 by 8, 4 by 6.

(c) 17 is prime. There can only be a 1 by 17 rectangle.

(d) $200 = 2^3 \cdot 5^2$. Possible lengths and widths are 1 by 200, 2 by 100, 4 by 50, 5 by 40, 8 by 25, 10 by 20.

16. Barbara runs every other day. Starting on day 1 she runs on the following days: 1, 3, ,5 ,7 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29. She goes to dance class every fifth day. Starting on day 1 she goes to dance class on the following days: 1, 6, 11, 16, 21, 26. She does both on days 1, 11, and 21 or 3 days in a 30-day month.

17. If he has one card left over each time, we can list the multiples of 2, 3, 4, 5, and 6 and add 1. That gives us: 3,5,7,9, … and 4,7,10,13, … and 6, 11, 16, 21, … and 7, 13, 19, 25, … . We know we are looking for an odd number (because of the divisibility by 2 plus 1), less than 100 that ends on 1 (because of the divisibility by 5 plus 1). We check 91, 81, 71, and finally 61. Jose has 61 cards.

18. Working backward we know that 100 coins is after Isabelle divided the pile by 3 and took one part so 100 is 2 parts and the total was 150. That is 150 coins plus the 2 she gave her dad = 152. The 152 coins is after Noelle divided the pile by 3 and took one part so 152 is 2 parts and the total was 228. That is 228 coins plus the 2 she gave her dad = 230. The 230 coins is after Juliette divided the pile by 3 and took one part so 230 is 2 parts and the total was 345. That is 345 coins plus the 2 she gave her dad = 347. There were 347 coins.

19. We can assign a variable to each of the colors: r for red, g for green and, b for blue. The product of the number of red beads by the total number of red and green beads is a number exactly 120 greater than the number of blue beads. This translates into $r(r + g) = b + 120$. We also know that r, g, and b are all prime numbers. Using the guess and check method we find that there are 2 green beads, 11 red beads and 23 blue beads.

20. **(a)** **0, 4, 6, or 8.** All would cause divisibility 2 or 5 at some point; 2 or 5 would allow superprimes in the leftmost position only.

(b) **1 and 9.** 9 is composite; 1 is not prime by definition.

(c) **23, 29, 31, 37, 53, 59, 71, 73, and 79,** following the above restrictions.

(d) Answer may vary; e.g., 233, 239, or 373.

21. **(a)** **(*i*)** $1 + 2 + 3 + 4 + 6 = 16$, so 12 is abundant.

(*ii*) $1 + 2 + 4 + 7 + 14 = 28$, so 28 is perfect and neither abundant or deficient.

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(iii) 1 + 5 + 7 = 13, so 35 is deficient.

(b) Answers vary. For example 14 is deficient and 18 is abundant.

Mathematical Connections 4-2: Review Problems

22. **(a)** **True**. $13|(13 \cdot 10)$.

(b) **True**. $11 \cdot 91 = 1001$.

(c) **True**. Theorem 4-2(d).

23. Use the standard divisibility tests for all except 7; for 7 just perform the division:

(a) Divisors are **2, 3, and 6**.

(b) Divisors are **2, 3, 5, 6, 9, and 10**.

24. If $15|n$ there exists a whole number a such that $15a = n$:

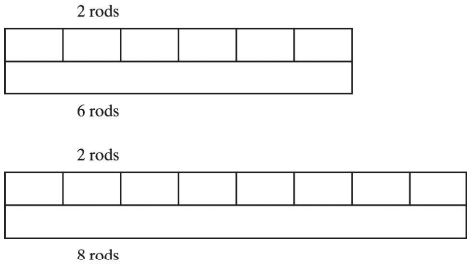
$(3 \cdot 5)a = n$

$3(5a) = n$

Since $5a$ is a whole number, $3|n$.

Assessment 4-3A: Greatest Common Divisor and Least Common Multiple

1. ***(i)*** The 2 rods can be used to build both the 6 rod and the 8 rod $\Rightarrow GCD(6,8) = \mathbf{2}$.



(ii) Four 6 rods (length 24) are the same length as three 8 rods (length 24)
$\Rightarrow LCM(6,8) = 24$.

2. **(a)** ***(i)*** $D_{18} = \{1,2,3,6,9,18\}$;
$D_{12} = \{1,2,3,4,6,12\}$;
$\Rightarrow GCD(18,12) = \mathbf{6}$.

(ii) $M_{18} = \{18,36,54,72,90,...\}$;
$M_{12} = \{12,24,36,...\}$;
$\Rightarrow LCM(18,12) = 36$.

(b) ***(i)*** $D_{20} = \{1,2,4,5,10,20\}$;
$D_{36} = \{1,2,3,4,6,9,12,18,36\}$;
$\Rightarrow GCD(20,36) = 4$.

(ii) $M_{20} = \{20,40,60,80,100,...,180,...\}$;
$M_{36} = \{36,72,108,144,180,...\}$;
$\Rightarrow LCM(24,36) = 180$.

(c) ***(i)*** $D_8 = \{1,2,4,8\}$;
$D_{24} = \{1,2,3,4,6,8,12,24\}$;
$D_{64} = \{1,2,4,8,16,32,64\}$;
$\Rightarrow GCD(8,24,64) = 8$.

(ii) $M_8 = \{8,16,24,...,192,...\}$;
$M_{24} = \{24,48,72,...,192,...\}$;
$M_{64} = \{64,128,192,...\}$;
$\Rightarrow LCM(8,24,64) = 192$.

(d) ***(i)*** $D_7 = \{1,7\}$;
$D_9 = \{1,3,9\}$;
$\Rightarrow GCD(7,9) = \mathbf{1}$.

(ii) $M_7 = \{7,14,21,...,63,...\}$;
$M_9 = \{9,18,27,...,63,...\}$;
$\Rightarrow LCM(7,9) = \mathbf{63}$.

3. **(a)** $132 = 2 \cdot 2 \cdot 3 \cdot 11 = 2^2 \cdot 3 \cdot 11$;
$504 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 =$
$2^3 \cdot 3^2 \cdot 7$; $\Rightarrow GCD(132,504) =$
$2^2 \cdot 3 = \mathbf{12}$. $LCM(132,504) =$
$2^3 \cdot 3^2 \cdot 7 \cdot 11 = \mathbf{5544}$.

(b) $65 = 5 \cdot 13$;
$1690 = 2 \cdot 5 \cdot 13 \cdot 13 = 2 \cdot 5 \cdot 13^2$;
$\Rightarrow GCD(65,1690) = 5 \cdot 13 = \mathbf{65}$.
$LCM(65,1690) = 2 \cdot 5 \cdot 13^2 = \mathbf{1690}$.

(c) $900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 2^2 \cdot 3^2 \cdot 5^2$;
$96 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^5 \cdot 3$;
$630 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 = 2 \cdot 3^2 \cdot 5 \cdot 7$;
$\Rightarrow GCD(900,96,630) = 2 \cdot 3 = \mathbf{6}$.
$LCM(900,96,630) = 2^5 \cdot 3^2 \cdot 5^2 \cdot 7$
$= \mathbf{50400}$.

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(d) $108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^2 \cdot 3^3$;
$360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^3 \cdot 3^2 \cdot 5$;
$\Rightarrow GCD(108,360) = 2^2 \cdot 3^2 = \mathbf{36}$.
$LCM(108,360) = 2^3 \cdot 3^3 \cdot 5 = \mathbf{1080}$.

4. $\rightarrow R$ symbolizes the reminder left after the indicated divisions.

(a) $GCD(2924,220) = GCD(220,64)$ since
$2924 \div 220 \rightarrow R\ 64$;
$GCD(220,64) = GCD(64,28)$ since
$220 \div 64 \rightarrow R\ 28$;
$GCD(64,28) = GCD(28,8)$ since
$64 \div 28 \rightarrow R\ 8$;
$GCD(28,8) = GCD(8,4)$ since
$28 \div 8 \rightarrow R\ 4$;
$GCD(8,4) = GCD(4,0)$ since
$8 \div 4 \rightarrow R\ 0$;
$\Rightarrow GCD(2924,220) = \mathbf{4}$.

(b) $GCD(14595,10856) = GCD(10856,3739)$
since $14595 \div 10856 \rightarrow R\ 3739$;
$GCD(10856,3739) = GCD(3739,3378)$
since $10856 \div 3739 \rightarrow R\ 3378$;
$GCD(3739,3378) = GCD(3378,361)$
since $3739 \div 3378 \rightarrow R\ 361$;
$GCD(3378,361) = GCD(361,129)$
since $3378 \div 361 \rightarrow R\ 129$;
$GCD(361,129) = GCD(129,103)$
since $361 \div 129 \rightarrow R\ 103$;
$GCD(129,103) = GCD(103,26)$
since $129 \div 103 \rightarrow R\ 26$;
$GCD(103,26) = GCD(26,25)$
since $103 \div 26 \rightarrow R\ 25$;
$GCD(26,25) = GCD(25,1)$
since $26 \div 25 \rightarrow R\ 1$;
$GCD(25,1) = GCD(1,0)$
since $25 \div 1 \rightarrow R\ 0$;
$\Rightarrow GCD(14595,10856) = \mathbf{1}$.

5. **(a)** ***(i)*** Intersection of sets:
$M_{24} = \{24,48,72,96,...\}$;
$M_{36} = \{36,72,108,...\}$;
$\Rightarrow LCM(24,36) = \mathbf{72}$.

(ii) Prime factorization:
$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2^3 \cdot 3$;
$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$;
$\Rightarrow LCM(24,36) = 2^3 \cdot 3^2 = \mathbf{72}$.

(b) ***(i)*** Intersection of sets:
$M_{72} = \{72,144,216,...,1440,...\}$;
$M_{90} = \{90,180,270,...,1440,...\}$;
$M_{96} = \{96,192,288,...,1440,...\}$;
$\Rightarrow LCM(72,90,96) = \mathbf{1440}$.

(ii) Prime factorization:
$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$;
$90 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$;
$96 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$
$= 2^5 \cdot 3$; $\Rightarrow LCM(72,90,96)$
$= 2^5 \cdot 3^2 \cdot 5 = \mathbf{1440}$.

(c) ***(i)*** Intersection of sets:
$M_{90} = \{90,180,270,...,630,...\}$;
$M_{105} = \{105,210,315,...,630,...\}$;
$M_{315} = \{315,630,945,...\}$;
$\Rightarrow LCM(90,105,315) = \mathbf{630}$.

(ii) Prime factorization:
$90 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$;
$105 = 3 \cdot 5 \cdot 7$;
$315 = 3 \cdot 3 \cdot 5 \cdot 7 = 3^2 \cdot 5 \cdot 7$;
$\Rightarrow LCM(90,105,315) = 2 \cdot 3^2 \cdot 5 \cdot 7$
$= \mathbf{630}$

(d) $16^{100} = (4^2)^{100} = 4^{200}$ and
$25^{100} = (5^2)^{100} = 5^{200}$
$\Rightarrow LCM(16^{100},25^{100}) = 4^{\mathbf{200}} \cdot \mathbf{5^{200}}$
$= \mathbf{(20)^{200}}$.

6. If the GCD is 17 then the product of the two numbers divided by the GCD will result in the LCM. $1734 \div 17 = 102$, so 102 is their LCM.

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7. If the GCD is 19 we can find the other factors of 57 by dividing 57 by 19. $57 \div 19 = 3$. The other number is the LCM divided by 3: $228 \div 3 = 76$. The other number is 76.

8. **(a)** $LCM\,(a, b) = \boldsymbol{ab}$. a and b have no common factors.

(b) $GCD\,(a, a) = \boldsymbol{a}$ and $LCM(a, a) = \boldsymbol{a}$. a has all factors in common with a.

(c) $GCD\,(a^2,\ a) = \boldsymbol{a}$ and $LCM\,(a^2,\ a) = \boldsymbol{a^2}$.

(d) $GCD\,(a, b) = \boldsymbol{a}$ and $LCM\,(a, b) = \boldsymbol{b}$, since $a|b$.

9. **(a)** **True**. If both a and b are even, then $GCD\,(a, b) \geq 2$.

(b) **True**. $GCD\,(a, b) = 2$ implies that $2|a$ and $2|b$.

(c) **False**. GCD could be any larger multiple of 2; e.g., $GCD\,(8, 20) = 4$.

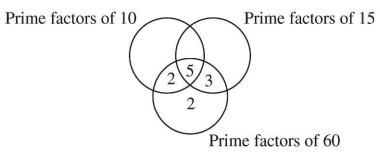
10. **(a)** $GCD\,(120, 75) = GCD\,(75, 45) = GCD\,(45, 30) = GCD(30, 15) = GCD(15, 0)$; $GCD(120, 75) = 15$.

$GCD\,(105, 15) = \text{GCD}\,(15, 0)$; $GCD\,(105, 15) = 15$.
Thus $GCD\,(120, 75, 105) = \mathbf{15}$.

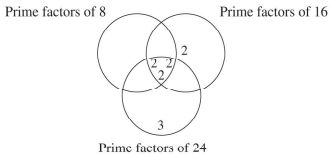
(b) $GCD\,(4618, 4619) = GCD(4618, 1) = 1$; $GCD\,(34578, 1) = 1$;
Thus $GCD\,(34578, 4618, 4619) = \mathbf{1}$.

11. 2 is the only prime factor of 4 and $2 \nmid 97,219,988,751$, so 1 is their only common divisor; i.e., they are relatively prime.

12. **(a)** The Venn diagram below shows 5 as the only prime factor common to 10, 15, and 60; 2 and 5 as common prime factors of 10 and 60; 3 and 5 as common prime factors of 15 and 60; 2 as the remaining prime factor of 60:



(b)



13. The prime factorization of 48 is $2^4 \cdot 3$. To find three pairs (a, b) such that their LCM is 48 is making sure that a and b have 2^4 and 3 in their LCM. For example: (1, 48), (2, 48), (3,48), etc.

14. This will be the set of all numbers between 1 and 49 that are relatively prime to 49. Since $49 = 7^2$, the set will contain all numbers between 1 and 49 **except** $1 \cdot 7, 2 \cdot 7, 3 \cdot 7, 4 \cdot 7, 5 \cdot 7,\ 6 \cdot 7,$ and $7 \cdot 7$.

15. **(a)** The real question is "what is $LCM\,(15, 40, 60)$?" since this is when the alarms will coincide: $LCM\,(15, 40, 60) = 120$, or **120 minutes;** i.e., 2 hours later, at 8:00 A.M.

(b) **No**. This would be equivalent to changing locations of clocks A and B in the room.

16. The **780th "like"**. $LCM(12, 13,20) = 780$.

17. There are 15 children. The candy will come out evenly for $LCM\,(15,12) = 60$; 60 candies $\div$ 12 candies per package $=$ **5 packages**.

18. The question is really, "What is $LCM\,(12, 18, 16)$?" since that is when the times will coincide $LCM\,(12, 18, 16) = \mathbf{144\ minutes}$.

19. GCD (42, 54) = 6. She can make 7 bags of chocolate chip cookies and 9 bags of sugar cookies. Each bag has 6 cookies in it.

20. LCM (100,18) = 900 inches or 75 feet before points P and Q touch the sidewalk at the same time again.

21. GCD(72, 42) = 6. The largest pieces she can cut are 6 yards long.

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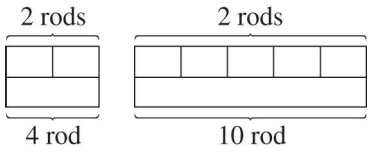
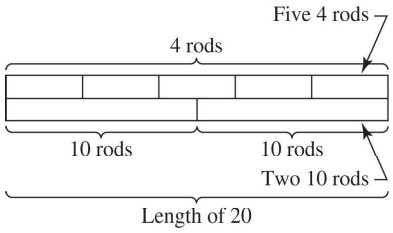
Assessment 4-3B

1. **(*i*)** $GCD\,(4, 10) = \mathbf{2}$**:**



(*ii*) $LCM\,(4, 10) = \mathbf{20}$:



2. **(a)** **(*i*)** $D_{12} = \{1, 2, 3, 4, 6, 12\}$

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$\Rightarrow GCD(12,30) = \mathbf{6}$.

(*ii*) $M_{12} = \{12, 24, 36, 48, 60, 72, 84, ...\}$

$M_{30} = \{30, 60, 90, 120, 150, 180, ...\}$

$\Rightarrow LCM(12,30) = 60$.

(b) **(*i*)** $D_{18} = \{1, 2, 3, 6, 9, 18\}$

$D_{58} = \{1, 2, 29, 58\}$

$\Rightarrow GCD(18,58) = 2$.

(*ii*) $M_{18} = \{18, 36, 54, 72, 90, 108, \ldots, 522, ...\}$

$M_{58} = \{58, 116, 174, 232, 290, ..., 522, ...\}$

$\Rightarrow LCM(18, 58) = 522$.

(c) **(*i*)** $D_6 = \{1, 2, 3, 6\}$

$D_{18} = \{1, 2, 3, 6, 9, 18\}$

$D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$\Rightarrow GCD(6, 18, 24) = \mathbf{6}$.

(*ii*) $M_6 = \{6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, ...\}$

$M_{18} = \{18, 36, 54, 72, 90, 108, ...\}$

$M_{24} = \{24, 48, 72, 96, 120, 144, ...\}$

$\Rightarrow LCM(6, 18, 24) = \mathbf{72}$.

(d) **(*i*)** $D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$D_{13} = \{1, 13\}$

$\Rightarrow GCD(36, 13) = \mathbf{1}$.

(*ii*) $M_{36} = \{36, 72, 108, ..., 468, ...\}$

$M_{13} = \{13, 26, 39, 52, 65, 78, ..., 468, ...\}$

$\Rightarrow LCM(36, 13) = 468$.

3. **(a)** $11 =$ prime number

$19 =$ prime number

$\Rightarrow GCD(11, 19) = \mathbf{1}$.

$LCM(11, 19) = 11 \cdot 19 = \mathbf{209}$.

(b) $140 = 2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$

$320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^6 \cdot 5$

$\Rightarrow GCD(140, 320) = 2^2 \cdot 5 = \mathbf{20}$

$LCM(140, 320) = 2^6 \cdot 5 \cdot 7 = \mathbf{2240}$.

(c) $800 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 = 2^5 \cdot 5^2$

$75 = 3 \cdot 5 \cdot 5 = 3 \cdot 5^2$

$450 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5 = 2 \cdot 3^2 \cdot 5^2$

$\Rightarrow GCD(800, 75, 450) = 5^2 = \mathbf{25}$

$LCM(800, 75, 450) = 2^5 \cdot 3^2 \cdot 5^2 = \mathbf{7200}$.

(d) $103 = 103$

$320 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^6 \cdot 5$

$\Rightarrow GCD(103, 320) = 1$

$LCM(103, 320) = 2^6 \cdot 5 \cdot 103 = 32,920$.

4. $\rightarrow R$ symbolizes the reminder left after the indicated divisions.

(a) $GCD(14560, 8250) = GCD(8250, 6310)$ since $14560 \div 8250 \rightarrow R\,6310$

$GCD(8250, 6310) = GCD(6310, 1940)$ since $8250 \div 6310 \rightarrow R\,1940$

$GCD(6310, 1940) = GCD(1940, 490)$ since $6310 \div 1940 \rightarrow R\,490$

$GCD(1940, 490) = GCD(490, 470)$ since $1940 \div 490 \rightarrow R\,470$

$GCD(490, 470) = GCD(470, 20)$ since $490 \div 470 \rightarrow R\,20$

$GCD(470, 20) = GCD(20, 10)$ since $470 \div 20 \rightarrow R\,10$

$GCD(20, 10) = GCD(10, 0)$ since $20 \div 10 \rightarrow R\,0$

$\Rightarrow GCD(14560, 8250) = \mathbf{10}$.

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(b) $GCD(8424, 2520) = GCD(2520, 864)$ since $8424 \div 2520 \rightarrow R\ 864$

$GCD(2520, 864) = GCD(864, 792)$ since $2520 \div 864 \rightarrow R\ 792$

$GCD(864, 792) = GCD(792, 72)$ since $864 \div 792 \rightarrow R\ 72$

$GCD(792, 72) = GCD(72, 0)$ since $792 \div 72 \rightarrow R\ 0$

$\Rightarrow GCD(8424, 2520) = \mathbf{72}$.

5. **(a)** ***(i)*** Intersection of sets:

$M_{25} = \{25, 50, 75, 100, ..., 900, ...\}$

$M_{36} = \{36, 72, 108, 144, ..., 900, ...\}$

$\Rightarrow LCM(25, 36) = \mathbf{900}$.

(ii) Prime factorization:

$25 = 5 \cdot 5 = 5^2$

$36 = 2 \cdot 2 \cdot 3 \cdot 3 = 2^2 \cdot 3^2$

$\Rightarrow LCM(25, 36) = 2^2 \cdot 3^2 \cdot 5^2 = \mathbf{900}$.

(b) Prime factorization:

$82 = 2 \cdot 41$

$90 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5$

$50 = 2 \cdot 5 \cdot 5 = 2 \cdot 5^2$

$\Rightarrow LCM(82, 90, 50) = 2 \cdot 3^2 \cdot 5^2 \cdot 41 = \mathbf{18{,}450}$.

(c) Prime factorization:

$80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 2^4 \cdot 5$

$105 = 3 \cdot 5 \cdot 7$

$315 = 3 \cdot 3 \cdot 5 \cdot 7 = 3^2 \cdot 5 \cdot 7$

$\Rightarrow LCM(80, 105, 315) = 2^4 \cdot 3^2 \cdot 5 \cdot 7 = \mathbf{5040}$.

(d) Prime factorization:

$8^{100} = (2^3)^{100} = 2^{300}$

$50^{100} = (2 \cdot 5^2)^{100} = 2^{100} \cdot 5^{200}$

$\Rightarrow LCM(8^{100}, 50^{100}) = \mathbf{2^{300} \cdot 5^{200}}$.

6. If the GCD is 14 then the product of the two numbers divided by the GCD will result in the LCM. $5880 \div 14 = 420$, so 420 is their LCM.

7. If the GCD is 11 we can find the other factors of 66 by dividing 66 by 11. $66 \div 11 = 6$. The other number is the LCM divided by 6: $330 \div 6 = 55$. The other number is 55.

8. **(a)** $GCD(a, b) = \mathbf{1}$ and $LCM(a, b) = \boldsymbol{ab}$. a and b have no factors in common.

(b) $\boldsymbol{a|b}$. If $GCD(a, b) = a$, then a must divide both a and b.

(c) $\boldsymbol{b|a}$. If $LCM(a, b) = a$, then $b \cdot n = a$, (where n may be any integer), so $a \div b = n$, or $b|a$.

9. **(a)** **False**. For $a \neq b$, $LCM(a,b) > GCD(a,b)$.

(b) **True**. Theorem 4-19.

(c) **True**. If $GCD(a, b) >$ a, it could not divide a.

(d) **True**. If $LCM(a, b) <$ a, it could not be a multiple.

10. **(a)** $GCD(180, 240) = GCD(180, 60) = GCD(60, 0) = 60$

$GCD(60, 306) = GCD(60, 6) = GCD(6, 0) = 6$

$GCD(60, 6) = GCD(6, 0) = 6$

$\Rightarrow GCD(180, 240, 306) = \mathbf{6}$.

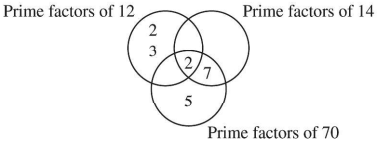
(b) $GCD(5284, 1250) = GCD(1250, 284) = GCD(284, 114) = GCD(114, 56) = GCD(56, 2) = GCD(2, 0) = 2$

$GCD(2, 1280) = GCD(2, 0) = 2$.

$\Rightarrow GCD(5284, 1250, 1280) = \mathbf{2}$.

11. 11 is its own only prime factor and $11 \not| 181{,}345{,}913$ $[11 \not| (1 + 1 + 4 + 9 + 3) - (8 + 3 + 5 + 1)]$, so 1 is their only common divisor; i.e., they are relatively prime.

12. **(a)** The Venn diagram below shows 2 as the only prime factor common to 12, 14, and 70; 2 as the only common prime factor of 12 and 70; 2 and 7 as common prime factors of 14 and 70; 3 as the remaining prime factor of 12; and 5 as the remaining prime factor of 70:



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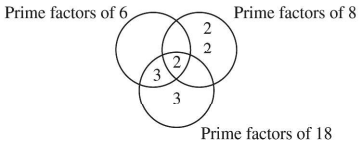
(b)



13. The prime factorization of 60 is $2^2 \cdot 3 \cdot 5$. To find three pairs (a, b) such that their LCM is 60 is making sure that a and b have 2^2, 3^2, and 5 in their LCM. For example: (1, 60), (2, 60), (3,60), etc.

14. GCD (25, x) $=$ 1 means that x can have no factors in common with 25, so x cannot have a factor of 5; the solution set is thus {1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24}.

15. The yellow light flashes 15 times per minute, or every 4 seconds (60 ÷ 15 = 4). The blue light flashes 6 times per minute, or every 10 seconds (60 ÷ 6 = 10). The red light flashes 12 times per minute or every 5 seconds (60 ÷ 12 = 5). The LCM (4,10,5) = 20. You must wait 20 seconds before the three lights flash on simultaneously again.

16. LCM (15,40) = 120. The 120th customer was the first to win both prizes.

17. GCD(245,238,84)= 7. Each bag will have 245 ÷ 7 = 35 white bouncy balls, 238 ÷ 7 = 34 yellow bouncy balls, and 84 ÷ 7 = 12 orange bouncy balls for a total of 35 + 34 + 12 = 81 bouncy balls.

18. $LCM\,(90, 75) = 450$ minutes, or $7\frac{1}{2}$ hours; 7:00 A.M. $+ 7\frac{1}{2}$ hours $=$ **2:30 P.M.**

19. $LCM(24, 45) = 360$; i.e., $360¢$ worth of cookies was sold; $360¢ \div 24¢$ per cookie $=$ **15 cookies.**

20. $LCM(40, 24, 60) = 120$. **Gear 1** must make $120 \div 40 =$ **3 revolutions**; **gear 2** must make $120 \div 24 =$ **5 revolutions**; and **gear 3** must make $120 \div 60 =$ **2 revolutions** to line up the arrows.

21. $GCD\,(300, 264) = 12$. He will have $300 \div 12 = 25$ piles of football cards and $264 \div 12 = 22$ piles of baseball cards. Each pile will have 12 cards.

Mathematical Connections 4-3: Review Problems

16. (a) **83,151, 83,451 or 83751**. Let the number be $83a51$; then $8 + 3 + a + 5 + 1 = 17 + a$ must be a multiple of 3; if a is a single digit the only possibilities for $17 + a$ to be a multiple of 3 are 1, 4, or 7. The greatest digit is 7.

(b) **86,691.** Let the number be $8a691$; then $(1 + 6 + 8) - (a + 9) = 6 - a$ must be a multiple of 11; if a is a single digit the only possibility is $6 - a = 0 \Rightarrow a = 6$.

(c) **10,396**. $10{,}306 = 23 \cdot 448 + 2$ so $10{,}3\_6 > 23 \cdot 448$. The second factor must end in 2 to yield 6 in the units position, thus $23 \cdot 452 = 10{,}396$.

17. (a) $2^3 \cdot 3^7$
(b) $3^2 \cdot 5 \cdot 727$
(c) $2^{10} \cdot 3^9$

18. **No**. $3|(2 + 2 + 2 + 3)$ so $3|2223$; it is thus not prime.

19. Answers may vary; $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2310$ is one.

20. The question is really "What is the *LCM* (2, 4, 6, 8, 10)?". Their factors are $\{2, 2^2,\ 2 \cdot 3,\ 2^3,\ 2 \cdot 5\}$, thus $LCM\,(2, 4, 6, 8, 10) = 2^3 \cdot 3 \cdot 5 = 120$.

21. **53**. The next prime squared (59^2) is greater than 3359.

Chapter 4 Review

1. (a) **False**. $12 \nmid 4$ since $4 = 12x$ has no whole number solution for x.

(b) **False**. $0 \nmid 8$ since $8 = 0 \cdot x$ has no whole number solution for x.

(c) **True**. 0 divided by any natural number is 0.

(d) **False**. For example, 12. The test works only when the two numbers have no common factors and 4 and 6 have a common factor of 2.

(e) **False**. For example, 9 is not divisible by 12 but is divisible by 3.

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2. **(a)** $m = 125{,}160$.
$2|m$ because $2|0$.
$3|m$ because $3|(1 + 2 + 5 + 1 + 6 + 0)$.
$4|m$ because $4|60$.
$5|m$ because $5|0$.
$6|m$ because $2|m$ and $3|m$.
$8|m$ because $8|160$.
$10|m$ because $10|0$.

(b) $m = 12{,}193$.
$2 \nmid m$ because $2 \nmid 3$.
$3 \nmid m$ because $3 \nmid (1+2+1+9+3)$.
$4 \nmid m$ because $4 \nmid 93$.
$5 \nmid m$ because $5 \nmid 3$.
$6 \nmid m$ because $2 \nmid m$ and $3 \nmid m$.
$8 \nmid m$ because $8 \nmid 193$.
$9 \nmid m$ because $9 \nmid (1+2+1+9+3)$.
$11 \nmid m$ because $11 \nmid (1 + 1 + 3) - (2 + 9)$.

3. **(a)** Write the number as $87a4$.
$6|87a4$ if $2|87a4$ and $3|87a4$;
$2|4$ so $2|87a4$, thus
$6|87a4$ if
$3|87a4 \Rightarrow 3|(8 + 7 + a + 4 = 19 + a)$;
$3|(21, 24, \text{or } 27)$ when $a = 2, 5, \text{or } 8$;
so $6|$**(87<u>2</u>4, 87<u>5</u>4, or 87<u>8</u>4)**. The greatest digit is 8.

(b) Write the number as $4a856$.
$24|4a856$ if $3|4a856$ and $8|4a856$;
$8|856$ so $8|4a856$, thus
$24|4a856$ if
$3|(4 + a + 8 + 5 + 6) = 23 + a$;
$3|(24, 27, \text{or } 30)$ when $a = 1, 4, \text{or } 7$;
so $24|$**(4<u>1</u>856, 4<u>4</u>856, or 4<u>7</u>856)**. The greatest digit is 7.

(c) Write the number as $87ab4$.
$29|87{,}000$ so $29|ab4$.
The only whole number to return 4 in the units position when multiplied by 9 is 6. So the possibilities are $6 \cdot 29, 16 \cdot 29,$ etc. until the product of $n \cdot 29 > 999$.
$6 \cdot 29 = 174; 16 \cdot 29 = 464; 26 \cdot 29 = 754;$
$36 \cdot 29 > 999$.
Thus $ab = 17, 46, \text{or } 75$.
So $29|$**(87<u>1</u>74, 87<u>4</u>64, or 87<u>7</u>54)**. The greatest digits are 75.

4. **(a)** The student's claim is true; examples may vary. For example.,
(i) $5|(3 + 4 + 5 + 6 + 7)$.
(ii) $5|(5 + 6 + 7 + 8 + 9)$.

(b) Let n be a whole number; then
$n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5(n + 2)$.
Thus the sum is divisible by 5.

5. ***(i)*** The number must be **divisible by 3 and 8**.
(ii) $3|4152$ and $8|4152$ so $24|4152$.

6. ***(i)*** Answers may vary; e.g., 16.
(ii) To obtain k divisors raise any prime to the $(k - 1)$st power; e.g., to obtain 5 divisors take $2^{5-1} = 16$; divisors are $2^0, 2^1, 2^2, 2^3,$ and 2^4.

7. $144 = (2^2 \cdot 3)^2 = 2^4 \cdot 3^2$. There are thus $(4 + 1) \cdot (2 + 1) = 15$ divisors: **1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72 and 144**.

8. $n = a \cdot 10^2 + b \cdot 10 + c$
$= a(99 + 1) + b(9 + 1) + c$
$= 99a + 9b + (a + b + c)$.
Since $9|99a$ and $9|9b$ then
$9 \mid [99a + 9b + (a + b + c)]$
if and only if $9|(a + b + c)$.

9. 1009 is prime, so $17 \nmid 1009$ but $17|17$.
$17 \nmid (1009 + 17)$ by Theorem 4-2(b).

10. **(a)** **Composite**. $3 \mid 147$.

(b) **Prime**. Primes through 19 do not divide 373; $23^2 > 373$.

11. **(a)** $7 \cdot 11 \cdot 13 \cdot 17 + 17 = 17(7 \cdot 11 \cdot 13 + 1)$, so 17 is a factor.

(b) $10! + k$ can be factored as in part (a), depending on the value of $k (2 \le k \le 10)$, thus composite.

12. First show that among any three consecutive odd whole numbers there is always one divisible by 3. Suppose that the first whole number in the triplet is not divisible by 3. By the Division Algorithm that

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whole number can be written in the form $3n + 1$ or $3n + 2$ for some whole number n.

Then the three consecutive odd whole numbers are $(3n + 1, 3n + 3, 3n + 5)$ or $(3n + 2, 3n + 4, 3n + 6)$.

In the first triplet $3|3n + 3$; in the second $3|3n + 6$. If the first whole number is greater than 3 and not divisible by 3 then the second or third must be divisible by 3 and so cannot be prime.

13. **(a)**

$$\begin{array}{r|ccc} 3 & 1 & 1 & 1 \\ \hline & 3 & 7 & \end{array} \Rightarrow 111 = 3 \cdot 37.$$

(b) $\Rightarrow 144 = 2^4 \cdot 3^2$.

$$\begin{array}{r|cc} 2 & 1 & 4 & 4 \\ \hline 2 & 7 & 2 \\ \hline 2 & 3 & 6 \\ \hline 2 & 1 & 8 \\ \hline & 3 & 9 \\ \hline & & 3 \end{array}$$

(c)

$$\begin{array}{r|ccc} 2 & 1 & 8 & 8 \\ \hline 2 & & 9 & 4 \\ \hline & & 4 & 7 \end{array} \Rightarrow 188 = 2^2 \cdot 47.$$

(d)

$$\begin{array}{r|ccc} 2 & 5 & 2 & 0 \\ \hline 2 & 2 & 6 & 0 \\ \hline 2 & 1 & 3 & 0 \\ \hline & 5 & 6 & 5 \\ \hline & & 1 & 3 \end{array} \Rightarrow 520 = 2^3 \cdot 5 \cdot 13.$$

14. **(a)** $10^{10} = (2 \cdot 5)^{10} = 2^{10} \cdot 5^{10}$.

(b) $89^4 = 89^4$.

(c) $8^3 \cdot 6^4 \cdot 13^2 = (2^3)^3 \cdot (2 \cdot 3)^4 \cdot 13^2 =$
$2^9 \cdot 2^4 \cdot 3^4 \cdot 13^2 = 2^{13} \cdot 3^4 \cdot 13^2$

(d) $2^3 \cdot 3^2 + 2^4 \cdot 3^3 \cdot 7 =$
$2^3 \cdot 3^2(1 + 2 \cdot 3 \cdot 7) =$
$\mathbf{2^3 \cdot 3^2 \cdot 43}$.

(e) $2^4 \cdot 3 \cdot 5^7 + 2^4 \cdot 5^6 =$
$2^4 \cdot 5^6(3 \cdot 5 + 1) = 2^4 \cdot 5^6 \cdot 16 =$
$2^4 \cdot 5^6 \cdot 2^4 = \mathbf{2^8 \cdot 5^6}$.

15. Product (all whole numbers ≤ 10) $=$
$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 =$
$1 \cdot 2 \cdot 3 \cdot 2^2 \cdot 5 \cdot (2 \cdot 3) \cdot 7 \cdot 2^3 \cdot 3^2 \cdot (2 \cdot 5)$.

Thus LCM (all whole numbers ≤ 10) $=$
$1 \cdot 2^3 \cdot 3^2 \cdot 5 \cdot 7 = \mathbf{2520}$.

16. **(a)** $24 = 2^3 \cdot 3$ and $52 = 2^2 \cdot 13$. Thus $GCD(24, 52) = 2^2 = \mathbf{4}$.

(b) $GCD(5767, 4453) = GCD(4453, 1314)$
$= GCD(1314, 511) = GCD(511, 292)$
$= GCD(292, 219) = GCD(219, 73)$
$= GCD(73, 0) \Rightarrow GCD(5767, 4453) = \mathbf{73}$.

17. **(a)** $LCM(2^3 \cdot 5^2 \cdot 7^3, 2 \cdot 5^3 \cdot 7^2 \cdot 13,$
$2^4 \cdot 5 \cdot 7^4 \cdot 29) =$
$\mathbf{2^4 \cdot 5^3 \cdot 7^4 \cdot 13 \cdot 29}$.

(b) $GCD(277, 278) = \text{GCD}(277, 1) = 1$
Therefore no common factors in 277 and 278.
Thus $LCM(277, 278) \cdot 1 = 277 \cdot 278$
$= 77{,}006$.

18. **No**. LCM and GCD are the same if the number are equal.

19. $LCM(a, b, c) = LCM(m, c)$, where $m = LCM(a, b)$. $LCM(a, b) = \frac{ab}{GCD(a,b)}$ and $LCM(m, c) = \frac{mc}{GCD(m,c)}$. Each of the above GCDs can be found using the Euclidean algorithm.

20. If $GCD(a, b) = 1 \Rightarrow \boldsymbol{LCM\,(a, b) = ab}$.
$GCD(a, b) \cdot LCM(a, b) = ab$; since $GCD\,(a, b) = 1$ then $LCM\,(a, b) = ab/1 = ab$.

21. **31 ¢** . The price must divide 3193 ¢; $31|3100$ and $31|93$ thus $31|3193$.

22. $9869 = 71 \cdot 139$ (both prime) implies **71 lattes** at \$1.39 each. She could not have sold 139 lattes at \$0.71 each, since she never sells for less than \$1.

23. LCM $(45, 30) = 90$ minutes. 8:00 A.M. $+$ 90 minutes $=$ **9:30 A.M.**

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24. The question is really "What is *GCD* (120, 144)?" since what is needed is the greatest number which will divide both 120 and 144: *GCD* (120, 144) = **24 coins**.

25. The runners will be at the starting place at the same time at *LCM* (3, 5) = **15 minutes**.

26. GCD(60,24) = 12. There will be 60 ÷12=5 bags of oranges and 24 ÷ 12 = 2 bags of apples. Each bag will have 12 pieces of fruit.

27. One revolution of gear 2, rotates 28 teeth. One revolution of gear 1, rotates 48 teeth. For each of the gears, the arrows are back in their original positions every multiple of 28 teeth for gear 2 and 48 teeth for gear 1. Thus, the arrows realign when *LCM* (48, 28) = 336 teeth have rotated. So gear 2 must make 336 ÷ 28 = **12 revolutions**.

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CHAPTER 5

INTEGERS

Assessment 5-1A: Addition and Subtraction of Integers

1. For every integer a, there exists a unique integer ^{-}a, such that $a + {}^{-}a = 0 = {}^{-}a + a$. ^{-}a is called the additive inverse of a.

(a) The unique integer $^{-}\mathbf{2}$ is the additive inverse of 2 because $2 + {}^{-}2 = 0$.

(b) The additive inverse of ^{-}a can be written as $^{-}(^{-}a) = a$, or the number of the opposite sign. Thus the additive inverse of $^{-}6$ is $^{-}(^{-}6) = 6$.

(c) The additive inverse of m is $^{-}\boldsymbol{m}$.

(d) The additive inverse of 0 is **0** because $0 + 0 = 0$.

(e) The additive inverse of ^{-}m is $^{-}(^{-}m) = \boldsymbol{m}$.

(f) The additive inverse of $(a + b)$ is $^{-}(a + b) = {}^{-}\boldsymbol{a} + {}^{-}\boldsymbol{b}$.

2. **(a)** $^{-}(^{-}2) = \mathbf{2}$ is the additive inverse of $^{-}2$.

(b) $^{-}(^{-}m) = \boldsymbol{m}$.

(c) $^{-}0 = \mathbf{0}$ since $^{-}0 + 0 = 0$.

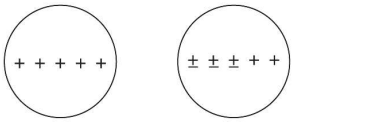
3. **(a)** Absolute value is the distance on a number line between the origin (0) and a specified number. Distance on the number line between 0 and $^{-}5$ is 5 units, so $|^{-}5| = \mathbf{5}$.

(b) Distance on the number line between 0 and 10 is 10 units, so $|10| = \mathbf{10}$.

(c) $^{-}|^{-}5|$ means the additive inverse of the absolute value of $^{-}5$, so $^{-}|^{-}5| = {}^{-}(5) = {}^{-}\mathbf{5}$.

(d) $^{-}|5| = {}^{-}(5) = {}^{-}\mathbf{5}$.

4. **(a)** Charged-field model:

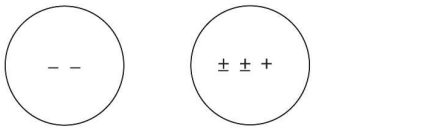


5 charge

Put in 3 negative charges. Net result: 2 positive charges; answer: 2

For the chip model, draw black chips to represent positive numbers and red chips to represent negative numbers. Each red chip neutralizes one black chip.

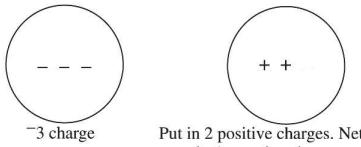
(b) Charged-field model:



$^{-}2$ charge

Put in 3 positive charges. Net result: 1 positive charge; answer: 1

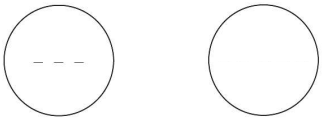
(c) Charged-field model:



$^{-}3$ charge

Put in 2 positive charges. Net result: 1 negative charge; answer: $^{-}1$

(d) Charged-field model:

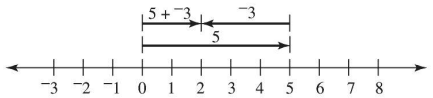


$^{-}3$ charge

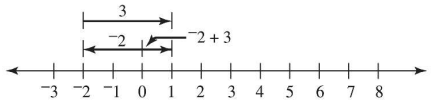
Put in 2 negative charges. Net result: 5 negative charges; answer: $^{-}5$

5. Movement on the number line in addition is always to the right for a positive number and to the left for a negative number.

(a)



(b)



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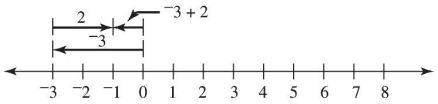
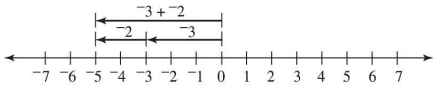
(c)



(d)



6. (a) $7 + {}^{-}13 = {}^{-}(\left|{}^{-}13\right| - \left|7\right|)$

$= {}^{-}(13 - 7)$

$= {}^{-}(6)$

$= {}^{-}6$

(b) ${}^{-}7 + {}^{-}13 = {}^{-}(\left|{}^{-}7\right| + \left|{}^{-}13\right|)$

$= {}^{-}(7 + 13)$

$= {}^{-}(20)$

$= {}^{-}20$

7. Using the theorem that $a - b = a + {}^{-}b$:

(a) $3 - {}^{-}2 = 3 + {}^{-}({}^{-}2) = 3 + 2 = \mathbf{5}$.

(b) ${}^{-}3 - 2 = {}^{-}3 + {}^{-}2 = {}^{-}\mathbf{5}$.

(c) ${}^{-}3 - {}^{-}2 = {}^{-}3 + {}^{-}({}^{-}2) = {}^{-}3 + 2 = {}^{-}\mathbf{1}$.

8. (a) $3 - ({}^{-}2) = n$ if and only if $3 = {}^{-}2 + n$.
Thus $\boldsymbol{n = 5}$.

(b) ${}^{-}3 - 2 = n$ if and only if ${}^{-}3 = 2 + n$.
Thus $\boldsymbol{n = {}^{-}5}$.

(c) ${}^{-}3 - {}^{-}2 = n$ if and only if ${}^{-}3 = {}^{-}2 + n$.
Thus $\boldsymbol{n = {}^{-}1}$.

9. (a) A drop in stock value is negative; an increase is positive, so a drop of 17 points followed by a gain of 20 points is represented by ${}^{-}17 + 20$, or a **net drop of 3 points.**

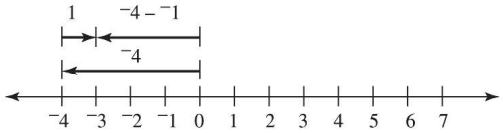
(b) ${}^{-}10°\text{C} + 8°\text{C} = {}^{-}\mathbf{2°C}$.

(c) $35{,}000 \text{ feet} + {}^{-}1000 \text{ feet} = 34{,}000 \textit{ feet}$.

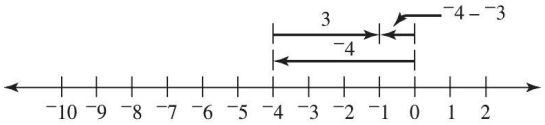
10. (a) $({}^{-}45) + ({}^{-}55) + ({}^{-}165) + ({}^{-}35) + ({}^{-}100)$
$+75 + 25 + 400 = {}^{-}400 + 500$, or **$100**.

(b) \$300 (beginning balance) + \$100 (in transactions) = **$400**.

11. (a) In the number-line model of ${}^{-}4 - {}^{-}1$, ${}^{-}4$ is depicted as a line that starts at 0 and moves 4 units to the left to ${}^{-}4$; the operation of *subtraction* now changes the direction of ${}^{-}1$ from moving left (as ${}^{-}1$ would during addition) to moving *right* 1 unit, ending at ${}^{-}3$. Thus ${}^{-}4 - {}^{-}1 = {}^{-}\mathbf{3}$:



(b) ${}^{-}4$ is depicted as a line that starts at 0 and moves 4 units to the left; *subtraction* of ${}^{-}3$ is a line that moves 3 units to the *right*, ending at ${}^{-}1$. Thus ${}^{-}4 - {}^{-}3 = {}^{-}\mathbf{1}$:



12. (a) Starting with subtraction that is already known:
${}^{-}4 - 2 = {}^{-}6$

Continuing the pattern:

$${}^{-}4 - 1 = {}^{-}5$$
$${}^{-}4 - 0 = {}^{-}4$$
$${}^{-}4 - {}^{-}1 = {}^{-}3.$$

(b) Starting with subtraction that is already known:

$$2 - 1 = 1$$
$$1 - 1 = 0$$
$$0 - 1 = {}^{-}1$$
$${}^{-}1 - 1 = {}^{-}2$$
$${}^{-}2 - 1 = {}^{-}3.$$

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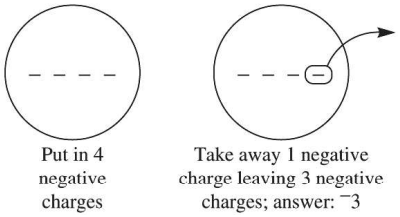
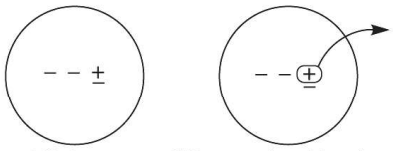
13. **(a)**



(a) Put in 4 negative charges. Take away 1 negative charge leaving 3 negative charges; answer: $^-3$

(b)



(b) $^-2$ charge. Take away 1 positive charge leaving 3 negative charges; answer: $^-3$

14. **(a)** $^-2 + (3 - 10) = {}^-2 + {}^-7 = \mathbf{^-9}$.

(b) $[8 - (^-5)] - 10 = [8 + {}^-(^-5)] - 10 = [8 + 5] - 10 = 13 - 10 = \mathbf{3}$.

(c) $(^-2 - 7) + 10 = (^-2 + {}^-7) + 10 = (^-9) + 10 = \mathbf{1}$.

15. **(a)** ***(i)*** $55 - 60$.

(ii) $55 + (^-60)$.

(iii) $T = 55 + (^-60) = \mathbf{^-5°F}$.

(b) ***(i)*** $200 - 220$.

(ii) $200 + (^-220)$.

(iii) Balance $= \$200 + {}^-\$220 = \mathbf{^-\$20}$.

16. **(a)** $3 - (2 - 4x) = 3 + {}^-(2 + {}^-4x)$
$= 3 + {}^-(2) + {}^-(^-4x)$
$= 3 + {}^-2 + 4x = \mathbf{1 + 4x}$.

(b) $x - (^-x - y) = x + {}^-(^-x + {}^-y)$
$= x + {}^-(^-x) + {}^-(^-y)$
$= x + x + y = \mathbf{2x + y}$.

17. It can be shown that for all integers a, b, and c;

$a - b + c = a + {}^-b + c$ Theorem 5 - 4
$= a + {}^-(b + {}^-c)$ Theorem 5 – 3 and Theorem 5 – 4
$= a - (b - c)$ Theorem 5 – 4

thus the original equation holds true for all integers.

18. **(a)** $W \cup I = \boldsymbol{I}$ $(W \subset I)$.

(b) $W \cap I = \boldsymbol{W}$ $(W \subset I)$.

(c) $I^+ \cup I^- = \boldsymbol{I} - \{\mathbf{0}\}$. 0 is neither positive nor negative, so it must be removed from the sets of integers.

(d) $W - I = \boldsymbol{\emptyset}$. $W \subset I$, so there are no elements in W that are not in I.

19. Answers may vary. The key idea for the approach below is to make the sum across all rows, columns, and diagonals equal to zero.

$^-3$	4	$^-1$
2	0	$^-2$
1	$^-4$	3

20. If $y = {}^-x - 1$:

(a) $y = {}^-(^-1) - 1 = 1 + {}^-1 = \mathbf{0}$, when $x = {}^-1$

(b) $y = {}^-(100) + {}^-1 = {}^-100 + {}^-1 = \mathbf{^-101}$, when $x = 100$

(c) $y = {}^-(^-2) - 1 = 2 + {}^-1 = \mathbf{1}$, when $x = {}^-2$

(d) $y = {}^-(^-a) - 1 = a - \mathbf{1}$, when $x = {}^-a$

(e) $3 = {}^-x - 1 \Rightarrow 3 + 1 = {}^-x - 1 + 1$
$\Rightarrow 4 = {}^-x \Rightarrow x = \mathbf{^-4}$.

21. **(a)** The common difference is 1. The n^{th} term is $n + (^-41)$. Set the n^{th} term equal to the last term and solve for n.
$n + (^-41) = 40 \Rightarrow n = 81$. There are 81 terms in the arithmetic sequence.

(b) The common ratio is 2. The n^{th} term is $2^{(n+4)}$. Set the n^{th} term equal to the last term and solve for n.
$2^{n+4} = 4096 \Rightarrow 2^{n+4}$
$= 2^{12} \Rightarrow n + 4 = 12 \Rightarrow n = 8$.
There are 8 terms in the geometric sequence.

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22. The sum of the first 100 negative integers = $^-1 + ^-2 + ^-3 + \ldots + ^-98 + ^-99 + ^-100$. Using the Gauss method from Chapter 1: $\frac{(^-1 + ^-100)100}{2} = ^-5050$. The sum is $^-5050$.

23. The sum of $^-40 + ^-39 + ^-38 + \ldots + 38 + 39 + 40$ Using the Gauss method from Chapter 1: $\frac{(^-40 + 40)81}{2} = 0$. The sum is 0.

24. Answers vary. If the east coast time is 0, then the Pacific time is $^-3$.

25. Write an equation:

$55\text{ mph} + 10\text{ mph over} + {}^-7\text{ mph under} + {}^-13\text{ mph under} = 45\text{ mph}.$

26. Leonine Trees shot the following: Bogey, Par, Eagle, Double Bogey, Birdie, Hole-in-one $1 + 0 + ^-2 + 2 + ^-1 + ^-3 = ^-3$. He is 3 under par for these holes.

27. **(a)** **All negative integers**. If x is negative, then its additive inverse, ^-x, is positive, i.e. $^-(^-x) = x$.

(b) **All positive integers**. If x is positive, then its additive inverse, ^-x, is negative.

(c) **All integers $< ^-1$**. If $^-x - 1 > 0 \Rightarrow {}^-x > 1 \Rightarrow x < ^-1$.

(d) $\boldsymbol{x = 2}$ **or** $\boldsymbol{x = ^-2}$. If $|x| = 2$, then x is a value on the number line that is 2 units away, in either direction, from 0.

28. **(a)** If $|x - 6| = 6$, then $x - 6 = 6$ or $x - 6 = ^-6$:

(i) If $x - 6 = 6 \Rightarrow x - 6 + 6 = 6 + 6 \Rightarrow \boldsymbol{x = 12}$, or

(ii) If $x - 6 = ^-6 \Rightarrow x - 6 + 6 = ^-6 + 6 \Rightarrow \boldsymbol{x = 0}$.

(b) $|x| + 2 = 10 \Rightarrow |x| + 2 - 2 = 10 - 2 \Rightarrow |x| = 8 \Rightarrow \boldsymbol{x = 8}$ **or** $\boldsymbol{x = ^-8}$.

(c) $|^-x| = |x|$ is **true for all integers**, since the distance from 0 to x on a number line is the same as the distance from 0 to ^-x.

29. To find the common difference (d) in an arithmetic sequence, subtract the first term from the second:

(a) $d = ^-3 - 0 = \boldsymbol{^-3}$. The next two terms are $\boldsymbol{^-12, ^-15}$.

(b) $d = x - (x + y) = \boldsymbol{^-y}$. The next two terms are $\boldsymbol{x - 2y}$, $\boldsymbol{x - 3y}$.

30. **(a)** **True**. Absolute value is always non-negative.

(b) **True**. Absolute value of the difference between the two elements is always non-negative.

(c) **True**. $^-x + ^-y = ^-(x + y)$, and $|^-(x + y)| = |x + y|$.

31. **(a)** $x + 7 = 3 \Rightarrow x + 7 - 7 = 3 - 7 \Rightarrow x = 3 + ^-7 \Rightarrow \boldsymbol{x = ^-4}$.

(b) $^-10 + x = ^-7 \Rightarrow ^-10 + 10 + x = ^-7 + 10 \Rightarrow \boldsymbol{x = 3}$.

(c) $^-x = 5 \Rightarrow ^-x + x = 5 + x \Rightarrow 0 = 5 + x \Rightarrow 0 + ^-5 = 5 + ^-5 + x \Rightarrow ^-5 = x \Rightarrow \boldsymbol{x = ^-5}$.

32. Estimates may vary.

(a) $327 + ^-52 - 398 \approx 325 + ^-50 - 400 \approx ^-125$. The exact answer is $^-123$.

(b) $^-1772 + 2005 - 503 \approx ^-1770 + 2000 - 500 \approx 270$. The exact answer is $^-270$.

(c) $996 - ^-10{,}007 - 102 \approx 1000 - ^-10{,}000 - 100 \approx 10{,}900$ The exact answer is 10,901.

(d) $^-303 - ^-1203 + 4997 \approx ^-300 - ^-1200 + 5000 \approx 5900$ The exact answer is 5897.

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Assessment 5-1B

1. For every integer a, there exists a unique integer ^{-}a, such that $a + {}^{-}a = 0 = {}^{-}a + a$. ^{-}a is called the additive inverse of a.

(a) The unique integer $\mathbf{^{-}3}$ is the additive inverse of 3 because $3 + {}^{-}3 = 0$.

(b) The additive inverse of ^{-}a can be written as $^{-}(^{-}a) = a$, or the number of the opposite sign. Thus the additive inverse of $^{-}4$ is $^{-}(^{-}4) = 4$.

(c) The additive inverse of q is $\mathbf{^{-}q}$.

(d) The additive inverse of 6 is $\mathbf{^{-}6}$ because $6 + {}^{-}6 = 0$.

(e) The additive inverse of ^{-}n is $^{-}(^{-}n) = \boldsymbol{n}$.

(f) The additive inverse of $(3 + x)$ is $^{-}(3 + x) = \mathbf{^{-}3} + \mathbf{^{-}}\boldsymbol{x}$.

2. **(a)** $^{-}(^{-}5)$ means the additive inverse of $^{-}5$, or **5**.

(b) $^{-}(^{-}x) = \boldsymbol{x}$.

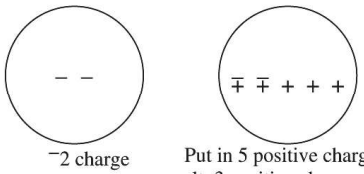
3. **(a)** Absolute value is the distance on a number line between the origin (0) and a specified number. Distance on the number line between 0 and $^{-}3$ is 3 units, so $|{}^{-}3| = \mathbf{3}$.

(b) Distance on the number line between 0 and 15 is 15 units, so $|15| = \mathbf{15}$.

(c) $^{-}|{}^{-}3|$ means the additive inverse of the absolute value of $^{-}3$, so $^{-}|{}^{-}3| = {}^{-}(3) = \mathbf{^{-}3}$.

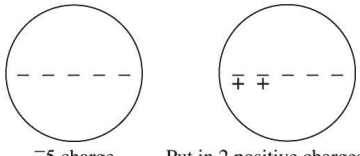
(d) $\mathbf{^{-}|6| = {}^{-}6}$

4. **(a)** Charged-field model:



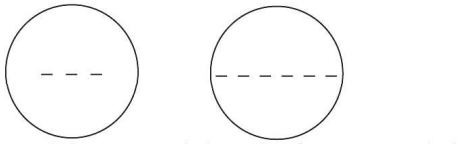
$^{-}2$ charge. Put in 5 positive charges. Net result: 3 positive charges; answer: 3

(b) Charged-field model:



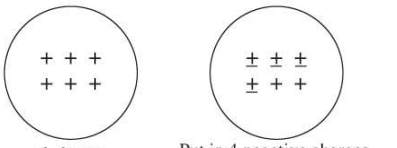
$^{-}5$ charge. Put in 2 positive charges. Net result: 3 negative charges; answer: $^{-}3$

(c) Charged-field model:



$^{-}3$ charge. Put in 3 negative charges: Net result: 6 negative charges; answer: $^{-}6$

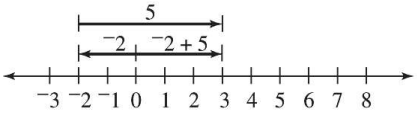
(d) Charged-field model:



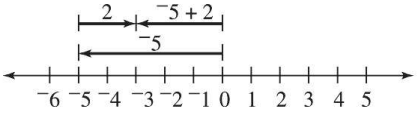
6 charge. Put in 4 negative charges. Net result: 2 positive charges; answer: 2

5. Movement on the number line in addition is always to the right for a positive number and to the left for a negative number.

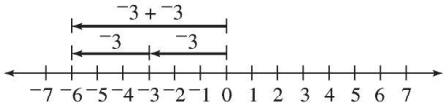
(a)



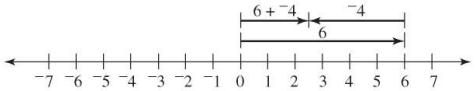
(b)



(c)



(d)



6. **(a)**
$$\begin{aligned} 5 + {}^{-}31 &= {}^{-}(|{}^{-}31| - |5|) \\ &= {}^{-}(31 - 5) \\ &= {}^{-}(26) \\ &= {}^{-}26 \end{aligned}$$

(b)
$$\begin{aligned} {}^{-}5 + {}^{-}31 &= {}^{-}(|{}^{-}5| + |{}^{-}31|) \\ &= {}^{-}(5 + 31) \\ &= {}^{-}(36) \\ &= {}^{-}36 \end{aligned}$$

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7. Using the theorem that $a - b = a + {}^{-}b$:

(a) ${}^{-}3 - 5 = {}^{-}3 + {}^{-}5 = \mathbf{{}^{-}8}$.

(b) $5 - {}^{-}3 = 5 + {}^{-}({}^{-}3) = 5 + 3 = \mathbf{8}$.

(c) ${}^{-}2 - ({}^{-}3) = {}^{-}2 + {}^{-}({}^{-}3) = -2 + 3 = \mathbf{1}$.

8. **(a)** ${}^{-}3 - 5 = n$ if and only if ${}^{-}3 = 5 + n$.
Thus $\boldsymbol{n = {}^{-}8}$.

(b) $5 - ({}^{-}3) = n$ if and only if $5 = {}^{-}3 + n$.
Thus $\boldsymbol{n = 8}$.

(c) ${}^{-}\mathbf{2} - ({}^{-}\mathbf{3}) = n$ if and only if ${}^{-}\mathbf{2} = {}^{-}\mathbf{3} + n$.
Thus $\boldsymbol{n = 1}$.

9. **(a)** ${}^{-}\$200 + \$100 + {}^{-}\$50 = {}^{-}\$250 + \$100 = {}^{-}\$150$, or a **drop in net worth of $150**.

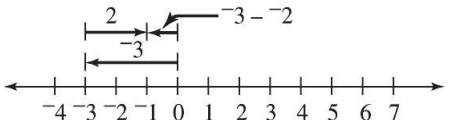
(b) ${}^{-}2 + 7 + 0 + {}^{-}8 = {}^{-}10 + 7 = {}^{-}3$, or a **loss of 3 yards**.

(c) $17 + {}^{-}8 + {}^{-}9 + 14 + 45 = 9 + {}^{-}9 + 14 + 45 = 0 + 14 + 45 = \mathbf{59\ points}$.

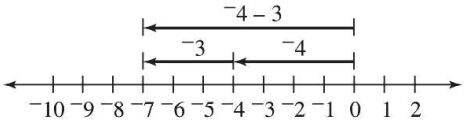
10. **(a)** ${}^{-}247 + {}^{-}11 + {}^{-}11 = {}^{-}258 + {}^{-}11 = \mathbf{{}^{-}269^{\circ}\,C}$.

(b) $98 - {}^{-}94 = 98 + {}^{-}({}^{-}94) = 98 + 94 = \mathbf{192^{\circ}F}$.

11. **(a)** In the number-line model of ${}^{-}3 - {}^{-}2$, ${}^{-}3$ is depicted as a line that starts at 0 and moves 3 units to the left to ${}^{-}3$; the operation of *subtraction* now changes the direction of ${}^{-}2$ from moving left (as ${}^{-}2$ world during addition) to moving *right* 2 units, ending at ${}^{-}1$. Thus ${}^{-}3 - {}^{-}2 = \mathbf{{}^{-}1}$:



(b) ${}^{-}4$ is depicted as a line that starts at 0 and moves 4 units to the left; subtraction of 3 is a line that moves 3 units to the left, ending at ${}^{-}7$. Thus ${}^{-}4 - 3 = \mathbf{{}^{-}7}$:



12. **(a)** Starting with subtraction that is already known:

$${}^{-}2 - 1 = {}^{-}3$$

Continuing the pattern:

$${}^{-}2 - 0 = {}^{-}2$$
$${}^{-}2 - {}^{-}1 = {}^{-}1$$
$${}^{-}2 - {}^{-}2 = 0$$
$${}^{-}2 - {}^{-}3 = \mathbf{1}.$$

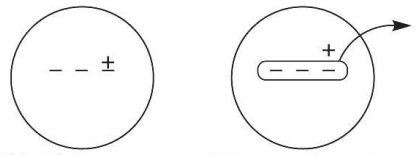
(b) Starting with subtraction that is already known:

$$3 - 2 = 1$$

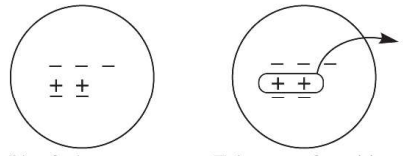
Continuing the pattern:

$$2 - 2 = 0$$
$$1 - 2 = {}^{-}1$$
$$0 - 2 = {}^{-}2$$
$${}^{-}1 - 2 = {}^{-}3$$
$${}^{-}2 - 2 = {}^{-}4$$
$${}^{-}3 - 2 = \mathbf{{}^{-}5}.$$

13. **(a)**



(b)



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14. **(a)** $^-2 - (7 + 10) = {}^-2 - (17) =$ $^-2 + {}^-17 = {}^-\mathbf{19}$.

(b) $8 - 11 - 10 = 8 + {}^-11 + {}^-10 =$ $^-3 + {}^-10 = {}^-\mathbf{13}$.

(c) $^-2 - 7 + 3 = {}^-2 + {}^-7 + 3 =$ $^-9 + 3 = {}^-\mathbf{6}$.

15. Using values picked from the graph:

(a) 10W-30 or 10W-40.

(b) 5W-30.

(c) 5W-30, 10W-30, or 10W-40.

(d) None of the oils shown.

(e) 10W-30 or 10W-40.

16. **(a)** $4x - 2 - 3x = 4x + {}^-(3x) + {}^-(2) =$ $x - 2$.

(b) $4x - (2 - 3x) = 4x + {}^-(2 + {}^-3x) =$ $4x + {}^-(2) + {}^-({}^-3x) = 4x + 3x + {}^-2 =$ $\mathbf{7}\boldsymbol{x} \mathbf{- 2}$.

17. **(a)** $^-x - y = {}^-x + {}^-y = {}^-y + {}^-x = {}^-y - x$.

(b) **No**. $^-x - y = {}^-x + {}^-y$ and $^-y - x =$ $^-y + {}^-x$, so the theorem used in part (a) is simply the commutative property of integer addition. In general, $x - y \neq y - x$.

18. **(a)** $W - I^+ = \{\mathbf{0}\}$ $(0 \notin I^+)$.

(b) $W - I^- = \boldsymbol{W}$. W and I^- are disjoint.

(c) $I \cap I = \boldsymbol{I}$ $(I \subseteq I)$.

(d) $I - W = I^-$

19. Adding all numbers yields $^-9$; dividing by 3 indicates that each row, column, and diagonal must total $^-3$. Possible solutions are:

8	$^-7$	$^-4$
$^-13$	$^-1$	11
2	5	$^-10$

or

2	$^-13$	8
5	$^-1$	$^-7$
$^-10$	11	$^-4$

20. If $y = {}^-3x - 2$:

(a) $y = {}^-3({}^-1) - 2 = 3 + {}^-2 = \mathbf{1}$, when $x = {}^-1$.

(b) $y = {}^-3(100) + {}^-2 = {}^-300 + {}^-2 = {}^-\mathbf{302}$, when $x = 100$.

(c) $y = {}^-3({}^-2) - 2 = 6 + {}^-2 = \mathbf{4}$, when $x = {}^-2$.

(d) $y = {}^-3({}^-a) - 2 = \boldsymbol{3a - 2}$, when $x = {}^-a$.

(e) $^-11 = {}^-3x - 2 \Rightarrow {}^-9 = {}^-3x \Rightarrow \boldsymbol{x = 3}$.

21. **(a)** The common difference is 1. The n^{th} term is $n + (^-28)$. Set the n^{th} term equal to the last term and solve for n.
$n + ({}^-28) = 52 \Rightarrow n = 80$. There are 80 terms in the arithmetic sequence.

(b) The common ratio is 3. The n^{th} term is $3^{(n+2)}$. Set the n^{th} term equal to the last term and solve for n. $3^{n+2} = 3^{10} \Rightarrow n + 2 = 10 \Rightarrow n = 8$. There are 8 terms in the geometric sequence.

22. The sum of $^-27 + {}^-26 + {}^-25 + \ldots + 50 + 51 + 52$
We know from 21(a) that there are 80 terms.
Using the Gauss method from Chapter 1:
$\frac{({}^-27 + 52)80}{2} = 1000$. The sum is 1000.

23. Write an equation:
$372 + {}^-9 + 4 + {}^-13 + {}^-24 = 330\,\text{lbs}$.

24. Write an equation:
$65 + {}^-4 = 61\,\text{seconds}.$.

25. Answers vary. If x represents the amount of memory used $x + {}^-3.6 - {}^-7.2 = x + 3.6\,\text{MB}$. She gained 3.6 MB of memory.

26. **(a)** There are **no values** of x for which this is true. If $^-|x| = 2$, then $|x| = {}^-2$, which is not possible.

(b) **All integers except 0.** Since $|x|$ is always non-negative, $^-|x|$ is always negative for all integers except 0; 0 is neither negative nor positive.

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(c) There are **no values** of x for which this is true. $^{-}|x|$ is always negative for all integers except 0, which is neither negative nor positive.

(d) $^{-}x + 1$ is positive means $-x + 1 > 0$.
$-x + 1 > 0 \Rightarrow -x > -1 \Rightarrow \boldsymbol{x < 1}$.

(e) $^{-}x - 1$ is negative means $-x - 1 < 0$.
$-x - 1 < 0 \Rightarrow -x < 1 \Rightarrow \boldsymbol{x > -1}$.

27. If $y = |x - 5|$:

(a) $y = |(10) - 5| = |10 + {}^{-}5| = |5| = \mathbf{5}$, when $x = 10$.

(b) $f(^{-}1) = |(^{-}1) - 5| = |{}^{-}1 + {}^{-}5| = |{}^{-}6| = \mathbf{6}$, when $x = {}^{-}1$.

(c) If $7 = |x - 5| \Rightarrow x - 5 = 7$ or $x - 5 = {}^{-}7$:

(i) $x - 5 = 7 \Rightarrow x = 7 + 5 \Rightarrow \boldsymbol{x = 12}$.

(ii) $x - 5 = {}^{-}7 \Rightarrow x = {}^{-}7 + 5 \Rightarrow x = {}^{-}\mathbf{2}$.

28. $y = \begin{cases} x - 6 & \text{if } x \geq 6 \\ {}^{-}x + 6 & \text{if } x < 6 \end{cases}$

29. **(a)** $d = 3 - 7 = {}^{-}\mathbf{4}$. The next two terms are $^{-}\mathbf{9}, {}^{-}\mathbf{13}$.

(b) $d = (1 - x) - (1 - 3x) = 1 - x - 1 + 3x = \mathbf{2}\boldsymbol{x}$. The next two terms are $\mathbf{1 + 3}\boldsymbol{x}$, $\mathbf{1 + 5}\boldsymbol{x}$.

30. **(a)** **True**. The square of integer is always non-negative.

(b) **False**. Let $x = {}^{-}2$. Then $|(^{-}2)^3| = |{}^{-}8| = 8 \neq (^{-}2)^3 = {}^{-}8$.

(c) **True**. $|x^3| = |x^2 \cdot x| = |x^2| \cdot |x| = x^2 \cdot |x|$.

31. **(a)** $^{-}x + 5 = 7 \Rightarrow {}^{-}x + 5 + {}^{-}5 = 7 + {}^{-}5 \Rightarrow {}^{-}x = 2 \Rightarrow \boldsymbol{x = {}^{-}2}$.

(b) $1 - x = {}^{-}13 \Rightarrow 1 + {}^{-}1 + {}^{-}x = {}^{-}13 + {}^{-}1 \Rightarrow {}^{-}x = {}^{-}14 \Rightarrow \boldsymbol{x = 14}$.

(c) $^{-}x - 8 = {}^{-}9 \Rightarrow {}^{-}x + {}^{-}8 + 8 = {}^{-}9 + 8 \Rightarrow {}^{-}x = {}^{-}1 \Rightarrow \boldsymbol{x = 1}$.

32. Estimates may vary depending upon the method used:

(a) Estimate:
$343 + {}^{-}42 - 402 \approx 300 - 400 = {}^{-}\mathbf{100}$
Actual: $^{-}101$.

(b) Estimate:
${}^{-}1992 + 3005 - 497 \approx {}^{-}2000 + 3000 - 500 = \mathbf{500.}$
Actual: 516.

(c) Estimate: $992 - {}^{-}10,003 - 101 \approx 1000 + 10,000 - 100 = \mathbf{10,900}$.
Actual: 10,894.

(d) Estimate: $^{-}301 - {}^{-}1303 + 4993 \approx {}^{-}300 + 1300 + 5000 = \mathbf{6000}$.
Actual: 5995.

Assessment 5-2A: Multiplication and Division of Integers

1. $3 \cdot {}^{-}1 = {}^{-}1 + {}^{-}1 + {}^{-}1 = {}^{-}3$

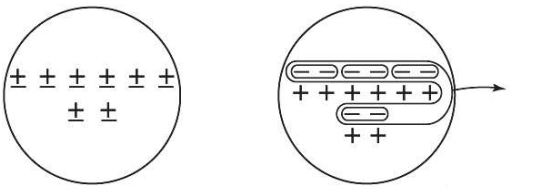
$2 \cdot {}^{-}1 = {}^{-}1 + {}^{-}1 = {}^{-}2$

$1 \cdot {}^{-}1 = {}^{-}1$

$0 \cdot {}^{-}1 = 0$

$^{-}1 \cdot {}^{-}1 = 1$.

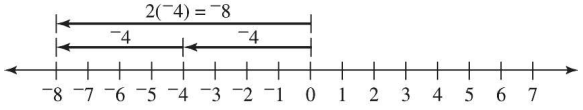
2.



0 charge

Take away 4 groups of 2 negative charges.
Net result is 8 positive charges; answer 8

3. Move four units to the left twice to arrive at the product $2(^{-}4) = {}^{-}8$:



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4. **(a)** Take away three groups of three negative charges, leaving nine positive charges; i.e., $\mathbf{(^{-}3)\cdot(^{-}3)=9}$.

(b) Take away five groups of two positive charges, leaving ten negative charges; i.e., $\mathbf{(5)(^{-}2)=^{-}10}$.

5. **(a)** The number of students will decrease by 30 per year over the next four years, or $4\cdot{}^{-}30={}^{-}120$.

(b) There were 30 more students per year four years ago, or $^{-}4\cdot{}^{-}30=120$.

(c) The number of students will decrease by 30 per year over the next n years, or $\boldsymbol{n\cdot{}^{-}30={}^{-}30n}$ students.

(d) There were 30 more students per year during the past n years, or $\boldsymbol{{}^{-}n\cdot{}^{-}30=30n}$ students.

6. **(a)** $^{-}40\div{}^{-}5\Rightarrow{}^{-}40={}^{-}5\cdot 8$
$\Rightarrow{}^{-}40\div{}^{-}5=8$.

(b) $^{-}143=13\cdot{}^{-}11\Rightarrow{}^{-}143\div 13={}^{-}\mathbf{11}$.

(c) $^{-}5\neq 0\cdot$ any integer $\Rightarrow{}^{-}5\div 0$ **is not defined** because no integer exists to satisfy the multiplication statement.

7. **(a)** $(^{-}10\div{}^{-}2)(^{-}2)=5\cdot{}^{-}2={}^{-}\mathbf{10}$.

(b) $(^{-}10\cdot 5)\div 5={}^{-}50\div 5={}^{-}\mathbf{10}$.

(c) $^{-}8\div(^{-}8+8)={}^{-}8\div 0\Rightarrow$ **not defined** because there is no integer that would make the multiplication statement true.

(d) $(^{-}6+6)\div(^{-}2+2)=0\div 0\Rightarrow$ **not defined** because too many integers can make the multiplication equation true.

(e) $|^{-}24|\div\left[4\cdot(9-15)\right]=24\div 4\cdot{}^{-}6={}^{-}1$. Order of operations specifies that multiplication and division be performed from left to right after the operations within the absolute value and parentheses are performed.

8. For each of the following, if $a\cdot b=c\Rightarrow c\div b=a$ and $c\div a=b\,(a\neq 0$ and $b\neq 0)$:

(a) ***(i)*** $^{-}6\cdot 5={}^{-}\mathbf{30}$.

(ii) $^{-}30\div 5={}^{-}\mathbf{6}$.

(iii) $^{-}30\div{}^{-}6=\mathbf{5}$.

(b) ***(i)*** $(^{-}5)\cdot(^{-}4)=\mathbf{20}$.

(ii) $20\div{}^{-}5={}^{-}\mathbf{4}$.

(iii) $20\div{}^{-}4={}^{-}\mathbf{5}$.

(c) ***(i)*** $^{-}3\cdot 0=\mathbf{0}$.

(ii) $0\div{}^{-}3=\mathbf{0}$.

(iii) $0\div 0$ is undefined.

9. **(a)** $4x\div 4=n$ if and only if $4x=4n$, where n is an integer. Then $4x=4n$ if and only if $\boldsymbol{n=x}$, so $4x\div 4=x$.

(b) $(^{-}xy)\div y=n$ if and only if $(^{-}xy)=yn$. Then $(^{-}xy)=yn$ if and only if $\boldsymbol{n={}^{-}x}\,(y\neq 0)$.

10. **(a)** $32^{\circ}\text{C}+(30\text{ minutes}\times{}^{-}1^{\circ}\text{ per minute})$
$=32^{\circ}+{}^{-}30^{\circ}=\mathbf{2^{\circ}C}$.

(b) $0^{\circ}\text{C}+(^{-}25\text{ minutes}\times{}^{-}2^{\circ}\text{ per minute})$
$=0^{\circ}+50^{\circ}=\mathbf{50^{\circ}\,C}$.

(c) $^{-}20^{\circ}\text{C}+(^{-}30\text{ minutes}\times{}^{-}2^{\circ}\text{ per minute})$
$={}^{-}20^{\circ}+60^{\circ}=\mathbf{40^{\circ}\,C}$.

(d) $25^{\circ}\text{C}+(^{-}20\text{ minutes}\times 2^{\circ}\text{ per minute})$
$=25^{\circ}+{}^{-}40^{\circ}={}^{-}\mathbf{15^{\circ}\,C}$.

11. $(^{-}12{,}000\text{ acres per year}\times 8\text{ years})=$ $^{-}96{,}000$ acres, or 96,000 acres lost.

12. $^{-}1(^{-}4-{}^{-}2)={}^{-}1\cdot{}^{-}4-{}^{-}1\cdot{}^{-}2=4-2=2$

13. **(a)** $(^{-}2)^{3}={}^{-}2\cdot{}^{-}2\cdot{}^{-}2=(^{-}2\cdot{}^{-}2)\,{}^{-}2=4\cdot{}^{-}2={}^{-}\mathbf{8}$.

(b) $(^{-}2)^{4}=(^{-}2\cdot{}^{-}2)\cdot(^{-}2\cdot{}^{-}2)=4\cdot 4=\mathbf{16}$.

(c) $(^{-}10)^{5}\div(^{-}10)^{2}={}^{-}100{,}000\div 100={}^{-}\mathbf{1000}$.

(d) $(^{-}3)^{5}\div(^{-}3)={}^{-}243\div{}^{-}3=\mathbf{81}$.

(e) $(^{-}1)^{50}=\mathbf{1}$ (i.e., $^{-}1$ taken to an even power).

(f) $(^{-}1)^{151}={}^{-}\mathbf{1}$ (i.e., $^{-}1$ taken to an odd power).

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(g) $^-2 + 3 \cdot 5 - 1 = {}^-2 + 15 - 1 = 13 - 1 =$ **12**. Note the order of operation; multiplication before addition.

(h) $10 - 3 \cdot 7 - 4(^-2) + 3 = 10 - 21 - {}^-8 + 3 = (10 + 8 + 3) - 21 = \mathbf{0}$.

(i) $(^-2)^{64} - 2^{64} = 2^{64} - 2^{64} = \mathbf{0}$.

(j) $^-2^8 + 2^8 = \mathbf{0}$.

14. (a) Always **negative**. x^2 is always positive, so its additive inverse is always negative.

(b) Always **positive**. Any non-zero number taken to an even power is always positive.

(c) Always **positive**. Any non-zero number taken to an even power is always positive.

(d) **Positive when x is negative**; **negative when x is positive**. i.e., the additive inverse of x^3, which may be positive or negative.

(e) **Positive when x is negative** (the additive inverse of a negative integer is positive). **Negative when x is positive** (the additive inverse of a positive integer is negative).

15. (a) $^-x^2 = x^2$ for $x = 0$.

(b) $^-x^3 = (^-x)^3$ for all integers.

16. If $48 \div x$ is an integer then x equals all positive and negative divisors of 48. The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48. The negative divisors of 48 are

$^-1$, $^-2$, $^-3$, $^-4$, $^-6$, $^-8$, $^-12$, $^-16$, $^-24$, and $^-48$.

17. (a) **Commutative property of multiplication**, or $a \cdot b = b \cdot a$.

(b) **Closure property of addition**. The sum of an integer plus another integer is an integer.

(c) **Associative property of multiplication**, or $a(b \cdot c) = (a \cdot b)c$.

(d) **Distributive property of multiplication over addition**, or $a(b + c) = ab + ac$.

18. (a) $(^-x)(^-y) = (^-1 \cdot x)(^-1 \cdot y)$
$= (^-1 \cdot {}^-1) \cdot (x \cdot y)$
$= 1(xy) = \boldsymbol{xy}$.

(b) $^-2x(^-y) = \mathbf{2xy}$.

(c) $^-2(^-x + y) + x + y = 2x - 2y + x + y = \mathbf{3x - y}$.

(d) $^-1 \cdot x = {}^-\boldsymbol{x}$.

19. (a) $^-2(x - y) = {}^-2x - {}^-2y = {}^-\mathbf{2x} + \mathbf{2y}$.

(b) $x(x - y) = x \cdot x - x \cdot y = \boldsymbol{x^2 - xy}$.

(c) $^-x(x - y) = {}^-x^2 - {}^-xy = {}^-\boldsymbol{x^2} + \boldsymbol{xy}$.

(d) $^-2(x + y - z) = {}^-2x + {}^-2y - {}^-2z = {}^-\mathbf{2x} - \mathbf{2y} + \mathbf{2z}$.

20. (a) $^-3x = 6 \Rightarrow \frac{^-3x}{3} = \frac{6}{^-3} \Rightarrow \boldsymbol{x = {}^-2}$.

(b) $^-2x = 0 \Rightarrow \frac{^-2x}{^-2} = \frac{0}{^-2} \Rightarrow \boldsymbol{x = 0}$.

(c) $x \div 3 = {}^-12 \Rightarrow 3(x \div 3) = 3(^-12)$
$\Rightarrow \boldsymbol{x = {}^-36}$.

(d) $x \div {}^-3 = {}^-2 \Rightarrow {}^-3(x \div {}^-3) = {}^-3(^-2)$
$\Rightarrow \boldsymbol{x = 6}$.

(e) $x \div (^-x) = {}^-1 \Rightarrow {}^-x(x \div {}^-x) = {}^-x(^-1)$
$\Rightarrow x = x$; i.e., the equation is satisfied by **all integers except 0** (since $0 \div 0$ is not defined).

(f) $^-3x - 8 = 7 \Rightarrow {}^-3x - 8 + 8 = 7 + 8$
$\Rightarrow {}^-3x = 15 \Rightarrow \frac{^-3}{^-3}x = \frac{15}{^-3} \Rightarrow \boldsymbol{x = {}^-5}$.

(g) $^-2(5x - 3) = 26 \Rightarrow {}^-10x + 6 = 26$
$\Rightarrow {}^-10x + 6 - 6 = 26 - 6 \Rightarrow {}^-10x = 20$
$\Rightarrow \frac{^-10}{^-10}x = \frac{20}{^-10} \Rightarrow \boldsymbol{x = {}^-2}$.

(h) $3x - x - 2x = 3 \Rightarrow 0 = 3 \Rightarrow$ **no solution**.

(i) $^-2(5x - 6) - 30 = {}^-x \Rightarrow {}^-10x + 12 - 30 = {}^-x \Rightarrow {}^-10x - 18 = {}^-x \Rightarrow {}^-18 = 9x \Rightarrow \boldsymbol{x = {}^-2}$.

(j) $x^2 = 4 \Rightarrow \boldsymbol{x = 2}$ or $\boldsymbol{x = {}^-2}$, since $2^2 = 4$ or $(^-2)^2 = 4$.

(k) $(x - 1)^2 = 9 \Rightarrow$

(i) $(x - 1) = 3 \Rightarrow \boldsymbol{x = 4}$, or

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(ii) $(x - 1) = {}^{-}3 \Rightarrow \boldsymbol{x = {}^{-}2}$.

(l) $(x - 1)^2 = (x + 3)^2 \Rightarrow$

(i) $(x - 1) = (x + 3) \Rightarrow 0 = 4 \Rightarrow$ not a viable solution.

(ii) $(x - 1) = {}^{-}(x + 3) \Rightarrow x - 1 = {}^{-}x - 3$
$\Rightarrow 2x = {}^{-}2 \Rightarrow \boldsymbol{x = {}^{-}1}$.

(m) $(x - 1)(x + 3) = 0 \Rightarrow$ either $(x - 1) = 0$ or $(x + 3) = 0$; i.e., the only way that the product of two numbers can be zero is that one or both must equal zero.

(i) **If** $(x - 1) = 0 \Rightarrow \boldsymbol{x = 1}$.

(ii) **If** $(x + 3) = 0 \Rightarrow \boldsymbol{x = {}^{-}3}$.

21. The difference-of-squares formula is:
$(a + b)(a - b) = a^2 - b^2$.

(a) $52 \cdot 48 = (50 + 2)(50 - 2) =$
$50^2 - 2^2 = 2500 - 4 = \mathbf{2496}$.

(b) $(5 - 100)(5 + 100) = 5^2 - 100^2 =$
$25 - 10{,}000 = {}^{-}\mathbf{9975}$.

(c) $({}^{-}x - y)({}^{-}x + y) = ({}^{-}x)^2 - y^2 =$
$\boldsymbol{x^2 - y^2}$.

22. To *factor* an expression means to find an equivalent expression that is a product; i.e., if $N = ab$, then a and b are factors of N. Factoring may be said to undo the distributive property of multiplication over addition or subtraction.

(a) $3x + 5x = x(3 + 5) = \boldsymbol{8x}$. The factor common to both terms, x, divides each and then multiplies their sum.

(b) $xy + x = x \cdot y + x \cdot 1 = \boldsymbol{x(y + 1)}$.

(c) $x^2 + xy = x \cdot x + x \cdot y = \boldsymbol{x(x + y)}$.

(d) $3xy + 2x - xz = \boldsymbol{x(3y + 2 - z)}$.

(e) $abc + ab - a = a(bc + b - 1) =$
$\boldsymbol{a[b(c + 1) - 1]}$.

(f) $16 - a^2 = 4^2 - a^2 = \boldsymbol{(4 + a)(4 - a)}$; i.e., the factorization of the difference-of-squares formula.

(g) $4x^2 - 25y^2 = (2x)^2 - (5y)^2$
$= \boldsymbol{(2x + 5y)(2x - 5y)}$.

23. $(a - b)^2 = (a - b)(a - b) = a(a - b)$
$+ {}^{-}b(a - b) = a^2 - ab + {}^{-}ba - {}^{-}b^2 =$
$a^2 - 2ab + b^2$.

24. **(a)** **(i)** Arithmetic sequence; **difference** $(d) =$
${}^{-}7 - {}^{-}10 = \mathbf{3}$.

(ii) The next two terms are: $5 + 3 = \mathbf{8}$ and $8 + 3 = \mathbf{11}$.

(iii) $a_n = a_1 + (n - 1)d = {}^{-}10 +$
$(n - 1) \cdot 3 = {}^{-}10 + 3n - 3 =$
$\boldsymbol{3n - 13}$.

(b) **(i)** Geometric sequence; **ratio** $(r) =$
${}^{-}4 \div {}^{-}2 = \mathbf{2}$

(ii) The next two terms are: ${}^{-}64 \cdot 2 = {}^{-}\mathbf{128}$ and ${}^{-}128 \cdot 2 = {}^{-}\mathbf{256}$.

(iii) $a_n = a_1(r)^{n-1} = {}^{-}2(2)^{n-1} = {}^{-}\boldsymbol{2^n}$.

(c) **(i)** Geometric sequence; $r = {}^{-}2^2 \div 2 = {}^{-}\mathbf{2}$.

(ii) The next two terms are: ${}^{-}2^6 \cdot {}^{-}2 = \mathbf{2^7}$ and $2^7 \cdot {}^{-}2 = {}^{-}\mathbf{2^8}$.

(iii) $a_n = 2({}^{-}2)^{n-1} = {}^{-}({}^{-}2)({}^{-}2)^{n-1} =$
${}^{-}({}^{-}2)^{n-1+1} = {}^{-}\boldsymbol{({}^{-}2)^n}$.

25. **(a)** ${}^{-}3, {}^{-}9, \_, \_, {}^{-}243$ is a geometric sequence with a common ratio of 3. The missing terms are ${}^{-}27$ and ${}^{-}81$.

(b) $\_, 32, {}^{-}16, 8, \_$ is a geometric sequence. The common ratio is $\boldsymbol{{}^{-}\frac{1}{2}}$. The missing terms are ${}^{-}64$ and ${}^{-}4$.

26. Write an equation: ${}^{-}5(6) + 3(8) = {}^{-}30 + 24 = {}^{-}6$. The net result is a descent of 6 feet.

27. Let n be the number of cups. The height of each cup is 7 and ${}^{-}6$ represents the unseen part of each cup when they are nested together. There are $(n\text{-}1)$ cups with unseen parts. The total height of n cups can then be written as:
$7n + ({}^{-}6)(n - 1) = 7n - 6n + 6 = n + 6$.

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Assessment 5-2B

1. $3 \cdot {}^{-}2 = {}^{-}2 + {}^{-}2 + {}^{-}2 = {}^{-}6$

$2 \cdot {}^{-}2 = {}^{-}2 + {}^{-}2 = {}^{-}4$

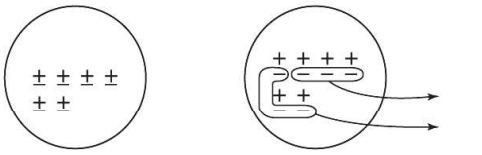
$1 \cdot {}^{-}2 = {}^{-}2$

$0 \cdot {}^{-}2 = 0$

${}^{-}1 \cdot {}^{-}2 = 2$

${}^{-}2 \cdot {}^{-}2 = 4.$

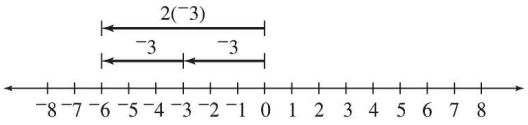
2.



0 charge

Take away 2 groups of 3 negative charges. Net result is 6 positive charges

3. Move three units to the left twice to arrive at the product $2({}^{-}3) = {}^{-}6$:



4. **(a)** Take away two groups of three negative charges, leaving six positive charges; i.e., $\mathbf{({}^{-}2) \cdot ({}^{-}3) = 6}$.

(b) Take away four groups of two positive charges, leaving eight negative charges; i.e., $\mathbf{({}^{-}4) \cdot 2 = {}^{-}8}$.

5. **(a)** $4({}^{-}20) = {}^{-}80$ **students**

(b) $({}^{-}5)({}^{-}20) =$ **100 students**

(c) $n({}^{-}20) = \mathbf{-20}n$ **students**

(d) $({}^{-}n)({}^{-}20) = \mathbf{20}n$ **students**

6. **(a)** $143 \div {}^{-}10 \Rightarrow 143 = {}^{-}10 \cdot x$. The quotient is undefined in the set of integers because there is no integer to multiply times $^{-}10$ to obtain 143.

(b) $0 \div ({}^{-}6) \Rightarrow 0 = {}^{-}6 \cdot 0 \Rightarrow 0 \div {}^{-}6 = 0.$

7. **(a)** Let $(a \div b)b = x$. Then $a \div b = x \div b \Rightarrow a = x$ and $(a \div b)b = \boldsymbol{a}$ for any integers a and b, $b \neq 0$.

(b) Let $(a \cdot b) \div b = x$. Then $a \cdot b = x \cdot b \Rightarrow a = x$ and $(a \cdot b) \div b = \boldsymbol{a}$ for any integers a and b, $b \neq 0$.

(c) $({}^{-}8 + 8) \div 8 = 0 \div 8 = \mathbf{0}$.

(d) $({}^{-}23 - {}^{-}7) \div 4 = {}^{-}16 \div 4 = {}^{-}4$ Order of operations specifies that the subtraction within the parentheses be performed first.

(e) $|{}^{-}28| \div [2|{}^{-}7|] = 28 \div [2(7)] = 28 \div 14 = \mathbf{2}$

8. For each of the following, if $a \cdot b = c \Rightarrow c \div b = a$ and $c \div a = b$ $(a \neq 0$ and $b \neq 0)$:

(a) ***(i)*** $^{-}5 \cdot 4 = {}^{-}20$

(ii) $^{-}20 \div 4 = {}^{-}5$

(iii) $^{-}20 \div {}^{-}5 = 4.$

(b) ***(i)*** $({}^{-}4) \cdot ({}^{-}3) = 12$

(ii) $12 \div ({}^{-}4) = {}^{-}3$

(iii) $12 \div ({}^{-}3) = {}^{-}4.$

(c) $0 \div 0$ is not defined. To understand why suppose there is one answer and name it x. If $0 \div 0 = x$, then $0 \cdot x = 0$. Notice that any real numbers could replace x and $0 \cdot x = 0$ would be true. For example, $0 \cdot 0 = 0$ and $0 \cdot 13 = 0$. Since the solution to $0 \cdot x = 0$ is not unique, $0 \div 0$ is not defined.

9. **(a)** $({}^{-}4x) \div x = n$ if and only if $({}^{-}4x) = xn$. Then $({}^{-}4x) = xn$ if and only if $\boldsymbol{n = {}^{-}4}$ $(x \neq 0)$.

(b) $({}^{-}10x + 5) \div 5 = n$ if and only if $^{-}10x + 5 = 5n$. $^{-}10x + 5 = 5n$ if and only if $5 \cdot ({}^{-}2x + 1) = 5n$. $5 \cdot ({}^{-}2x + 1) = 5n$ if and only if $\boldsymbol{n = {}^{-}2x + 1}$.

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10. (a) $^-5°\text{C} + (m \text{ minutes} \times d° \text{ per minute}) = {}^-5° + md° = \mathbf{(^-5 + md)°\ C}$.

(b) $0°\text{C} + (^-m \text{ minutes} \times {}^-d° \text{ per minute}) = 0° + md° = \mathbf{md°C}$.

(c) $20°\text{C} + (^-m \text{ minutes} \times d° \text{ per minute}) = 20° + {}^-md° = \mathbf{(20 - md)°\ C}$.

11. (a) $4 \cdot {}^-11 = {}^-44$, or **44 yards lost**.

(b) $^-66 \div 11 = {}^-6$, or **6 yards lost on average**.

12. $^-1(4 - {}^-2) = {}^-1 \cdot 4 - {}^-1 \cdot {}^-2 = {}^-4 - 2 = {}^-6$

13. (a) $10 - 3 - 12 = 7 - 12 = \mathbf{^-5}$.

(b) $10 - (3 - 12) = 10 - (^-9) = 10 + 9 = \mathbf{19}$.

(c) $(^-3)^2 = {}^-3 \cdot {}^-3 = \mathbf{9}$. A negative integer taken to an even power is positive.

(d) $^-3^2 = {}^-(3 \cdot 3) = \mathbf{^-9}$.

(e) $^-5^2 + 3(^-2)^2 = {}^-25 + 3 \cdot 4 = {}^-25 + 12 = \mathbf{^-13}$.

(f) $^-2^3 = {}^-(2 \cdot 2 \cdot 2) = \mathbf{^-8}$.

(g) $(^-2)^5 = {}^-2 \cdot {}^-2 \cdot {}^-2 \cdot {}^-2 \cdot {}^-2 \cdot = \mathbf{^-32}$. A negative integer taken to an odd power is negative.

(h) $^-2^4 = {}^-(2 \cdot 2 \cdot 2 \cdot 2) = \mathbf{^-16}$.

(i) $^-2^{63} + 2^{64} = 2^{63}(^-1 + 2) = \mathbf{2^{63}}$.

(j) $7^5 + {}^-7^5 = 0$.

14. (a) Always **negative**. The additive inverse of a positive integer.

(b) Always **positive**. The product of either four positive or four negative integers is positive; i.e., any non-zero integer taken to an even power is positive.

(c) Always **positive**. Any non-zero integer taken to an even power is positive.

(d) **Positive when x is positive; negative when x is negative**.

(e) **Positive when x is negative; negative when x is positive** (i.e., additive inverses).

15. (a) **Closure property of multiplication**. The product of two integers is an integer.

(b) **Zero multiplication property**. For all integers a, $a \cdot 0 = 0 = 0 \cdot a$.

(c) **Distributive property of multiplication over addition**, or $a(b + c) = ab + ac$.

(d) **Commutative property of multiplication**, or $a \cdot b = b \cdot a$.

16. (a) $x - 2(^-y) = x - {}^-2y = \mathbf{x + 2y}$.

(b) $a - (a - b)(^-1) = a - ({}^-a + b) = \mathbf{2a - b}$.

(c) $y - (y - x)(^-2) = y - (^-2y + 2x) = \mathbf{3y - 2x}$.

(d) $^-(x - y) + x = {}^-x - {}^-y + x = \mathbf{y}$.

17. (a) $^-x(x - y - 3) = {}^-x^2 - {}^-xy - {}^-3x = \mathbf{{}^-x^2 + xy + 3x}$.

(b) $(^-5 - x)(5 + x) = {}^-5(5 + x) + {}^-x(5 + x) = {}^- 25 + {}^-5x + {}^-5x + {}^-x^2 = \mathbf{^-25 - 10x - x^2}$.

(c) $(x - y - 1)(x + y + 1) = x(x + y + 1) - y(x + y + 1) - 1(x + y + 1) = x^2 + xy + x + {}^-xy + {}^-y^2 + {}^-y + {}^-x + {}^-y + {}^-1 = \mathbf{x^2 - y^2 - 2y - 1}$.

18. (a) $x^2 = 9 \Rightarrow (^-3)^2 = 9$ or $(3)^2 = 9$ $\Rightarrow \mathbf{x = {}^-3 \text{ or } x = 3}$.

(b) $x^2 = {}^-9 \Rightarrow$ **no solution**. x^2 is always positive.

(c) $^-x \div {}^-x = 1 \Rightarrow x = x$; i.e., the equation is satisfied by **all integers except 0**.

(d) $^-x^2$ is negative $\Rightarrow x$ may be **any integer except 0**. 0 is neither positive nor negative.

(e) $^-(1 - x) = x - 1 \Rightarrow {}^-1 - {}^-x = x - 1 \Rightarrow x$ may be **any integer**.

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(f) $x - 3x = {}^{-}2x \Rightarrow {}^{-}2x = {}^{-}2x \Rightarrow x$ may be **any integer**.

(g) ${}^{-}3(x + 2) = {}^{-}3x + 6 \Rightarrow {}^{-}3x + {}^{-}6 = {}^{-}3x + 6 \Rightarrow {}^{-}6 = 6 \Rightarrow$ **no solution**.

(h) $(2x - 1)^2 = (1 - 2x)^2$. True for **all integers**, since regardless of the value of x, the two sides of this equation will be additive inverses, and any number squared is non-negative.

(i) $x^3 = {}^{-}2^9 \Rightarrow x^3 = ({}^{-}2^3)^3$(i.e., ${}^{-}2^3 \cdot {}^{-}2^3 \cdot {}^{-}2^3 \cdot = {}^{-}2^9) \Rightarrow \boldsymbol{x = {}^{-}2^3 = {}^{-}8}$.

(j) ${}^{-}6x > {}^{-}x + 20 \Rightarrow {}^{-}5x > 20 \Rightarrow \boldsymbol{x < {}^{-}4}$. Note that the direction of the inequality changed under division by a negative number.

(k) ${}^{-}5(x - 3) > {}^{-}5 \Rightarrow x - 3 < 1 \Rightarrow \boldsymbol{x < 4}$.

(l) $x > {}^{-}2 \Rightarrow 5x > {}^{-}10 \Rightarrow 3 - 5x < 3 - {}^{-}10 \Rightarrow \boldsymbol{3 - 5x < 13}$.

(m) $x < 0 \Rightarrow 7x < 0 \Rightarrow 2 - 7x > 2 - 0 \Rightarrow \boldsymbol{2 - 7x > 2}$.

19. (a) $(2 + 3x)(2 - 3x) = 2^2 - (3x)^2 = \boldsymbol{4 - 9x^2}$.

(b) $(x - 1)(1 + x) = (x - 1)(x + 1) = x^2 - 1^2 = \boldsymbol{x^2 - 1}$.

(c) $213^2 - 13^2 = (213 + 13)(213 - 13) = 226 \cdot 200 = \mathbf{45{,}200}$.

20. To *factor* an expression means to find an equivalent expression that is a product; i.e., if $N = ab$, then a and b are factors of N. factoring may be said to undo the distributive property of multiplication over addition or subtraction.

(a) $ax + 2x = \boldsymbol{x(a + 2)}$.

(b) $ax - 2x = \boldsymbol{x(a - 2)}$.

(c) $3x - 4x + 7x = x(3 - 4 + 7) = \boldsymbol{6x}$.

(d) $3x^2 + xy - x = \boldsymbol{x(3x + y - 1)}$.

(e) $(a + b)(c + 1) - (a + b) = (a + b)[(c + 1) - 1] = \boldsymbol{(a + b)c}$.

(f) $x^2 - 9y^2 = x^2 - (3y)^2 = \boldsymbol{(x + 3y)(x - 3y)}$.

(g) $(x^2 - y^2) + x + y = (x + y)(x - y) + (x + y) = (x + y)[(x - y) + 1] = \boldsymbol{(x + y)(x - y + 1)}$.

21. $(a - 1)^2 = (a + {}^{-}1)(a + {}^{-}1)$

$= (a + {}^{-}1)a + (a + {}^{-}1)({}^{-}1)$

$= a^2 + {}^{-}1 \cdot a + a \cdot {}^{-}1 + {}^{-}1 \cdot {}^{-}1$

$= a^2 + {}^{-}a + {}^{-}a + 1$

$= a^2 + {}^{-}2a + 1$

$= a^2 - 2a + 1$

22. (a) ***(i)*** Arithmetic sequence; $\boldsymbol{d} = 7 - 10 = \mathbf{{}^{-}3}$.

(ii) The next two terms are: ${}^{-}5 + {}^{-}3 = \mathbf{{}^{-}8}$ and ${}^{-}8 + {}^{-}3 = \mathbf{{}^{-}11}$.

(iii) $a_n = 10 + (n - 1) \cdot {}^{-}3 = 10 - 3n + 3 = \mathbf{{}^{-}3n + 13}$.

(b) ***(i)*** Geometric sequence; $\boldsymbol{r} = 4 \div {}^{-}2 = \mathbf{{}^{-}2}$.

(ii) The next two terms are: $64 \cdot {}^{-}2 = \mathbf{{}^{-}128}$ and ${}^{-}128 \cdot {}^{-}2 = \mathbf{256}$.

(iii) $a_n = {}^{-}2({}^{-}2)^{n-1} = \mathbf{({}^{-}2)^n}$.

23. (a) ${}^{-}x = x^2$ for $x = {}^{-}1, 0$.

(b) ${}^{-}x^2 = ({}^{-}x)^3$ for $x = 0, 1$.

24. If $54 \div x$ is an integer then x equals all positive and negative divisors of 54. The factors of 54 are 1, 2, 3, 6, 9, 18, 27, and 54. The negative divisors of 54 are ${}^{-}1$, ${}^{-}2$, ${}^{-}3$, ${}^{-}6$, ${}^{-}9$, ${}^{-}18$, ${}^{-}27$, and ${}^{-}54$.

25. (a) ${}^{-}4, {}^{-}16, \_, \_, {}^{-}1024$ is a geometric sequence with a common ratio of 4. The missing terms are ${}^{-}64$ and ${}^{-}256$.

(b) $\_, 27, {}^{-}9, 3, \_$ is a geometric sequence. The common ratio is $\mathbf{{}^{-}\frac{1}{3}}$. The missing terms are ${}^{-}81$ and ${}^{-}1$.

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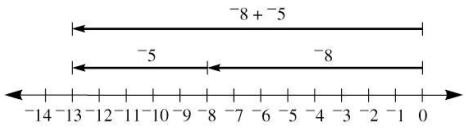
26. Write an equation: $^{-}4(6)+2(8)=^{-}24+16=^{-}8$. The net result is a descent of 8 feet.

27. Let n be the number of cups. The height of each cup is 8 and $^{-}6$ represents the unseen part of each cup when they are nested together. There are (n-1) cups with unseen parts. The total height of n cups can then be written as:
$8n+(^{-}6)(n-1)=8n-6n+6=2n+6.$

Mathematical Connections 5-2: Review Problems

18. Plot $^{-}$**8** on the number line then move 5 more units to the left to obtain $^{-}$**13**.



19. **(a)** Since $^{-}5+5=0$, **5** is the additive inverse.
(b) Since $7+^{-}7=0$, **7** is the additive inverse.
(c) Since $0+0=0$, **0** is the additive inverse.

20. **(a)** 14.
(b) 21.
(c) -4.
(d) 22.

21. **(a)** $|x-7|=0$ for $x=7$.
(b) $|x-7|<0$ has no solution because the absolute value can only be positive.
(c) $|x-7|>0$ is true for all integers other than 7.

22. Yes. When two integers are subtracted, their difference will always e another inter.

Chapter 5 Review

1. **(a)** The additive inverse of 3 is $^{-}$**3**.
(b) The additive inverse of ^{-}a is $^{-}(^{-}a)=\boldsymbol{a}$.
(c) The additive inverse of $^{-}2+3$ is $^{-}(^{-}2+3)=^{-}1$
(d) The additive inverse of $x+y$ is $^{-}(x+y)=^{-}x+^{-}y=^{-}\boldsymbol{x}-\boldsymbol{y}$.
(e) The additive inverse of $^{-}x+y$ is $^{-}(^{-}x+y)=x+^{-}y=\boldsymbol{x}-\boldsymbol{y}$.
(f) The additive inverse of $^{-}x-y$ is $^{-}(^{-}x-y)=x-^{-}y=\boldsymbol{x}+\boldsymbol{y}$.
(g) $(^{-}2)^5=^{-}32$, thus the additive inverse is **32**.
(h) $^{-}2^5=^{-}32$, thus the additive inverse is **32**.

2. **(a)** $(^{-}2+^{-}8)+3=(^{-}10)+3=^{-}$**7**.
(b) $^{-}2-(^{-}5)+5=^{-}2+5+5=$ **8**.
(c) $^{-}3(^{-}2)+2=6+2=$ **8**.
(d) $^{-}3(^{-}5+5)=^{-}3(0)=$ **0**.
(e) $^{-}40\div^{-}5=$ **8**.
(f) $(^{-}25\div5)(^{-}3)=(^{-}5)(^{-}3)=$ **15**.

3. **(a)** $^{-}x+3=0\Rightarrow{}^{-}x+3-3=0-3\Rightarrow{}^{-}x=^{-}3\Rightarrow{}^{-}x(^{-}1)=^{-}3(^{-}1)\Rightarrow x=3.$
(b) $^{-}2x=10\Rightarrow\frac{^{-}2x}{^{-}2}=\frac{10}{^{-}2}\Rightarrow x=^{-}5.$
(c) $0\div(^{-}x)=0\Rightarrow\frac{0}{^{-}x}(^{-}x)=0(^{-}x)\Rightarrow$ $0=0$; i.e., x may be **any integer except 0**.
(d) $^{-}x\div0=^{-}1\Rightarrow$ **no integer solution**. Division by 0 is undefined.
(e) $3x-1=^{-}124\Rightarrow3x=^{-}123\Rightarrow x=\frac{^{-}123}{3}$ $=^{-}$**41**
(f) $^{-}2x+3x=x\Rightarrow x(^{-}2+3)=x$ $\Rightarrow x=x$; i.e., x may be **any integer**.

4. $2\cdot{}^{-}3=(^{-}3)+(^{-}3)=^{-}6;$
$1\cdot{}^{-}3=^{-}3;$
$0\cdot{}^{-}3=0;$ and if the pattern continues:
$^{-}1\cdot{}^{-}3=3;$
$^{-}2\cdot{}^{-}3=6.$

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5. **(a)** Removing 5 black chips from $10 \Rightarrow 10 - 5 = \mathbf{5}$.

(b) When the 2 red chips are removed 3 black ones remain, thus $1 - {}^{-}2 = \mathbf{3}$.

6. **(a)** ${}^{-}1x = {}^{-}1 \cdot x = {}^{-}\boldsymbol{x}$.

(b) $({}^{-}1)(x - y) = {}^{-}1 \cdot x - {}^{-}1 \cdot y = {}^{-}x - {}^{-}y$
$= {}^{-}x + y = \boldsymbol{y - x}$.

(c) $2x - (1 - x) = 2x - 1 - {}^{-}x = 2x - 1 + x$
$= \mathbf{3}\boldsymbol{x} - \mathbf{1}$.

(d) $({}^{-}x)^2 + x^2 = ({}^{-}x \cdot {}^{-}x) + x^2 = x^2 + x^2$
$= \mathbf{2}\boldsymbol{x}^{\mathbf{2}}$.

(e) $({}^{-}x)^3 + x^3 = ({}^{-}x \cdot {}^{-}x \cdot {}^{-}x) + x^3$
$= {}^{-}x^3 + x^3 = \mathbf{0}$.

(f) $({}^{-}3 - x)(3 + x) = ({}^{-}3 - x)3 + ({}^{-}3 - x)x$
$= {}^{-}9 - 3x + {}^{-}3x - x^2 = {}^{-}9 - 6x - x^2$
$= {}^{-}\boldsymbol{x}^{\mathbf{2}} \boldsymbol{- 6x - 9}$.

(g) $({}^{-}2 - x)({}^{-}2 + x) = ({}^{-}2)^2 - x^2 = \mathbf{4} \boldsymbol{- x}^{\mathbf{2}}$.

7. **(a)** $x - 3x = x(1 - 3) = {}^{-}\mathbf{2}\boldsymbol{x}$.

(b) $x^2 + x = \boldsymbol{x(x + 1)}$.

(c) $x^2 - 36 = x^2 - 6^2 = \boldsymbol{(x + 6)(x - 6)}$.

(d) $81y^4 - 16x^4 = (9y^2 + 4x^2)(9y^2 - 4x^2)$
$= \boldsymbol{(9y^2 + 4x^2)(3y + 2x)(3y - 2x)}$.

(e) $5 + 5x = \mathbf{5(1 + \boldsymbol{x})}$.

(f) $(x - y)(x + 1) - (x - y)$
$= (x - y)[(x + 1) - 1] = \boldsymbol{(x - y)x}$.

8. **(a)** **False**. $|x|$ is not positive when $x = 0$.

(b) **False**. $|x + y| \neq |x| + |y|$ when x and y are of opposite sign.

(c) **False**. Let $a = 3$ and $b = {}^{-}4$. Then $3 < {}^{-}({}^{-}4)$ and $3 \not< 0$.

(d) **True**. $x - y$ and $y - x$ both represent the difference between x and y. Squaring that difference produces a positive number in either case.

9. Answer may vary:

(a) $2 \div 1 \neq 1 \div 2$.

(b) $3 - (4 - 5) \neq (3 - 4) - 5$.

(c) $1 \div 2 \notin I$.

(d) $8 \div (4 - 2) \neq (8 \div 4) - (8 \div 2)$.

10. **(a)** $x + 3 = {}^{-}x - 17 \Rightarrow 2x = {}^{-}20$
$\Rightarrow x = {}^{-}\mathbf{10}$.

(b) $2x = {}^{-}2^{100} \Rightarrow x = \frac{{}^{-}2^{100}}{2} \Rightarrow x = {}^{-}\mathbf{2}^{\mathbf{99}}$.

(c) $2^{10}x = 2^{99} \Rightarrow x = \frac{2^{99}}{2^{10}} \Rightarrow x = \mathbf{2}^{\mathbf{89}}$.

(d) ${}^{-}x = x \Rightarrow {}^{-}2x = 0 \Rightarrow x = \mathbf{0}$.

(e) $|{}^{-}x| = 3 \Rightarrow x = 3$ or $x = {}^{-}\mathbf{3}$.

(f) $|x| = {}^{-}x \Rightarrow x = 0, {}^{-}1, {}^{-}2, \ldots$; i.e., $\boldsymbol{x \leq 0}$.

(g) $|x| > 3 \Rightarrow x = {}^{-}4, {}^{-}5, {}^{-}6, \ldots$, or $x = 4, 5, 6, \ldots$; i.e., $\boldsymbol{x > 3}$ **or** $\boldsymbol{x < {}^{-}3}$.

(h) $(x - 1)^2 = 100 \Rightarrow$

(i) $x - 1 = 10 \Rightarrow x = \mathbf{11}$, or

(ii) $x - 1 = {}^{-}10 \Rightarrow x = {}^{-}\mathbf{9}$.

11. **(a)** If $a_n = ({}^{-}1)^n$:

$a_1 = ({}^{-}1)^1 = {}^{-}\mathbf{1}$;
$a_2 = ({}^{-}1)^2 = \mathbf{1}$;
$a_3 = ({}^{-}1)^3 = {}^{-}\mathbf{1}$;
$a_4 = ({}^{-}1)^4 = \mathbf{1}$;
$a_5 = ({}^{-}1)^5 = {}^{-}\mathbf{1}$;
$a_6 = ({}^{-}1)^6 = \mathbf{1}$.

(b) If $a_n = ({}^{-}2)^n$:

$a_1 = ({}^{-}2)^1 = {}^{-}\mathbf{2}$;
$a_2 = ({}^{-}2)^2 = \mathbf{4}$;
$a_3 = ({}^{-}2)^3 = {}^{-}\mathbf{8}$;
$a_4 = ({}^{-}2)^4 = \mathbf{16}$;
$a_5 = ({}^{-}2)^5 = {}^{-}\mathbf{32}$;
$a_6 = ({}^{-}2)^6 = \mathbf{64}$.

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(c) If $a_n = {}^{-}2 - 3n$:

$a_1 = {}^{-}2 - 3 \cdot 1 = {}^{-}\mathbf{5}$;

$a_2 = {}^{-}2 - 3 \cdot 2 = {}^{-}\mathbf{8}$;

$a_3 = {}^{-}2 - 3 \cdot 3 = {}^{-}\mathbf{11}$;

$a_4 = {}^{-}2 - 3 \cdot 4 = {}^{-}\mathbf{14}$;

$a_5 = {}^{-}2 - 3 \cdot 5 = {}^{-}\mathbf{17}$;

$a_6 = {}^{-}2 - 3 \cdot 6 = {}^{-}\mathbf{20}$.

12. (a) Geometric; $r = {}^{-}1$.

(b) Geometric; $r = {}^{-}2$.

(c) Arithmetic; $d = 3$.

13. (a) Fibonacci-type sequence. The missing terms are 1, 1, 2.

(b) Geometric sequence with common ratio ${}^{-}2$. The missing terms are 392, ${}^{-}784$, 1568.

(c) Arithmetic sequence with common difference 147. The missing terms are 392, 539, 686.

14. (a) ${}^{-}1+1+0+1+1+2+3=7$.

(b) ${}^{-}49+98+{}^{-}196+392+{}^{-}784+1568=1029$.

(c) ${}^{-}49+98+245+392+539+686=1911$.

15. (a) This is an arithmetic sequence with common difference ${}^{-}2$. The n^{th} term is ${}^{-}2n+103$. We set the last term equal to the n^{th} term and solve for n: ${}^{-}2n+103={}^{-}103 \Rightarrow {}^{-}2n={}^{-}206 \Rightarrow n=103$. There are 103 terms total. Subtracting the 5 given terms leaves 98 terms missing by the indicated ellipse.

(b) Each term is the previous term multiplied by ${}^{-}5$. This gives: $5, {}^{-}25, 125, {}^{-}625, 3125, {}^{-}15625$. There are 6 terms total. Subtracting the 4 given terms leaves 2 terms missing by the indicated ellipse.

16. (a) The common difference is ${}^{-}2$.

(b) The common ratio is ${}^{-}5$.

17. The questions answered correctly contribute ${}^{+}4$ points, and the questions answered incorrectly contribute ${}^{-}7$ points. Terry's score is $87(4) + 46({}^{-}7) = 348 - 322 = \mathbf{26\ points}$.

18. Oregon, Nevada, Iowa, California, Arizona, Ohio, West Virginia, Alabama.

19. (a) ${}^{-}40 - ({}^{-}62) = -40 + 62 = \mathbf{22}$ degrees Celsius.

(b) $8180 - 1100 = \mathbf{7080\ ft}$.

(c) The pattern above suggest that as one travels South from the North Pole elevations must be higher to achieve negative temperatures. Since ${}^{-}11 - ({}^{-}40) = 29$ degrees Celsius, we might guess that since Hawaii is closer to the equator than Alaska, the elevation where ${}^{-}11^{\circ}\text{C}$ was recorded was 7000 or 8000 feet higher than the elevation where ${}^{-}40^{\circ}\text{C}$ was recorded in Arizona. A reasonable guess is that the elevation in Hawaii is 14,000 to 15,000 ft.

20. (a) ${}^{-}8 - ({}^{-}282) = \mathbf{274\ ft}$.

(b) Answers may vary, since several states are at sea level. The most accurate answers are Death Valley, **California** (-282 ft) and **Louisiana**, which as a lowest point at -8 ft.

21. If sea level is considered 0 m elevation, then the average depth at the Pacific Ocean is $-\mathbf{3963\ m}$.

22. (a) **degree Celsius** $\approx$ **degree Kelvin** $-\mathbf{273^{\circ}}$.

(b) degree Kelvin $\approx 100^{\circ} + 273^{\circ} = \mathbf{373^{\circ}}$.

(c) degree Kelvin $\approx {}^{-}40^{\circ} + 273^{\circ} = \mathbf{233^{\circ}}$.

23. (a) If we consider the number of forces on the battlefield before the Union and Confederate force arrived as 0, we can describe the change in the number of forces engaged in the battle as ${}^{+}75,000$, ${}^{+}82,289$, ${}^{-}23,049$, and ${}^{-}28,063$.

(b) ${}^{-}23,049 + {}^{-}28,063 = {}^{-}51,112$. Since the ${}^{-}$ denotes casualties, there were **51,112** casualties.

24. If we interpret 0 to be the ground floor, -2 would mean that the elevator is below ground level, which might be described as a 2nd level basement.

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25. **(a)** $^{-}2x < 14 \Rightarrow {}^{-}x < 7 \Rightarrow x > {}^{-}7$

$3x > {}^{-}15 \Rightarrow x > {}^{-}5$

The common solution is $x > {}^{-}5$.

(b) $x^2 < 9 \Rightarrow {}^{-}3 < x < 3$

$x^2 \le 1 \Rightarrow {}^{-}1 \le x \le 1$

The common solution is $^{-}1 \le x \le 1$

(c) $|x| > {}^{-}4$ and $|x| < {}^{-}1$ have no common solutions.

26. Write an equation:

$6 = -(-x + 30 - 14)$

$6 = x - 30 + 14$

$6 = x - 16$

$22 = x$

27. The difference of squares formula is:

$a^2 - b^2 = (a - b)(a + b)$. Applying this to $96 \cdot 104$:

$(100 - 4)(100 + 4) = 100^2 - 4^2$

$= 10,000 - 16 = 9984.$

28. **(a)** $1^2 - 4 = {}^{-}3$

$2^2 - 4 = 0$

$3^2 - 4 = 5$

$4^2 - 4 = 12$

$5^2 - 4 = 21$

(b) $^{-}5(1) - (^{-}3) = {}^{-}2$

$^{-}5(2) - (^{-}3) = {}^{-}7$

$^{-}5(3) - (^{-}3) = {}^{-}12$

$^{-}5(4) - (^{-}3) = {}^{-}17$

$^{-}5(5) - (^{-}3) = {}^{-}22$

(c) $^{-}(1)^2 - 4 = {}^{-}5$

$^{-}(2)^2 - 4 = {}^{-}8$

$^{-}(3)^2 - 4 = {}^{-}13$

$^{-}(4)^2 - 4 = {}^{-}20$

$^{-}(5)^2 - 4 = {}^{-}29$

29. **(a)** $(^{-}1)^{2n}$ will always be equal to 1 for all integer values of n because $2n$ will always be even.

(b) $(^{-}1)^{2n+1}$ will always be equal to $^{-}1$ for all integer values of n because $2n+1$ will always be odd.

30. **(a)** The n^{th} term is $(^{-}1)^{n+1}(n)$. The sequence is neither arithmetic or geometric but the signs alternate letting us know to use a $^{-}1$ in the n^{th} term. Each term number is also equal to the number in the term so we multiply the $(^{-}1)$ by n. Finally we want the odd term numbers to be positive and the even terms numbers to be negative so we raise the $^{-}1$ to the power $n+1$ to allow for the correct signs.

(b) The sum of the first 100 terms of the sequence is:

$1 + {}^{-}2 + 3 + \ldots + {}^{-}98 + 99 + {}^{-}100$

$= (1 + 3 + 5 + \ldots + 99) + (^{-}2 + {}^{-}4 + {}^{-}6 + \ldots + {}^{-}100)$

$= \dfrac{(1 + 99)50}{2} + \dfrac{(^{-}2 + {}^{-}100)50}{2}$

$= 2500 + {}^{-}2550$

$= {}^{-}50$

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CHAPTER 6

RATIONAL NUMBERS AND PROPORTIONAL REASONING

Assessment 6-1A: The Set of Rational Numbers

1. **(a)** The solution to $8x = 7$ is $\frac{7}{8}$.

(b) Joe ate seven of the eight apple slices.

(c) The ratio of boys to girls in this math class is seven to eight.

2. Assume the shaded regions are symmetrical:

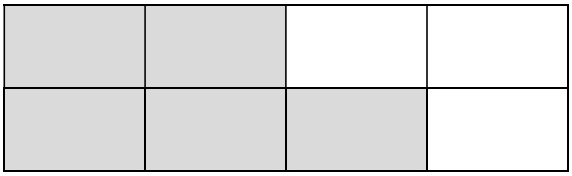
(a) $\frac{1}{6}$. The ratio of shaded to unshaded areas is one to six.

(b) $\frac{1}{4}$. The ratio of shaded to unshaded areas is one to four.

(c) $\frac{2}{6} = \frac{1}{3}$. The circle is in six parts; two are shaded.

(d) $\frac{7}{12}$. Seven of the twelve dots are shaded.

3. Divide the whole into equal parts:



$\frac{5}{8}$ is shaded out of the whole.

4. The diagrams illustrate the Fundamental Law of Fractions; i.e., the value of a fraction does not change if its numerator and denominator are multiplied by the same nonzero number.

(a) $\frac{2}{3}$. Two of the three parts are shaded.

(b) $\frac{4}{6} = \frac{2}{3}$. Four of the six parts are shaded.

(c) $\frac{6}{9} = \frac{2}{3}$. Six of the nine parts are shaded.

(d) $\frac{8}{12} = \frac{2}{3}$. Eight of the twelve parts are shaded.

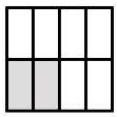
5. Assume the shaded regions are symmetrical:

(a) **No**. The parts are not of equal area.

(b) **Yes**. The sections are equal in area; three of the four are shaded.

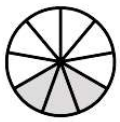
(c) **Yes**. The circle parts are equal in area; one of the two is shaded.

6. **(a)** One block of a square sectioned into four equal pieces is the same area as two blocks of a square sectioned into eight equal pieces; i.e., $\frac{1}{4} = \frac{2}{8}$:



$\frac{2}{8}$

(b) One arc of a circle sectioned into three equal pieces is the same area as three arcs of a circle sectioned into nine equal pieces; i.e., $\frac{1}{3} = \frac{3}{9}$:



$\frac{3}{9}$

(c) One part of a hexagon sectioned into two equal pieces is the same area as three parts of a hexagon sectioned into six equal pieces; i.e., $\frac{1}{2} = \frac{3}{6}$:

$\frac{3}{6}$

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7. **(a)** $\frac{\text{Dots in circle}}{\text{Total dots}} = \frac{9}{24} = \mathbf{\frac{3}{8}}$.

(b) $\frac{\text{Dots in rectangle}}{\text{Total dots}} = \frac{12}{24} = \mathbf{\frac{1}{2}}$.

(c) $\frac{\text{Dots in intersection}}{\text{Total dots}} = \frac{4}{24} = \mathbf{\frac{1}{6}}$.

(d) $\frac{\text{Dots in rectangle but outside circle}}{\text{Total dots}} = \frac{8}{24} = \mathbf{\frac{1}{3}}$.

8. **(a)** $A \cap B = \frac{4}{43}$. There are 4 people in both an arts class and a botany class out of the 43 people in the universal set $(17 + 4 + 13 + 9 = \quad 43)$.

(b) $A - B = \frac{17}{43}$. There are 21 people in an arts class but 4 of those are also in a botany class out of 43 people in the universal set $(21 - 4 = 17)$.

(c) $U = \frac{43}{43} = 1$. There are 43 people in the universal set. As a fraction of the universal set it would be 43 out of 43.

(d) $\overline{A \cap B} = \frac{43-4}{43} = \frac{39}{43}$. There are 43 people in the universal set and 4 people in set A and set B, so there are $43 - 4 = 39$ people who are not in A and B out of the the 43 people in the universal set.

9. Answers may vary (as long as the numerator and denominator of the given fractions are multiplied by the same number); some possibilities are:

(a) $\frac{4}{18}, \frac{6}{27}, \frac{10}{45}$.

(b) $\frac{^-4}{10}, \frac{2}{^-5}, \frac{^-10}{25}$.

(c) $\frac{0}{6}, \frac{0}{9}, \frac{0}{12}$.

(d) $\frac{2a}{4}, \frac{3a}{6}, \frac{4a}{8}$.

10. **(a)** $\frac{156}{93} = \frac{3 \cdot 52}{3 \cdot 31} = \mathbf{\frac{52}{31}}$.

(b) $\frac{27}{45} = \frac{9 \cdot 3}{9 \cdot 5} = \mathbf{\frac{3}{5}}$.

(c) $\frac{^-65}{91} = \frac{^-5 \cdot 13}{7 \cdot 13} = \mathbf{\frac{^-5}{7}}$.

11. **(a)** **Undefined**. Division by 0 is undefined.

(b) **Undefined**. Division by 0.

(c) **0**. $\frac{0}{5} = 0$ because $0 \cdot 5 = 0$.

(d) **Cannot be simplified**. $(2 + a)$ and a have no common factors other than 1.

(e) **Cannot be simplified**. $(15 + x)$ and $3x$ have no common factors other than 1.

12. In both parts (a) and (b), a restriction must be maintained so that the denominator cannot be zero.

(a) $\frac{a^2 - b^2}{3a + 3b} = \frac{(a+b)(a-b)}{3(a+b)} = \mathbf{\frac{a-b}{3}} (a \neq {}^-b)$.

(b) $\frac{14x^2y}{63xy^2} = \frac{7 \cdot 2 \cdot x \cdot x \cdot y}{7 \cdot 9 \cdot x \cdot y \cdot y} = \mathbf{\frac{2x}{9y}} (x, y \neq 0)$.

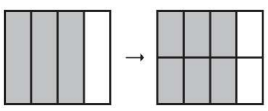
13. **(a)** **Equal**. $\frac{375}{1000} = \frac{125 \cdot 3}{125 \cdot 8} = \frac{3}{8}$ or $375 \cdot 8 = 1000 \cdot 3$.

(b) **Equal**. $\frac{18}{54} = \frac{18}{3 \cdot 18} = \frac{1}{3}$ and $\frac{23}{69} = \frac{1 \cdot 23}{3 \cdot 23} = \frac{1}{3}$ or $18 \cdot 69 = 54 \cdot 23$.

14. **(a)** **Not equal**. $16 = 2^4; 18 = 2 \cdot 3^2 \Rightarrow LCM(16,18) = 2^4 \cdot 3^2 = 144$. Then $\frac{10}{16} = \frac{10 \cdot 9}{16 \cdot 9} = \frac{90}{144}$ and $\frac{12}{18} = \frac{12 \cdot 8}{18 \cdot 8} = \frac{96}{144}$; $\frac{90}{144} \neq \frac{96}{144}$.

(b) **Not equal**. $86 = 2 \cdot 43; 215 = 5 \cdot 43 \Rightarrow LCM(86,215) = 2 \cdot 5 \cdot 43 = 430$. Then $\frac{^-21}{86} = \frac{^-21 \cdot 5}{86 \cdot 5} = \frac{^-105}{430}$ and $\frac{^-51}{215} = \frac{^-51 \cdot 2}{215 \cdot 2} = \frac{^-102}{430}$; $\frac{^-105}{430} \neq \frac{^-102}{430}$.

15. The shaded area takes in three of the four columns and six of the eight small rectangles. Since the area in each case is the same, $\frac{3}{4} = \frac{6}{8}$.



16. To obtain equal fractions, multiply numerator and denominator by the same number. All fractions equal to $\frac{3}{4}$ will then be of the form $\frac{3n}{4n}$. $3n + 4n = 84 \Rightarrow n = 12$; thus $\frac{3}{4} = \frac{3 \cdot 12}{4 \cdot 12} = \mathbf{\frac{36}{48}}$.

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17. Mr. Gomez had $16 - 14 = 2$ gallons left. $\frac{2}{16} = \frac{1}{8}$ of a tank remained; the needle points to the 1st division of 8 as shown:

(b) He used 14 gallons: $\frac{14}{16} = \frac{7}{8}$.

18. **(a)** For equal fractions $\frac{a}{b} = \frac{c}{d}$, $ad = bc$.
If $\frac{2}{3} = \frac{x}{16}$, then $2 \cdot 16 = 3 \cdot x \Rightarrow$
$32 = 3x \Rightarrow \boldsymbol{x = \frac{32}{3}}$.

(b) $3 \cdot x = 4 \cdot {}^{-}27 \Rightarrow 3x = {}^{-}108 \Rightarrow$
$\boldsymbol{x = {}^{-}36}$.

19. **(a)** $\frac{7}{8} > \frac{5}{6}$. $LCD(8,6) = 24 \Rightarrow \frac{7}{8} = \frac{21}{24}$ and $\frac{5}{6} = \frac{20}{24}$. Or (noting that if $\frac{a}{b} > \frac{c}{d}$ then $ad > bc$) $7 \cdot 6 > 8 \cdot 5$.

(b) $\frac{{}^{-}7}{8} < \frac{{}^{-}4}{5}$. $LCD\,(8,5) = 40 \Rightarrow \frac{{}^{-}7}{8} = \frac{{}^{-}35}{40}$ and $\frac{{}^{-}4}{5} = \frac{{}^{-}32}{40}$ (note that ${}^{-}35 < {}^{-}32$), or ${}^{-}7 \cdot 5 < 8 \cdot {}^{-}4$.

20. **(a)** $\frac{11}{13}, \frac{11}{16}, \frac{11}{22}$. When rational numbers have the same numerators, those with larger denominators, are of lesser value.

(b) $\frac{{}^{-}1}{5}, \frac{{}^{-}19}{36}, \frac{{}^{-}17}{30}$. $LCD(5,36,30) = 180 \Rightarrow$
$\frac{{}^{-}1}{5} = \frac{{}^{-}36}{180}, \frac{{}^{-}19}{36} = \frac{{}^{-}95}{180}$, and $\frac{{}^{-}17}{30} = \frac{{}^{-}102}{180}$.
${}^{-}36 > {}^{-}95 > {}^{-}102$.

21. Answers may vary; one method is to convert the given fractions into equal fractions having larger common denominators, thus creating spaces between them:

(a) $\frac{3}{7} = \frac{9}{21}$ and $\frac{4}{7} = \frac{12}{21}$. Two rational numbers between them could then be $\frac{10}{21}$ and $\frac{11}{21}$.

(b) $\frac{{}^{-}7}{9} = \frac{{}^{-}28}{36}$ and $\frac{{}^{-}8}{9} = \frac{{}^{-}32}{36}$. Two rational numbers between them could then be $\frac{{}^{-}30}{36}$ and $\frac{{}^{-}31}{36}$.

22. **(a)** ***(i)*** $6 \text{ oz}\left(\frac{1 \text{ lb}}{16 \text{ oz}}\right) = \frac{6}{16} \text{ lb} = \boldsymbol{\frac{3}{8}}$ **pound**.

(ii) $6 \text{ oz}\left(\frac{1 \text{ lb}}{16 \text{ oz}}\right)\left(\frac{1 \text{ ton}}{2000 \text{ lb}}\right) = \frac{6}{32{,}000} \text{ ton} = \boldsymbol{\frac{3}{16{,}000}}$ **ton**.

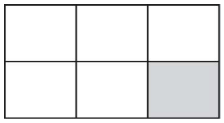
(b) $1 \text{ dime}\left(\frac{1 \text{ dollar}}{10 \text{ dime}}\right) = \boldsymbol{\frac{1}{10}}$ **dollar**.

(c) $15 \text{ min}\left(\frac{1 \text{ hr}}{60 \text{ min}}\right) = \frac{15}{60} \text{hr} = \boldsymbol{\frac{1}{4}}$ **hour**.

(d) $8 \text{ hr}\left(\frac{1 \text{ day}}{24 \text{ hr}}\right) = \frac{8}{24} \text{ day} = \boldsymbol{\frac{1}{3}}$ **day**.

23. If $a, b,$ and c are integers and $b < 0$, then $\frac{a}{b} > \frac{c}{b}$ if, and only if, $a > c$. This is false. A counterexample would be $\frac{2}{{}^{-}5} > \frac{3}{{}^{-}5}$ but 2<3.

24. Even though the figure is divided in an inconsistent manner, we could redraw the figure as follows.



The shaded region is $\frac{1}{6}$ of the entire figure.

25. **(a)** Answers vary. For example $\frac{1}{3} < \frac{2}{4} < \frac{3}{5} < \frac{4}{6}$. The terms in this sequence go in ascending order: $\frac{20}{60} < \frac{30}{60} < \frac{36}{60} < \frac{40}{60}$.

(b) Answers vary. For example $\frac{{}^{-}3}{3} < \frac{{}^{-}4}{5} < \frac{{}^{-}5}{7} < \frac{{}^{-}6}{9}$. The terms in this sequence go in ascending order: $\frac{{}^{-}315}{315} < \frac{{}^{-}252}{315} < \frac{{}^{-}225}{315} < \frac{{}^{-}210}{315}$.

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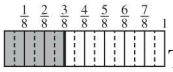
26. n cannot be zero otherwise there would be division by zero which is undefined. $\frac{0}{0}$ is undefined.

27. $n = \frac{n}{1}$ for all integers n, so every integer can be written as a fraction.

28.  The drawing is not to scale.

29. $\frac{1}{5}$ of $10 = \frac{1}{5} \cdot 10 = 2$. Two light bulbs were not shining.

30. $\frac{2}{5}$ were men so $\frac{3}{5}$ were women.

Assessment 6-1B

1. **(a)** The solution to $10x = 7$ is $\frac{7}{10}$.

(b) Joe ate seven of the ten apple slices.

(c) The ratio of boys to girls in this math class is seven to ten.

2. Assume the shaded regions are symmetrical:

(a) $\frac{1}{3}$. The figure highlights one of three parts.

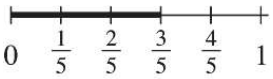
(b) $\frac{9}{12} = \frac{3}{4}$. Nine of the twelve dots are shaded.

(c) $\frac{5}{16}$. Five of the sixteen small triangles are shaded.

(d) $\frac{2}{16} = \frac{1}{8}$. If the large square were to be cut into pieces all the same size as those now shaded, there would be a total of sixteen, two of which are shaded.

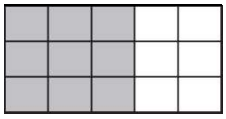
3. If the whole is divided into equal parts we have three triangles out of four shaded. $\frac{3}{4}$ of the whole is shaded.

4. **(a)** The figure highlights three of the five parts of the bar:



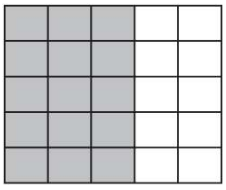
(b) Three of the five dots are filled:

(c) Nine of the fifteen blocks are shaded $= \frac{3}{5}$ of the area:

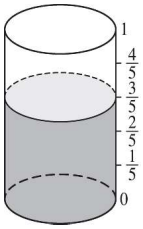


(d) Six of the ten dots are filled $= \frac{3}{5}$ of the dots:

(e) Fifteen of the 25 blocks are shaded $= \frac{3}{5}$ of the area:



(f) $\frac{3}{5}$ of the volume is shaded:

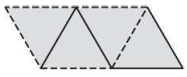


5. **(a)** **No**. The parts are not of equal area.

(b) **Yes**. The square can be divided into eight equal triangles, one of which is shaded.

(c) **Yes**. The circle can be divided into eight equal segments, two of which are shaded.

6. **(a)** Divide the figure into three equal parts, then add another equal part. The shaded area is then the whole:



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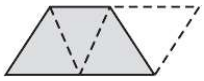
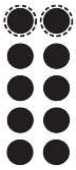
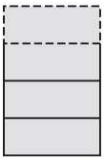
(b) Divide the figure into four equal parts, then remove one of the equal parts. The shaded area is then the whole:



(c) If two circles are $\frac{1}{5}$ or $\frac{2}{10}$ of the whole, then the whole is ten shaded circles:



(d) If the rectangle is $\frac{1}{4}$ of the whole, then add three other equal rectangles:



7. **(a)** $\frac{\text{Dots outside circle}}{\text{Total dots}} = \frac{15}{24} = \mathbf{\frac{5}{8}}$.

(b) $\frac{\text{Dots outside rectangle}}{\text{Total dots}} = \frac{12}{24} = \mathbf{\frac{1}{2}}$.

(c) $\frac{\text{Dots in union}}{\text{Total dots}} = \mathbf{\frac{17}{24}}$.

(d) $\frac{\text{Dots in circle but outside of rectangle}}{\text{Total dots}} = \mathbf{\frac{5}{24}}$.

8. **(a)** $A \cup B = \frac{17+4+13}{43} = \frac{34}{43}$. There are 34 people in either an algebra class or a biology class out of the 43 people in the universal set (17 + 4 + 13 + 9 = 43).

(b) $A - \overline{B} = \frac{4}{43}$. There are 21 people total in an algebra class and 17 of those are not in a biology class out of 43 people in the universal set (21 – 17 = 4).

(c) $\varnothing = \frac{0}{43} = 0$. There are 43 people in the universal set. There are no people in the empty set.

(d) $\overline{A \cup B} = \frac{43}{43} - \frac{17+4+13}{43} = \frac{43}{43} - \frac{34}{43} = \frac{9}{43}$. There are 43 people in the universal set and 9 people not in the union of set A and set B.

9. Answers may vary.

(a) $\mathbf{\frac{2}{6}, \frac{3}{9}, \frac{4}{12}}$.

(b) $\mathbf{\frac{8}{10}, \frac{12}{15}, \frac{16}{20}}$.

(c) $\mathbf{\frac{-6}{14}, \frac{-9}{21}, \frac{-12}{28}}$.

(d) $\mathbf{\frac{2a}{6}, \frac{3a}{9}, \frac{4a}{12}}$.

10. **(a)** $\frac{0}{68} = \frac{0.68}{1.68} = \mathbf{\frac{0}{1}}$.

(b) $\frac{84^2}{91^2} = \frac{(7 \cdot 12)^2}{(7 \cdot 13)^2} = \frac{7^2 \cdot 12^2}{7^2 \cdot 13^2} = \frac{12^2}{13^2} = \mathbf{\frac{144}{169}}$.

(c) $\frac{662}{703}$ is **already in reduced form**. There are no factors common to both numerator and denominator.

11. **(a)** **Cannot be simplified**. $\frac{6+x}{3x} \neq \frac{2+x}{x}$ because there are no factors common to each term.

(b) $\frac{2^6+2^5}{2^4+2^7} = \frac{2^5(2^1+1)}{2^4(1+2^3)} = \frac{2^5(3)}{2^4(9)} = \frac{32 \cdot 3}{16 \cdot 9} = \frac{2 \cdot 16 \cdot 3}{16 \cdot 3 \cdot 3} = \mathbf{\frac{2}{3}}$.

(c) $\frac{2^{100}+2^{98}}{2^{100}-2^{98}} = \frac{2^{98}(2^2+1)}{2^{98}(2^2-1)} = \frac{2^{98} \cdot 5}{2^{98} \cdot 3} = \mathbf{\frac{5}{3}}$.

12. In parts (a) and (b) both, a restriction must be maintained so that the denominator cannot be zero.

(a) $\frac{a^2+ab}{a+b} = \frac{a(a+b)}{a+b} = \mathbf{\frac{a}{1}} = a, (a + b \neq 0)$.

(b) $\frac{a}{3a+ab} = \frac{a}{a(3+b)} = \mathbf{\frac{1}{3+b}} (a \neq 0, b \neq -3)$.

13. **(a)** $\frac{6}{16} = \frac{6}{4^2} = \frac{6}{4^2} \cdot \frac{25}{25} \cdot \frac{25}{25} = \frac{6 \cdot 25 \cdot 25}{4 \cdot 25 \cdot 4 \cdot 25} = \frac{3{,}750}{10{,}000}$. The fractions are **equal**.

(b) **Not equal**. $\frac{17}{27}$ is in its reduced form and $\frac{25}{45} = \frac{5 \cdot 5}{5 \cdot 9} = \frac{5}{9}$ or $17 \cdot 45 \neq 27 \cdot 25$.

14. **(a)** **Equal**. $\frac{3}{-12} = \frac{-3}{12} = \frac{-3 \cdot 12}{12 \cdot 12} = \frac{-36}{144}$.

(b) **Not equal**.
$\frac{-51}{215} = \frac{-51 \cdot 2}{215 \cdot 2} = \frac{-102}{430}$; $\frac{-105}{430} \neq \frac{-21}{430}$.

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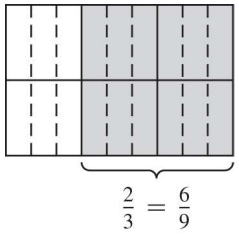
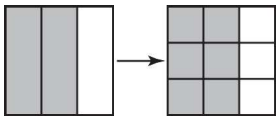
15.



Alternative:



16. **Yes**. $\frac{3}{8} = \frac{3 \cdot 4}{8 \cdot 4} = \frac{12}{32} > \frac{11}{32}$, so the board is thick enough. Shaving $\frac{1}{32}$ **inch** will bring it to the required thickness.

17. **Meter A by 3 minutes**. Meter A has $\frac{4}{10}$ of one hour $=$ 24 minutes remaining; meter B has $\frac{7}{10}$ of $\frac{1}{2}$ hour $= \frac{7}{20}$ of one hour $=$ 21 minutes remaining.

18. **(a)** For equal fractions $\frac{a}{b} = \frac{c}{d}$, $ad = bc$.
If $\frac{2}{3} = \frac{x}{18}$, then $2 \cdot 18 = 3 \cdot x \Rightarrow 36 = 3x \Rightarrow \boldsymbol{x = 12}$.

(b) **Any nonzero rational number except 0**.
$3 \cdot x^2 = x \cdot 3x \Rightarrow 3x^2 = 3x^2$; but 0 cannot be a solution because of division by x and x^2 in the original equation.

19. **(a)** $\frac{1}{^-7} < \frac{1}{^-8}$. $LCD(7, 8) = 56 \Rightarrow$
$\frac{1}{^-7} = \frac{^-1}{7} = \frac{^-8}{56}$ and $\frac{1}{^-8} = \frac{^-1}{8} = \frac{^-7}{56}$,
or $1 \cdot {}^-8 < {}^-7 \cdot 1$.

(b) $\frac{2}{5} = \frac{4}{10} \cdot 2 \cdot 10 = 5 \cdot 4$.

(c) $\frac{0}{7} = \frac{0}{17} \cdot \frac{0}{7} = 0 = \frac{0}{17}$.

20. Answers may vary; one method is to convert the given fractions into equal fractions having larger common denominators, thus creating spaces between them:

(a) $\frac{5}{6} = \frac{1000}{1200}$ and $\frac{83}{100} = \frac{996}{1200}$. Two rational numbers between them could then be $\frac{997}{1200}$ and $\frac{998}{1200}$.

(b) $\frac{^-1}{3} = \frac{^-4}{12}$ and $\frac{3}{4} = \frac{9}{12}$. Two rational numbers between them could then be $\mathbf{0}$ and $\mathbf{\frac{1}{2}}$.

21. **(a)** $12 \text{ oz}\left(\frac{1 \text{ lb}}{16 \text{ oz}}\right) = \frac{12}{16} \text{ lb} = \mathbf{\frac{3}{4}}$ **lb**.

(b) $1 \text{ nickel}\left(\frac{1 \text{ dollar}}{20 \text{ nickel}}\right) = \mathbf{\frac{1}{20}}$ **dollar**.

(c) $25 \text{ min}\left(\frac{1 \text{ hr}}{60 \text{ min}}\right) = \frac{25}{60} \text{ hr} = \mathbf{\frac{5}{12}}$ **hr**.

(d) $16 \text{ hr}\left(\frac{1 \text{ day}}{24 \text{ hr}}\right) = \frac{16}{24} \text{ day} = \mathbf{\frac{2}{3}}$ **day**.

22. **(a)** $\mathbf{2\frac{7}{8}}$ **inch**, or 2 inches plus 14 of the 16 divisions between 2 and 3; $2\frac{14}{16}$ inch $= 2\frac{7}{8}$ inch.

(b) $\mathbf{2\frac{3}{8}}$ **inch**, or 2 inches plus 6 of the 16 divisions between 2 and 3; $2\frac{6}{16}$ inch $= 2\frac{3}{8}$ inch.

(c) $\mathbf{1\frac{3}{8}}$ **inch**, or 1 inch plus 6 of the 16 divisions between 1 and 2; $1\frac{6}{16}$ inch $= 1\frac{3}{8}$ inch.

(d) $\mathbf{\frac{15}{16}}$ **inch**, or 15 of the 16 divisions between 0 and 1 inch.

23. **(a)** Answers vary. For example $\frac{1}{3} > \frac{0}{4} > \frac{^-1}{5} > \frac{^-2}{6}$. The terms in this sequence go in descending order: $\frac{20}{60} > \frac{0}{60} > \frac{^-12}{60} > \frac{^-20}{60}$.

(b) Answers vary. For example $\frac{^-3}{6} > \frac{^-4}{5} > \frac{^-5}{4} > \frac{^-6}{3}$. The terms in this sequence go in descending order: $\frac{^-30}{60} > \frac{^-48}{60} > \frac{^-75}{60} > \frac{^-120}{60}$.

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24. $n = \frac{n}{1}$ where n is an integer. Because the set of integers is infinite, so is this representation.

25.

$n = \frac{n}{1}$ for all negative integers n,
so every negative integer can be written as a fraction with a positive denominator of 1.

26. If $1\text{mm} = \frac{1}{1000}\text{m}$ then 1000mm=1m and 5m=5000mm.

27. $\frac{3}{5}$ were shining so $\frac{2}{5}$ were not shining.

28. $\frac{1}{5}$ were men so $\frac{4}{5}$ were women.

29.

(a)	**(b)**
Area of $a = \frac{1}{4}$	Area of $a = 1$
Area of $b = \frac{1}{4}$	Area of $b = 1$
Area of $c = \frac{1}{16}$	Area of $c = \frac{1}{4}$
Area of $d = \frac{1}{8}$	Area of $d = \frac{1}{2}$
Area of $e = \frac{1}{16}$	Area of $e = \frac{1}{4}$
Area of $f = \frac{1}{8}$	Area of $f = \frac{1}{2}$
Area of $g = \frac{1}{8}$	Area of $g = \frac{1}{2}$

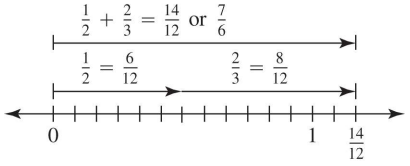
Assessment 6-2A: Addition, Subtraction, and Estimation with Rational Numbers

1. **(a)** Three possible methods are illustrated below:

(i) $\frac{1}{2} + \frac{2}{3} = \frac{1\cdot3 + 2\cdot2}{2\cdot3} = \frac{3+4}{6} = \frac{7}{6}$; or

(ii) $LCD(2,3) = 6 \Rightarrow \frac{1}{2} + \frac{2}{3} = \frac{1}{2}\cdot\frac{3}{3} + \frac{2}{3}\cdot\frac{2}{2} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$; or

(iii)



(b) $LCD(12,3) = 12 \Rightarrow \frac{4}{12} - \frac{2}{3} = \frac{4}{12} - \frac{2}{3}\cdot\frac{4}{4} = \frac{4}{12} - \frac{8}{12} = \frac{^-4}{12} = \mathbf{\frac{^-1}{3}}$.

(c) $\frac{5}{x} + \frac{-3}{y} = \frac{5\cdot y - x\cdot 3}{x\cdot y} = \mathbf{\frac{5y-3x}{xy}}$.

(d) $LCD(2x^2y, 2xy^2, x^2) = 2x^2y^2 \Rightarrow$

$\frac{^-3}{2x^2y} + \frac{5}{2xy^2} + \frac{7}{x^2} = \frac{^-3}{2x^2y}\cdot\frac{y}{y} + \frac{5}{2xy^2}\cdot\frac{x}{x} + \frac{7}{x^2}\cdot\frac{2y^2}{2y^2} = \mathbf{\frac{^-3y + 5x + 14y^2}{2x^2y^2}}$.

(e) $\frac{5}{6} + 2\frac{1}{8} = \frac{5}{6} + \frac{17}{8} = \frac{5\cdot8 + 6\cdot17}{6\cdot8} = \frac{40+102}{48} = \frac{142}{48} = \frac{71}{24} = \mathbf{2\frac{23}{24}}$.

(f) $^-4\frac{1}{2} - 3\frac{1}{6} = {}^-4\frac{3}{6} - 3\frac{1}{6} = {}^-7\frac{4}{6} = {}^-7\frac{2}{3}$ $\left(\text{or } \mathbf{\frac{^-23}{3}}\right)$.

(g) $7\frac{1}{4} + 3\frac{5}{12} - 2\frac{1}{3} = \frac{29}{4} + \frac{41}{12} - \frac{7}{3} = \frac{87}{12} + \frac{41}{12} - \frac{28}{12} = \frac{128}{12} - \frac{28}{12} = \frac{100}{12} = 8\frac{4}{12} = 8\frac{1}{3}$

2. **(a)** Two possible methods are illustrated below:

(i) $\frac{56}{3} = \frac{3\cdot18+2}{3} = \frac{3\cdot18}{3} + \frac{2}{3} = 18 + \frac{2}{3} = \mathbf{18\frac{2}{3}}$; or

(ii) $56 \div 3 = 18$, remainder $2 \Rightarrow \frac{56}{3} = \mathbf{18\frac{2}{3}}$.

(b) $-\frac{293}{100} = -\left(\frac{2\cdot100+93}{100}\right) = {}^-\mathbf{2\frac{93}{100}}$.

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3. **(a)** $6\frac{3}{4} = \frac{6}{1} + \frac{3}{4} = \frac{6\cdot 4 + 1\cdot 3}{1\cdot 4} = \frac{24+3}{4} = \mathbf{\frac{27}{4}}$.

(b) $^{-}3\frac{5}{8} = {}^{-}\left(\frac{3}{1} + \frac{5}{8}\right) = {}^{-}\left(\frac{3\cdot 8 + 1\cdot 5}{1\cdot 8}\right) = {}^{-}\left(\frac{24+5}{8}\right) = \mathbf{\frac{^{-}29}{8}}$.

4. **(a)** $\frac{15}{46} \approx \frac{15}{45} = \mathbf{\frac{1}{3}}$. 45 is smaller than the actual denominator so the estimate is **high**.

(b) $\frac{7}{41} \approx \frac{7}{42} = \mathbf{\frac{1}{6}}$. 42 is larger than the actual denominator so the estimate is **low**.

(c) $\frac{62}{80} \approx \frac{60}{80} = \mathbf{\frac{3}{4}}$. 60 is smaller than the actual numerator so the estimate is **low**.

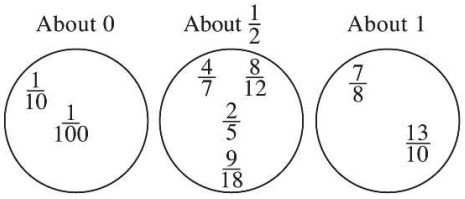
(d) $\frac{9}{19} \approx \frac{9}{18} = \mathbf{\frac{1}{2}}$. The denominator in the estimate is smaller than the actual denominator, so the estimate is **high**.

5. **(a)** **Beavers**. $\frac{10}{19} > \frac{1}{2}$.

(b) **Ducks**. $\frac{10}{22} < \frac{1}{2}$ by the least margin.

(c) **Bears**. $\frac{8}{23} > \frac{1}{3}$ by the least margin.

6.



Note that $\frac{8}{12} = \frac{3}{4}$ so it could be placed in either the "about $\frac{1}{2}$" or the "about 1" oval.

7. **(a)** $\mathbf{\frac{1}{2}}$**; high**. $\frac{19}{38} = \frac{1}{2} \Rightarrow \frac{19}{39} < \frac{1}{2}$.

(b) **0; low**. $\frac{3}{197} > 0$.

(c) $\mathbf{\frac{3}{4}}$**; high**. $\frac{150}{200} = \frac{3}{4} \Rightarrow \frac{150}{201} < \frac{3}{4}$.

(d) **1; high**. $\frac{8}{9} < 1$.

8. **(a)** **2**. Each addend is about $\frac{1}{2}$; thus the best approximation would be $4 \cdot \frac{1}{2} = 2$.

(b) $\mathbf{\frac{3}{4}}$. $\frac{30}{41}$ is about $\frac{3}{4}$; the other two addends are negligible compared to $\frac{3}{4}$.

9. Possible thought processes could be:

(a) $\frac{4}{4} - \frac{3}{4} = \mathbf{\frac{1}{4}}$.

(b) $\left(3 + 2 + \frac{3}{8} + \frac{2}{8}\right) - 5\frac{5}{8} = 5\frac{5}{8} - 5\frac{5}{8} = \mathbf{0}$.

10. **(a)** Region ***A***. $\frac{20}{8} = \frac{10}{4}$ is between 2 and 3.

(b) Region ***H***. $\frac{36}{8} = \frac{18}{4}$ is between 4 and 5.

(c) Region ***T***. $\frac{60}{16} = \frac{15}{4}$ is between 3 and 4.

(d) Region ***H***. $\frac{18}{4}$ is between 4 and 5.

11. About $4 \cdot 3 = \mathbf{12}$. The numbers cluster around 3.

12. The entire student population is represented by 1; subtract to obtain the senior's fraction; i.e., seniors make up $1 - \frac{2}{5} - \frac{1}{4} - \frac{1}{10}$ of the class. Thus $1 - \frac{2}{5} - \frac{1}{4} - \frac{1}{10} = \frac{20}{20} - \frac{8}{20} - \frac{5}{20} - \frac{2}{20} = \frac{5}{20} = \mathbf{\frac{1}{4}}$.

13. $\frac{1}{3} + 2\frac{3}{4} + 3\frac{1}{2} = \frac{4}{12} + 2\frac{9}{12} + 3\frac{6}{12} = 5\frac{19}{12} = 5 + 1\frac{7}{12} = \mathbf{6\frac{7}{12}}$ **yards**.

14. The amount of fabric to be used is $1\frac{7}{8} + 2\frac{3}{8} + 1\frac{2}{3} = 1\frac{21}{24} + 2\frac{9}{24} + 1\frac{16}{24} = 4\frac{46}{24} = 5\frac{22}{24}$ yards. The amount left over is $8\frac{3}{4} - 5\frac{22}{24} = 8\frac{18}{24} - 5\frac{22}{24} = 7\frac{42}{24} - 5\frac{22}{24} = 2\frac{20}{24} = \mathbf{2\frac{5}{6}}$ **yards**.

15. Examples may vary:

(a) Closure property: If two rational numbers are added, the sum should also be rational; e.g., $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$, which is rational.

(b) Commutative property: $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$; e.g., $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = \frac{3}{4} + \frac{1}{2}$.

(c) Associative property: $\frac{a}{b} + \left(\frac{c}{d} + \frac{c}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{c}{f}$; e.g., $\frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) = \frac{13}{12} = \left(\frac{1}{2} + \frac{1}{3}\right) + \frac{1}{4}$.

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16. Let the sequence be $1, a_2, a_3, a_4, a_5, a_6, 2$.

Thus $2 = 1 + (7-1)d \Rightarrow 6d = 1 \Rightarrow d = \frac{1}{6}$.

Then $a_2 = 1 + \frac{1}{6} = \frac{7}{6}$,

$a_3 = \frac{7}{6} + \frac{1}{6} = \frac{8}{6}$,

$a_4 = \frac{8}{6} + \frac{1}{6} = \frac{9}{6}$,

$a_5 = \frac{9}{6} + \frac{1}{6} = \frac{10}{6}$,

and $a_6 = \frac{10}{6} + \frac{1}{6} = \frac{11}{6}$.

The sequence is $\mathbf{1}, \mathbf{\frac{7}{6}}, \mathbf{\frac{8}{6}}, \mathbf{\frac{9}{6}}, \mathbf{\frac{10}{6}}, \mathbf{\frac{11}{6}}, \mathbf{2}$.

17. **(a)** ***(i)*** $\frac{1}{4} + \frac{1}{3\cdot 4} = \frac{1}{4} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$.

(ii) $\frac{1}{5} + \frac{1}{4\cdot 5} = \frac{1}{5} + \frac{1}{20} = \frac{4}{20} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4}$.

(iii) $\frac{1}{6} + \frac{1}{5\cdot 6} = \frac{1}{6} + \frac{1}{30} = \frac{5}{30} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$.

(b) $\mathbf{\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}}$.

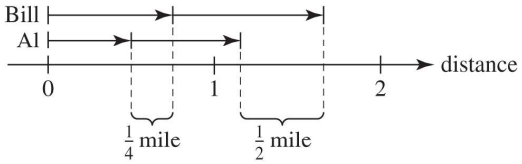
18. **(a)** $x + 2\frac{1}{2} = 3\frac{1}{3} \Rightarrow x = 3\frac{1}{3} - 2\frac{1}{2} = 2\frac{8}{6} - 2\frac{3}{6}$. $\boldsymbol{x = \frac{5}{6}}$.

(b) $x - 2\frac{2}{3} = \frac{5}{6} \Rightarrow x = \frac{5}{6} + \frac{16}{6}$.

$\boldsymbol{x = \frac{21}{6} = 3\frac{1}{2}}$.

19. In 10 minutes, the difference of the two runners' distances is $\frac{7}{8} - \frac{5}{8} = \frac{2}{8} = \frac{1}{4}$ mile. In 20 minutes the difference will be twice as large. Answer: $2\left(\frac{1}{4}\right) = \frac{2}{4} = \mathbf{\frac{1}{2}}$ **mile**.

Alternative:



20. **No**. To make both recipes, you need $1\frac{3}{4} + 1\frac{1}{2} = 1 + \frac{3}{4} + 1 + \frac{1}{2} = 2 + \frac{3}{4} + \frac{1}{2} = 2 + 1\frac{1}{4} = 3\frac{1}{4}$ cups. $3\frac{1}{4} - 3 = \frac{1}{4}$ cups less than what is needed.

21. Answers vary.

(a)
$$\frac{76,432 + 76,503 + 78,672}{150,155 + 149,904 + 154,144} = \frac{231,607}{454,203} \approx \frac{23}{45}$$

(b)
$$\frac{73,723 + 73,401 + 75,472}{150,155 + 149,904 + 154,144} = \frac{222,596}{454,203} \approx \frac{22}{45}$$

(c) $\frac{231,607}{12,742,886} \approx \frac{2}{127}$

22. $\frac{633,000}{1,193,000} \approx \frac{63}{119}$ and $\frac{1,186,000}{1,259,000} \approx \frac{118}{125}$. Since $\frac{118}{125} > \frac{63}{119}$ there are more 16- and 17-year olds in school.

23. $40^\circ + 5^\circ = 45^\circ$ and $\frac{45^\circ}{360^\circ} = \frac{1}{8}$ so rent and saving represent $\frac{1}{8}$ of the total.

24. $\frac{37}{100} + \frac{3}{10} + \frac{19}{100} + \frac{7}{100} = \frac{37}{100} + \frac{30}{100} + \frac{19}{100} + \frac{7}{100} = \frac{93}{100}$ so $1 - \frac{93}{100} = \frac{7}{100}$ is unaccounted for.

Assessment 6-2B

1. Various methods may be used:

(a) $\frac{^-1}{2} + \frac{2}{3} \Rightarrow \text{LCD}(2,3) = 6 \Rightarrow \frac{^-1}{2} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{2}{2} = \frac{^-3}{6} + \frac{4}{6} = \mathbf{\frac{1}{6}}$.

(b) $\frac{5}{12} - \frac{2}{3} = \frac{5\cdot 3 - 2\cdot 12}{12\cdot 3} = \frac{15 - 24}{36} = \frac{^-9}{36} = \mathbf{\frac{^-1}{4}}$.

(c) $\frac{5}{4x} + \frac{^-3}{2y} = \frac{5\cdot 2y + {}^-3\cdot 4x}{4x\cdot 2y} = \frac{10y - 12x}{8xy} = \mathbf{\frac{5y - 6x}{4xy}}$.

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(d) $\frac{-3}{2x^2y^2} + \frac{5}{2xy^2} + \frac{7}{x^2y} \Rightarrow$
$\text{LCD}(2x^2y^2, 2xy^2, x^2y) =$
$2x^2y^2 \Rightarrow \frac{-3}{2x^2y^2} + \frac{5}{2xy^2} \cdot \frac{x}{x} +$
$\frac{7}{x^2y} \cdot \frac{2y}{2y} = \frac{\mathbf{-3+5x+14y}}{\mathbf{2x^2y^2}}$.

(e) $\frac{5}{6} - 2\frac{1}{8} = \frac{5}{6} - \frac{17}{8} = \frac{5\cdot 8 - 6\cdot 17}{6\cdot 8} =$
$\frac{40-102}{48} = \frac{-62}{48} = \mathbf{\frac{-31}{24}} = \mathbf{-1\frac{7}{24}}$.

(f) $-4\frac{1}{2} + 3\frac{1}{6} \Rightarrow \text{LCD}(2,6) = 6 \Rightarrow$
$-4\frac{1}{2} \cdot \frac{3}{3} + 3\frac{1}{6} = -4\frac{3}{6} + 3\frac{1}{6} =$
$-1\frac{2}{6} = \mathbf{-1\frac{1}{3}} = \mathbf{\frac{-4}{3}}$.

(g) $5\frac{1}{3} + 5\frac{5}{6} - 3\frac{1}{9} = \frac{16}{3} + \frac{35}{6} - \frac{28}{9} =$
$\frac{96}{18} + \frac{105}{18} - \frac{56}{18} = \frac{201}{18} - \frac{56}{18} = \frac{145}{18} = 8\frac{1}{18}$

2. **(a)** $14 \div 5 = 2$, remainder $4 \Rightarrow \frac{14}{5} = \mathbf{2\frac{4}{5}}$.

(b) $-47 \div 8 = -5$, remainder $7 \Rightarrow -\frac{47}{8} =$
$\mathbf{-5\frac{7}{8}}$.

3. **(a)** $7\frac{1}{2} = \frac{7}{1} + \frac{1}{2} = \frac{7\cdot 2 + 1\cdot 1}{1\cdot 2} = \frac{14+1}{2} = \mathbf{\frac{15}{2}}$.

(b) $-4\frac{2}{3} = -\left(\frac{4}{1} + \frac{2}{3}\right) = -\left(\frac{4\cdot 3 + 1\cdot 2}{1\cdot 3}\right) =$
$-\left(\frac{12+2}{3}\right) = \mathbf{\frac{-14}{3}}$.

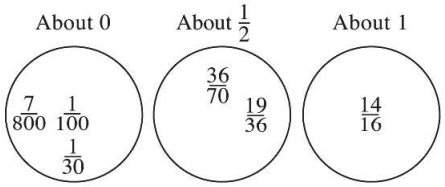
4. $\frac{\boxed{2}}{\boxed{6}} + \frac{\boxed{5}}{\boxed{8}} = \frac{23}{24}$. A denominator of 24 in the sum eliminates 5 as a denominator in either of the addends; 2 in either would give an answer > 1; trial and error yields 6 paired with 2 and 8 paired with 5.

5. **(a)** **Tigers**. $\frac{9}{28} < \frac{1}{3}$ by the least margin.

(b) **Lions**. $\frac{7}{27} > \frac{1}{4}$ by the least margin.

(c) **Wildcats** and **Badgers**. $\frac{6}{25}$ and $\frac{5}{21} < \frac{1}{4}$.

6.



7. **(a)** **1; low**. $\frac{113}{100} > 1$.

(b) **0; low**. $\frac{3}{1978} > 0$.

(c) $\mathbf{\frac{3}{4}}$**; low**. $\frac{150}{198} > \frac{150}{200} = \frac{3}{4}$.

(d) **1; high**. $\frac{8}{9} < 1$.

8. **(a)** The first addend is about 0, the next two addends are about $\frac{1}{2}$, and the third is about 1. **2** best approximates the sum.

(b) The third addend is about 0 and the second is close to one-fourth. The first is estimated at about three-forths. **1** is the best approximation.

9. Possible thought processes could be:

(a) $(5+1) - \frac{7}{8} = 5 + \left(\frac{8}{8} - \frac{7}{8}\right) = \mathbf{5\frac{1}{8}}$.

(b) $\left(2 + 4 + 3 + \frac{6}{10} + \frac{1}{10} + \frac{3}{10}\right) =$
$9 + \frac{10}{10} = \mathbf{10}$.

10. **(a)** Region ***M***. $\frac{9}{8} = 1\frac{1}{8}$ is between 1 and 2.

(b) Region ***A***. $\frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$ is between 2 and 3.

(c) Region ***T***. $\frac{50}{16} = \frac{25}{8} = 3\frac{1}{8}$ is between 3 and 4.

(d) Region ***H***. $\frac{17}{4} = 4\frac{1}{4}$ is between 4 and 5.

11. The entire student population is represented by 1; subtract to obtain the senior's fraction; i.e., seniors make up $1 - \frac{1}{4} - \frac{1}{5} - \frac{1}{10}$ of the class. Thus
$1 - \frac{1}{4} - \frac{1}{5} - \frac{1}{10} = \frac{20}{20} - \frac{5}{20} - \frac{4}{20} - \frac{2}{20} = \mathbf{\frac{9}{20}}$.

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12. **(a)** $\frac{1}{5}$ (Japan) $-\frac{1}{6}$ (Canada) $= \frac{6}{30} - \frac{5}{30} = \mathbf{\frac{1}{30}}$.

(b) $\frac{7}{20}$ (United States) $-\frac{1}{4}$ (England) $=$ $\frac{7}{20} - \frac{5}{20} = \frac{2}{20} = \mathbf{\frac{1}{10}}$.

(c) $\frac{7}{20}(2012) - \frac{1}{3}(2009) = \frac{21}{60} - \frac{20}{60} = \mathbf{\frac{1}{60}}$.

(d) **No**. The total number of dollars might have been greater in 2012 than in 2014, but the fraction of the total dollars $\left(\text{i.e., } \frac{1}{10} \text{ versus } \frac{1}{20}\right)$ was greater in 2014.

13. $3\frac{1}{2} - 1\frac{3}{4} = 3\frac{2}{4} - 1\frac{3}{4} = \left(2\frac{2}{4} + \frac{4}{4}\right) - 1\frac{3}{4}$ $= 2\frac{6}{4} - 1\frac{3}{4} = \mathbf{1\frac{3}{4}}$ **cups**.

14. $38\frac{1}{4} - \left(15\frac{3}{4} + \frac{3}{8}\right) = 38\frac{2}{8} - \left(15\frac{6}{8} + \frac{3}{8}\right) =$ $38\frac{2}{8} - 16\frac{1}{8} = \mathbf{22\frac{1}{8}}$ **inches**.

15. **(a)** Team **4** collected $35\frac{3}{16} + 41\frac{1}{2} =$ $35\frac{3}{16} + 41\frac{8}{16} = \mathbf{76\frac{11}{16}}$ **pounds**.

(b) Collections in April were $28\frac{3}{4} + 32\frac{7}{8} +$ $28\frac{1}{2} + 35\frac{3}{16} = 28\frac{12}{16} + 32\frac{14}{16} + 28\frac{8}{16} +$ $35\frac{3}{16} = 123\frac{37}{16} = 125\frac{5}{16}$ pounds.

Collections in May were $33\frac{1}{3} + 28\frac{5}{12} +$ $25\frac{3}{4} + 41\frac{1}{2} = 33\frac{4}{12} + 28\frac{5}{12} + 25\frac{9}{12} +$ $41\frac{6}{12} = 127\frac{24}{12} = 129$ pounds.

The difference was $129 - 125\frac{5}{16} =$ $128\frac{16}{16} - 125\frac{5}{16} = \mathbf{3\frac{11}{16}}$ **pounds**.

16. Let the sequence be $1, a_2, a_3, a_4, a_5, 3$. Thus $3 = 1 + (6-1)d \Rightarrow 5d = 2 \Rightarrow d = \frac{2}{5}$. Then

$a_2 = 1 + \frac{2}{5} = \frac{7}{5}$,

$a_3 = \frac{7}{5} + \frac{2}{5} = \frac{9}{5}$,

$a_4 = \frac{9}{5} + \frac{2}{5} = \frac{11}{5}$, and

$a_5 = \frac{11}{5} + \frac{2}{5} = \frac{13}{5}$.

The sequence is $\mathbf{1, \frac{7}{5}, \frac{9}{5}, \frac{11}{5}, \frac{13}{5}, 3}$.

17. **(a)** $x - \frac{5}{6} = \frac{2}{3} \Rightarrow x = \frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} =$ $\frac{9}{6} = \frac{3}{2}$. $\boldsymbol{x = 1\frac{1}{2}}$.

(b) $x - \frac{7}{2^3 \cdot 3^2} = \frac{5}{2^2 \cdot 3^2} \Rightarrow x = \frac{5}{2^2 \cdot 3^2} +$ $\frac{7}{2^3 \cdot 3^2} = \frac{5}{2^2 \cdot 3^2} \cdot \frac{2}{2} + \frac{7}{2^3 \cdot 3^2} =$ $\frac{10+7}{2^3 \cdot 3^2} \Rightarrow \boldsymbol{x = \frac{17}{2^3 \cdot 3^2} = \frac{17}{72}}$.

18. Write the equivalent operations of using (take away) paint and adding paint:

$$\frac{3}{4} - \frac{1}{3} + \frac{1}{2} = \frac{9}{12} - \frac{4}{12} + \frac{6}{12} = \mathbf{\frac{11}{12}} \textbf{ cup}.$$

19. **(a)** $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \mathbf{\frac{3}{4}}$.

(b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \mathbf{\frac{7}{8}}$.

(c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} =$ $\frac{14}{16} + \frac{1}{16} = \mathbf{\frac{15}{16}}$.

(d) **No**. Notice that in each of the sums in a,b, and c, the numerator is one less than the denominator. In fact, $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots +$ $\frac{1}{2^n} = \frac{2^n - 1}{2^n}$.

Alternative: Note that each new sum in the pattern is created by adding a value that is $\frac{1}{2}$ the distance between the previous sum and 1. For example, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$. $1 - \frac{15}{16} = \frac{1}{16}$, so this sum is $\frac{1}{16}$ units from 1. The next sum in the pattern is $1 + \frac{1}{4} +$ $\frac{1}{8} + \frac{1}{16} + \frac{1}{32}$. Since $\frac{1}{32}$ is $\frac{1}{2}$ of $\frac{1}{16}$, the new sum is less than 1.

20. Answers vary.

(a) $\frac{73,723}{150,155} \approx \frac{74}{150} \approx \frac{148}{300}$ and $\frac{75,472}{154,144} \approx \frac{75}{154} \approx \frac{150}{300}$ so approximately $\frac{2}{300}$ or $\frac{1}{150}$ more.

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(b) $\dfrac{73,723+73,401}{150,155+149,904}=\dfrac{147,124}{300,059}\approx\dfrac{147}{300}\approx\dfrac{1}{2}$

(c)
$\dfrac{73,723+73,401+75,472}{12,742,886}=\dfrac{222,596}{12,742,886}\approx\dfrac{2}{127}.$

21. $\dfrac{633,000}{1,193,000}\approx\dfrac{63}{119}$ and $\dfrac{1,186,000}{1,259,000}\approx\dfrac{118}{125}$. Since $\dfrac{63}{119}<\dfrac{118}{125}$ there are fewer 3- and 4-year olds in school.

22. $50^\circ+7^\circ=57^\circ$ and $\dfrac{360^\circ}{360^\circ}-\dfrac{57^\circ}{360^\circ}=\dfrac{303}{360}=\dfrac{101}{120}$ so the fractional part not represented by rent and saving is $\dfrac{101}{120}$ of the total.

23. $\dfrac{37}{100}=\dfrac{3}{10}+x\Rightarrow\dfrac{37}{100}=\dfrac{30}{100}+\dfrac{7}{100}$ so $\dfrac{7}{100}$ more coal than natural gas is used.

Mathematical Connections 6-2: Review Problems

15. **(a)** $\frac{14}{21}=\frac{7\cdot2}{7\cdot3}=\mathbf{\frac{2}{3}}$.

(b) $\frac{117}{153}=\frac{9\cdot13}{9\cdot17}=\mathbf{\frac{13}{17}}$.

(c) $\frac{5^2}{7^2}=\mathbf{\frac{25}{49}}$, which cannot be further simplified.

(d) $\frac{a^2+a}{1+a}=\frac{a(a+1)}{a+1}=\frac{a}{1}=\boldsymbol{a}$.

(e) $\frac{a^2+1}{a+1}$ **cannot be further simplified**. There are no factors common to both the numerator and denominator.

(f) $\frac{a^2-b^2}{a-b}=\frac{(a+b)(a-b)}{a-b}=\boldsymbol{a+b}$ (if $a\neq b$).

16. **(a)** **Equal**. $\frac{a^2b^2}{b^3}=\frac{a^2\cdot b^2}{b\cdot b^2}=\frac{a^2}{b}$ $(b\neq0)$.

(b) **Not equal**. $377\cdot401=151{,}177\neq400\cdot378=151{,}200$.

(c) **Equal**. $\frac{0}{10}=0=\frac{0}{^-10}$.

(d) **Not equal**. $a(b+1)=ab+a\neq b(a+1)=ab+b$.

17. Yes. $\dfrac{27}{103}=\dfrac{54}{206}$ so there could be 54 bones in both hands or 27 bones in each hand.

18. There are infinitely many fractions equivalent to $\dfrac{3}{5}$ because the equivalent fractions can be written in the form $\dfrac{3n}{5n}$ with n an integer.Since the set of integers is infinite, the set of fractions equivalent to $\dfrac{3}{5}$ is infinite.

19. There are $1-\dfrac{3}{5}=\dfrac{5}{5}-\dfrac{3}{5}=\dfrac{2}{5}$ of the cookies left for Suzanne.

20. $\dfrac{-1}{10}=\dfrac{^-10}{100}$ and $\dfrac{^-10}{100}$ is 10^{th} away from 0 and $\dfrac{-1}{100}$ is 1^{th} away from 0 so $\dfrac{^-1}{100}$ is greater than $\dfrac{^-1}{10}$.

Assessment 6-3A: Multiplication, Division, and Estimation with Rational Numbers

1. **(a)** The blue-shaded vertical region represents $\frac{1}{3}$ of the total area; the yellow-shaded horizontal region represents $\frac{1}{4}$ of the total area. The blue-yellow region therefore represents $\frac{1}{4}$ of $\frac{1}{3}$, or the product of the two fractions. Since one of the twelve blocks is blue-yellow it represents $\mathbf{\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}}$.

(b) The blue-shaded vertical region represents $\frac{3}{5}$ of the total area; the yellow-shaded horizontal region represents $\frac{2}{4}$ of the total area. The blue-yellow region therefore represents $\frac{2}{4}$ of $\frac{3}{5}$, or the product of the two fractions. Since six of the twenty blocks are blue-yellow it represents $\mathbf{\frac{2}{4}\cdot\frac{3}{5}=\frac{6}{20}}$.

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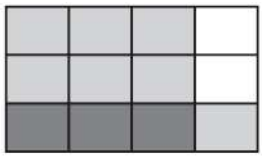
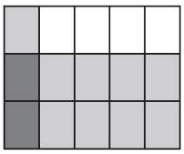
2. **(a)** There are 12 boxes in the figure below. The dark-shaded region represents $\frac{9}{12}$ of $\frac{4}{12}$, or $\frac{3}{4} \cdot \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$:



(b) There are 15 boxes in the figure below. The dark-shaded region represents $\frac{3}{15}$ of $\frac{10}{15}$, or $\frac{1}{5} \cdot \frac{2}{3} = \mathbf{\frac{2}{15}}$:



3. **(a)** $\frac{49}{65} \cdot \frac{26}{98} = \frac{1274}{6370} = \frac{1 \cdot 1274}{5 \cdot 1274} = \mathbf{\frac{1}{5}}$.

(b) $\frac{a}{b} \cdot \frac{b^2}{a^2} = \frac{ab^2}{a^2 b} = \frac{b \cdot ab}{a \cdot ab} = \mathbf{\frac{b}{a}}$.

(c) $\frac{xy}{z} \cdot \frac{z^2 a}{x^3 y^2} = \frac{axyz^2}{x^3 y^2 z} = \frac{az \cdot xyz}{x^2 y \cdot xyz} = \mathbf{\frac{az}{x^2 y}}$.

4. **(a)**
$$4\tfrac{1}{2} \cdot 2\tfrac{1}{3} = \left(4 + \tfrac{1}{2}\right) \cdot \left(2 + \tfrac{1}{3}\right)$$
$$= 4\left(2 + \tfrac{1}{3}\right) + \tfrac{1}{2}\left(2 + \tfrac{1}{3}\right)$$
$$= 8 + \tfrac{4}{3} + 1 + \tfrac{1}{6}$$
$$= 9 + \tfrac{8}{6} + \tfrac{1}{6} = \mathbf{10\tfrac{1}{2}}.$$

(b)
$$3\tfrac{1}{3} \cdot 2\tfrac{1}{2} = \left(3 + \tfrac{1}{3}\right) \cdot \left(2 + \tfrac{1}{2}\right)$$
$$= 3\left(2 + \tfrac{1}{2}\right) + \tfrac{1}{3}\left(2 + \tfrac{1}{2}\right)$$
$$= 6 + \tfrac{3}{2} + \tfrac{2}{3} + \tfrac{1}{6}$$
$$= 6 + \tfrac{9}{6} + \tfrac{4}{6} + \tfrac{1}{6} = \mathbf{8\tfrac{1}{3}}.$$

5. **(a)** Multiplicative inverse of $\frac{-1}{3}$ is $\frac{3}{-1} = \mathbf{-3}$.

(b) Multiplicative inverse of $3\frac{1}{3} = \frac{10}{3}$ is $\mathbf{\frac{3}{10}}$.

(c) Multiplicative inverse of $\frac{x}{y}$ is $\mathbf{\frac{y}{x}}(x, y \neq 0)$.

(d) Multiplicative inverse of $-7 = \frac{-7}{1}$ is $\frac{1}{-7} = \mathbf{\frac{-1}{7}}$.

6. **(a)** $\frac{2}{3}x = \frac{7}{6} \Rightarrow x = \frac{7}{6} \div \frac{2}{3} = \frac{7}{6} \cdot \frac{3}{2} \Rightarrow \boldsymbol{x = \frac{7}{4}}$.

(b) $\frac{3}{4} \div x = \frac{1}{2} \Rightarrow \frac{3 \cdot 1}{4 \cdot x} = \frac{1}{2} \Rightarrow 3 \cdot 1 \cdot 2 = 4 \cdot x \cdot 1 \Rightarrow 4x = 6 \Rightarrow \boldsymbol{x = \frac{6}{4} = \frac{3}{2}}$.

(c) $\frac{5}{6} + \frac{2}{3}x = \frac{3}{4} \Rightarrow \frac{2}{3}x = \frac{3}{4} - \frac{5}{6} = \frac{-1}{12} \Rightarrow x = \frac{-1}{12} \div \frac{2}{3} = \frac{-1}{12} \cdot \frac{3}{2} \Rightarrow \boldsymbol{x = \frac{-1}{8}}$.

(d) $\frac{2x}{3} - \frac{1}{4} = \frac{x}{6} + \frac{1}{2} \Rightarrow \frac{2}{3}x - \frac{1}{6}x = \frac{1}{2} + \frac{1}{4} \Rightarrow \frac{1}{2}x = \frac{3}{4} \Rightarrow x = \frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1} \Rightarrow \boldsymbol{x = \frac{3}{2}}$.

7. Answers may vary; e.g.,

(a) $\frac{1}{2} \div \frac{1}{4} \neq \frac{1}{4} \div \frac{1}{2}$.

(b) $\left(\frac{2}{3} \div \frac{1}{2}\right) \div \frac{3}{5} \neq \frac{2}{3} \div \left(\frac{1}{2} \div \frac{3}{5}\right)$

8. Possible thought processes are described; all provide exact answers:

(a) $3 \cdot 8 = 24, \frac{1}{4} \cdot 8 = 2,$ and $24 + 2 = \mathbf{26}$.

(b) $7 \cdot 4 = 28, \frac{1}{4} \cdot 4 = 1,$ and $28 + 1 = \mathbf{29}$.

(c) $9 \cdot 10 = 90, \frac{1}{5} \cdot 10 = 2,$ and $90 + 2 = \mathbf{92}$.

(d) $8 \cdot 2 = 16, 8 \cdot \frac{1}{4} = 2,$ and $16 + 2 = \mathbf{18}$.

9. **(a)** **20**. $3\frac{11}{12} \cdot 5\frac{3}{100} \approx 4 \cdot 5 = 20$.

(b) **16**. $2\frac{1}{10} \cdot 7\frac{7}{8} \approx 2 \cdot 8 = 16$.

(c) **1**. $\frac{1}{101}$ and $\frac{1}{103}$ are approximately equal.

10. **(a)** **Less than 1**. $\frac{13}{14} \cdot \frac{17}{19}$ is the product of two proper fractions; i.e., each is less than 1. The product of two positive proper fractions will be smaller than either, and therefore will always be less than 1.

(b) **Less than 1**. $3\frac{2}{7} \div 5\frac{1}{9}$ is a positive number divided by a larger positive number. Their quotient, a proper fraction, would always be less than 1.

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(c) **Greater than 2**. If the quotient was 2, then checking would show that $2\left(2\frac{3}{100}\right) = 4\frac{6}{100}$, which is less than $4\frac{1}{3}$. Thus the true quotient must be greater than 2.

11. **(d)**, between \$33 and \$40. Estimation gives a cost of approximately
$6 \cdot \$4.00 + 3 \cdot \$3.00 = \$24.00 + \$9.00 = \$33.00$
All rounding was down, so the estimate is low.

12. Let p be the student population. The 6000 students living in dorms are $\frac{5}{8}$ of p; i.e., $6000 = \frac{5}{8}p \Rightarrow p = \frac{8}{5}(6000) = \mathbf{9600\ students}$.

13. Alberto has $\frac{5}{9}$ of the stock; Renatta has $\frac{1}{2} \cdot \frac{5}{9} = \frac{5}{18}$ of the stock. Thus $1 - \frac{5}{9} - \frac{5}{18} = \mathbf{\frac{1}{6}}$ **of the stock** is not owned by them.

14. Let p be the original price. Then $p - \frac{1}{4}p = 180 \Rightarrow \frac{3}{4}p = 180 \Rightarrow p = \frac{4}{3}(180) = \mathbf{\$240}$.

15. Let a be the amount of money in the account. After spending \$50 there was $a - 50$. He spent $\frac{3}{5}$ of that, or $\frac{3}{5}(a - 50)$, leaving him $\frac{2}{5}(a - 50)$; half goes back into the bank, or $\frac{1}{2} \cdot \frac{2}{5}(a - 50) = \frac{1}{5}(a - 50)$. The other half was \$35; or $\frac{1}{5}(a - 50) = 35 \Rightarrow \frac{1}{5}a - 10 = 35 \Rightarrow \frac{1}{5}a = 45 \Rightarrow a = 5 \cdot 45 = \mathbf{\$225}$.

16. Al's marbles are halved three times in the process of Dani receiving 4 marbles, i.e., if m is the number of marbles Al had originally, then $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot m = 4 \Rightarrow \frac{1}{8}M = 4 \Rightarrow M = 8 \cdot 4 = \mathbf{32\ marbles}$.

17. If a is any number and m and n are natural numbers, then $a^m \cdot a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, and $a^{-m} = \frac{1}{a^m}$:

(a) $3^{-7} \cdot 3^{-6} = 3^{-7+-6} = 3^{-13} = \frac{1}{3^{13}} = \left(\mathbf{\frac{1}{3}}\right)^{\mathbf{13}}$.

(b) $3^7 \cdot 3^6 = 3^{7+6} = \mathbf{3^{13}}$.

(c) $5^{15} \div 5^4 = 5^{15-4} = \mathbf{5^{11}}$.

(d) $5^{15} \div 5^{-4} = 5^{15-\,-4} = 5^{15+4} = \mathbf{5^{19}}$.

(e) $(^-5)^{-2} = \frac{1}{(^-5)^2} = \mathbf{\frac{1}{5^2}} = \left(\mathbf{\frac{1}{5}}\right)^{\mathbf{2}}$.

(f) $\frac{a^2}{a^{-3}} = a^{2-\,-3} = a^{2+3} = \boldsymbol{a^5}$.

18. **(a)** $\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{3+7} = \left(\mathbf{\frac{1}{2}}\right)^{\mathbf{10}}$.

(b) $\left(\frac{1}{2}\right)^9 \div \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^{9-6} = \left(\mathbf{\frac{1}{2}}\right)^{\mathbf{3}}$.

(c) $\left(\frac{2}{3}\right)^5 \cdot \left(\frac{4}{9}\right)^2 = \left(\frac{2}{3}\right)^5 \cdot \left[\left(\frac{2}{3}\right)^2\right]^2 = \left(\frac{2}{3}\right)^{5+4} = \left(\mathbf{\frac{2}{3}}\right)^{\mathbf{9}}$.

(d) $\left(\frac{3}{5}\right)^7 \div \left(\frac{3}{5}\right)^7 = \left(\frac{3}{5}\right)^{7-7} = \left(\frac{3}{5}\right)^0 = \mathbf{1}$.

19. Counterexamples may vary:

(a) **False**. $2^3 \cdot 3^2 = 8 \cdot 9 = 72 \neq (2 \cdot 3)^{3+2} = 6^5 = 7776$.

(b) **False**. $2^3 \cdot 3^2 = 72 \neq (2 \cdot 3)^{2 \cdot 3} = 6^6 = 46{,}656$.

(c) **False**. $2^3 \cdot 3^3 = (2 \cdot 3)^3 = 216 \neq (2 \cdot 3)^{2 \cdot 3} = 46{,}656$.

(d) **False**. Any number (except 0) to the 0 power $= 1$; 0^0 is undefined.

(e) **False**. $(2 + 3)^2 = (2 + 3)(2 + 3) = 25 \neq 2^2 + 3^2 = 13$.

(f) **False**. $(2 + 3)^{-2} = \frac{1}{(2+3)^2} = \frac{1}{25} \neq \frac{1}{2^2} + \frac{1}{3^2} = \frac{13}{36}$.

20. **(a)** $2^n = 32 \Rightarrow 2^n = 2^5 \Rightarrow \boldsymbol{n = 5}$.

(b) $n^2 = 36 \Rightarrow n^2 = (\pm 6)^2 \Rightarrow \boldsymbol{n = 6}$ or $\mathbf{^-6}$.

(c) $2^n \cdot 2^7 = 2^5 \Rightarrow 2^{n+7} = 2^5 \Rightarrow n + 7 = 5 \Rightarrow \boldsymbol{n = \mathbf{^-2}}$.

(d) $2^n \cdot 2^7 = 8 \Rightarrow 2^{n+7} = 2^3 \Rightarrow n + 7 = 3 \Rightarrow \boldsymbol{n = \mathbf{^-4}}$.

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21. **(a)** $3^x \le 9 \Rightarrow 3^x \le 3^2 \Rightarrow \boldsymbol{x \le 2}$, where x is an integer.

(b) $25^x < 125 \Rightarrow (5^2)^x < 5^3 \Rightarrow$
$5^{2x} < 5^3 \Rightarrow 2x < 3 \Rightarrow \boldsymbol{x < \frac{3}{2}}$,
$\Rightarrow x < 2$
where x is an integer.

(c) $3^{2x} > 27 \Rightarrow 3^{2x} > 3^3 \Rightarrow 2x > 3 \Rightarrow$
$\boldsymbol{x > \frac{3}{2}} \Rightarrow x \ge 2$ where x is an integer.

(d) $4^x > 1 \Rightarrow 4^x > 4^0$
$\Rightarrow x > 0 \Rightarrow x \ge 1$, where x is an integer.

22. **(a)** $\left(\boldsymbol{\frac{1}{2}}\right)^{\boldsymbol{3}} > \left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ while $\left(\frac{1}{2}\right)^4 = \frac{1}{16}; \frac{1}{8}$ is greater.

(b) $\left(\frac{3}{4}\right)^8 > \left(\frac{3}{4}\right)^{10} . \left(\frac{3}{4}\right)^8 \div \left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$, which is greater than $1 \Rightarrow$ an improper fraction $\Rightarrow \left(\frac{3}{4}\right)^8$ is greater.

(c) $\left(\boldsymbol{\frac{4}{3}}\right)^{\boldsymbol{10}} > \left(\frac{4}{3}\right)^8 . \left(\frac{4}{3}\right)^{10} \div \left(\frac{4}{3}\right)^8 = \left(\frac{4}{3}\right)^2$, which is greater than $1 \Rightarrow \left(\frac{4}{3}\right)^{10}$ is greater.

(d) $\left(\boldsymbol{\frac{4}{5}}\right)^{\boldsymbol{10}} > \left(\frac{3}{4}\right)^{10} . \frac{4}{5} \div \frac{3}{4} > 1 \Rightarrow \frac{4}{5} > \frac{3}{4} \Rightarrow \left(\frac{4}{5}\right)^{10} > \left(\frac{3}{4}\right)^{10}$.

(e) $32^{50} > 4^{100} \Rightarrow 32^{50} = (2^5)^{50} = 2^{250}$
$\Rightarrow 4^{100} = (2^2)^{100} = 2^{200}$
$2^{250} > 2^{200}$
$(^-3)^{-75} > (^-27)^{-15}$

(f) $(^-27)^{-15} = (^-3^3)^{-15} = (^-3)^{-45}$
$(^-3)^{-75} > (^-3)^{-45}$

23. If either $\frac{a}{b}$ and $\frac{c}{d}$ is negative, it can be written with the negative signs associated with the numerators. Thus, $\frac{a}{b} < \frac{c}{d} \Rightarrow ad < bc$. Adding ad to both sides of the inequalities results in $2ad < ad + bc$. Adding bc to both sides of the inequalities results in $ad + bc < 2bc$. Dividing both sides of the inequalities by $2bd$ gives us $\frac{a}{b} < \frac{1}{2}\left(\frac{a}{b} + \frac{c}{d}\right) < \frac{c}{d}$.

24. **(a)** $1 - \frac{17}{25} = \frac{25}{25} - \frac{17}{25} = \frac{8}{25}$. There were $\frac{8}{25}$ of the students who were female.
(b) No, the number of male students depends on the total number of students. If there were 25 students total then there would be 17 male students.

25. **(a)** The restaurant tax minus the retail tax is approximately $120,000,000 - 60,000,000 = 60,000,000$ which is approximately $\frac{60,000,000}{670,000,000} \approx \frac{1}{11}$ of the total tax.
(b) The sales taxes add up to 187,656,061 ≈ 190,000,000. So $\frac{190,000,000}{680,000,000} \approx \frac{2}{7}$ would be lost.

26. 2004 -> 2005: -2
2005 -> 2006: -27
2006 -> 2007: +12
2007 -> 2008: +5
2008 -> 2009: -42
2009 -> 2010: -12
2010 -> 2011: -24
2011 -> 2012: -20
2012 -> 2013: +16

(a) The greatest fractional increase from the previous year happened from 2012 to 2013.
(b) The greatest fractional decrease from the previous year happened from 2008 to 2009.

27. $\frac{1}{33} + \frac{2}{7} = \frac{7}{231} + \frac{66}{231} = \frac{73}{231}$ is the fractional part of the deer population that is in Mississippi and in Montana.

28. $\frac{1}{4} \div 8 = \frac{1}{32}$ of the entire estate is what each cousin receives.

Assessment 6-3B

1. **(a)**

The figure above represents $\frac{2}{3}$. Divide this figure horizontally into two equal parts. This divides $\frac{2}{3}$ into 4 pieces of 6. Only 2 of the 6 have both shadings. This represents $\boldsymbol{\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6}}$.

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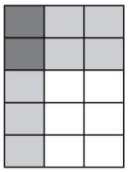
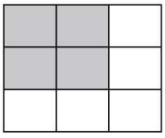
(b) The figure shows $\frac{1}{3}$ divided into two equal parts. $\mathbf{\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}}$.

2. **(a)** There are 15 boxes in the figure below. The dark-shaded region represents $\frac{6}{15}$ of $\frac{5}{15}$, or $\frac{2}{5} \cdot \frac{1}{3} = \mathbf{\frac{2}{15}}$:



(b) There are 9 boxes in the figure below. The dark-shaded region represents $\frac{6}{9}$ of $\frac{6}{9}$, or $\frac{2}{3} \cdot \frac{2}{3} = \mathbf{\frac{4}{9}}$.



3. **(a)** $2\frac{1}{3} \cdot 3\frac{3}{4} = \frac{7}{3} \cdot \frac{15}{4} = \frac{105}{12} = \frac{35 \cdot 3}{4 \cdot 3} = \frac{35}{4} = \mathbf{8\frac{3}{4}}$.

(b) $\frac{22}{7} \cdot 4\frac{2}{3} = \frac{22}{7} \cdot \frac{14}{3} = \frac{308}{21} = \frac{44 \cdot 7}{3 \cdot 7} = \frac{44}{3} = \mathbf{14\frac{2}{3}}$.

(c) $\frac{^-5}{2} \cdot 2\frac{1}{2} = \frac{^-5}{2} \cdot \frac{5}{2} = \frac{^-25}{4} = \mathbf{^-6\frac{1}{4}}$.

(d) $2\frac{3}{4} \cdot 2\frac{1}{3} = \frac{11}{4} \cdot \frac{7}{3} = \frac{77}{12} = \mathbf{6\frac{5}{12}}$.

(e) $\frac{a^2}{b^3} \cdot \frac{b^2}{a^3} = \frac{a^2b^2}{a^3b^3} = \frac{a^2b^2 \cdot 1}{a^2b^2 \cdot ab} = \mathbf{\frac{1}{ab}}$.

(f) $\frac{x^3y^2}{z} \cdot \frac{z}{x^2y} = \frac{x^3y^2z}{x^2yz} = \frac{x^2yzxy}{x^2yz} = \mathbf{xy}$.

4. **(a)** $2\frac{1}{3} \cdot 4\frac{3}{5} = \left(2 + \frac{1}{3}\right) \cdot \left(4 + \frac{3}{5}\right) = 2\left(4 + \frac{3}{5}\right) + \frac{1}{3}\left(4 + \frac{3}{5}\right) = 8 + \frac{6}{5} + \frac{4}{3} + \frac{3}{15} = \frac{120+18+20+3}{15} = \frac{161}{15} = \mathbf{10\frac{11}{15}}$.

(b) $\left(\frac{x}{y} + 1\right)\left(\frac{y}{x} - 1\right) = \frac{x}{y}\left(\frac{y}{x} - 1\right) + 1\left(\frac{y}{x} - 1\right) = 1 - \frac{x}{y} + \frac{y}{x} - 1 = \mathbf{\frac{y^2 - x^2}{xy}}$.

(c) $248\frac{2}{5} \cdot 100\frac{1}{8} = \left(248 + \frac{2}{5}\right) \cdot \left(100 + \frac{1}{8}\right) = 24{,}800 + 31 + 40 + \frac{1}{20} = \mathbf{24{,}871\frac{1}{20}}$.

5. **(a)** $\mathbf{\frac{7}{6}}$. $\frac{6}{7} \cdot \frac{7}{6} = 1$.

(b) $\mathbf{\frac{1}{8}}$. $8 \cdot \frac{1}{8} = 1$.

(c) $4\frac{1}{5} = \frac{21}{5}$. $\frac{21}{5} \cdot \mathbf{\frac{5}{21}} = 1$.

(d) $^-1\frac{1}{2} = \frac{^-3}{2}$. $\frac{^-3}{2} \cdot \left(\mathbf{\frac{^-2}{3}}\right) = 1$.

6. **(a)** $\frac{2}{3}x = \frac{11}{6} \Rightarrow x = \frac{11}{6} \div \frac{2}{3} = \frac{11}{6} \cdot \frac{3}{2} \Rightarrow x = \frac{33}{12} = \mathbf{\frac{11}{4}} = \mathbf{2\frac{3}{4}}$.

(b) $\frac{3}{4} \div x = \frac{1}{3} \Rightarrow \frac{3 \cdot 1}{4 \cdot x} = \frac{1}{3} \Rightarrow 3 \cdot 1 \cdot 3 = 4 \cdot x \cdot 1 \Rightarrow 4x = 9 \Rightarrow \mathbf{x = \frac{9}{4} = 2\frac{1}{4}}$.

(c) $\frac{5}{6} - \frac{2}{3}x = \frac{3}{4} \Rightarrow \frac{2}{3}x = \frac{5}{6} - \frac{3}{4} = \frac{1}{12} \Rightarrow x = \frac{1}{12} \div \frac{2}{3} = \frac{1}{12} \cdot \frac{3}{2} \Rightarrow \mathbf{x = \frac{1}{8}}$.

(d) $\frac{2x}{3} + \frac{1}{4} = \frac{x}{6} - \frac{1}{2} \Rightarrow \frac{2}{3}x - \frac{1}{6}x = ^-\left(\frac{1}{2}\right) + ^-\left(\frac{1}{4}\right) \Rightarrow \frac{1}{2}x = ^-\left(\frac{3}{4}\right)$
$\Rightarrow x = ^-\left(\frac{3}{4}\right) \div \frac{1}{2} = ^-\left(\frac{3}{4}\right) \cdot \frac{2}{1}$
$\Rightarrow \mathbf{x = \frac{^-3}{2} = ^-1\frac{1}{2}}$.

7. Let the fraction be $\frac{a}{b}$. Then $\frac{a+b}{b} = 3\left(\frac{a}{b}\right)$ or $\frac{a+b}{b} = \frac{3a}{b}$. The denominators are equal, so $a + b = 3a \Rightarrow b = 2a$. The fraction then becomes $\frac{a}{2a} = \mathbf{\frac{1}{2}}$.

8. Possible thought processes are described; all provide exact answers:

(a) $3 \cdot 8 = 24$ and $\frac{1}{2} \cdot 8 = 4$. So $3\frac{1}{2} \cdot 8 = 24 + 4 = \mathbf{28}$.

(b) $7 \cdot 4 = 28$ and $\frac{3}{4} \cdot 4 = 3$. So $7\frac{3}{4} \cdot 4 = 28 + 3 = \mathbf{31}$.

(c) $9 \cdot 6 = 54$ and $\frac{1}{5} \cdot 6 = \frac{6}{5} = 1\frac{1}{5}$. So $9\frac{1}{5} \cdot 6 = 54 + 1\frac{1}{5} = \mathbf{55\frac{1}{5}}$.

(d) $8 \cdot 2 = 16$ and $8 \cdot \frac{1}{3} = \frac{8}{3} = 2\frac{2}{3}$. So $8 \cdot 2\frac{1}{3} = 16 + 2\frac{2}{3} = \mathbf{18\frac{2}{3}}$.

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(e) $3 \div \frac{1}{2} = 3 \cdot 2 = \mathbf{6}$.

(f) $3\frac{1}{2} \div \frac{1}{2} = 3\frac{1}{2} \cdot 2 = \mathbf{7}$.

(g) $3 \div \frac{1}{3} = 3 \cdot 3 = \mathbf{9}$.

(h) $4\frac{1}{2} \div 2 = 4\frac{1}{2} \cdot \frac{1}{2} \Rightarrow 4 \cdot \frac{1}{2} = 2$, $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, and $2 + \frac{1}{4} = \mathbf{2\frac{1}{4}}$.

9. **(a)** **2**. $20\frac{2}{3} \div 9\frac{7}{8} \approx 20 \div 10 = 2$.

(b) **24**. $3\frac{1}{20} \cdot 7\frac{77}{100} \approx 3 \cdot 8 = 24$.

(c) **1**. $\frac{1}{10^3} \div \frac{1}{1001} \approx \frac{1}{1000} \div \frac{1}{1000} = 1$.

10. **(a)** **Greater than 2**. If the quotient was 2, then checking would show that $2\left(2\frac{13}{100}\right) = 4\frac{26}{100}$, which is less than $4\frac{1}{3}$. Thus the true quotient must be greater than 2.

(b) **Less than 4**. $16 \div 4\frac{3}{18}$ is 16 divided by a number more than 4.

(c) **Greater than 4**. $16 \div 3\frac{8}{9}$ is 16 divided by a number less than 4.

11. If n is the number, then $3n - \frac{7}{18} = 2n + \frac{5}{12} \Rightarrow n = \frac{5}{12} + \frac{7}{18} = \frac{5 \cdot 18 + 12 \cdot 7}{12 \cdot 18} = \mathbf{\frac{29}{36}}$.

12. Let f be the original number of faculty members. Then $f - \frac{1}{5}f = 320 \Rightarrow \frac{4}{5}f = 320 \Rightarrow \left(\frac{5}{4}\right)\frac{4}{5}f = \left(\frac{5}{4}\right)320 \Rightarrow f = \mathbf{400\ members}$.

13. **(a)** Let u be the number of uniforms to be made. Assuming no waste, $u = 29\frac{1}{2} \div \frac{3}{4} = \frac{59}{2} \cdot \frac{4}{3} = \frac{236}{6} = \frac{118}{3} = 39\frac{1}{3}$, or **39 uniforms** can be made.

(b) Enough material for $\frac{1}{3}$ uniform will be left over. Each uniform needs $\frac{3}{4}$ yard material, so $\frac{1}{3} \cdot \frac{3}{4} = \frac{3}{12} = \mathbf{\frac{1}{4}\ yard}$ will remain.

14. **(a)** Increasing a salary by $\frac{1}{10}$ means the salary will be $1\frac{1}{10} = \frac{11}{10}$ of its previous value. With two such raises Martha will make $\left(100{,}000 \cdot \frac{11}{10}\right) \cdot \frac{11}{10} = 110{,}000 \cdot \frac{11}{10} =$ **$121,000**.

(b) $99,000 is $\frac{11}{10}$ of **what Aaron made last year**, i.e., $99{,}000 = \frac{11}{10}s$ (where s is last year's salary) $\Rightarrow s = \frac{10}{11}(99{,}000) =$ **$90,000**.

(c) Let s be Juanita's salary two years ago. Then $363{,}000 = \frac{11}{10}\left(\frac{11}{10}s\right)$, since two raises brought her to that point. $\frac{121}{100}s = 363{,}000 \Rightarrow s = \frac{100}{121}(363{,}000) =$ **$300,000**.

15. Let b be the number of pages in the book. Jasmine has read $\frac{3}{4}b$, so she has $b - \frac{3}{4}b = \frac{1}{4}b$ yet to read. If 82 pages $= \frac{1}{4}b$ then multiply each side of the equation by 3 to yield $246 = \frac{3}{4}b$, or she has read **246 pages**.

16. **(a)** Peter, $\frac{1}{2} \cdot 60 =$ **30 minutes**.
Paul, $\frac{5}{12} \cdot 60 =$ **25 minutes**.
Mary, $\frac{1}{3} \cdot 60 =$ **20 minutes**.

(b) $LCD(30, 25, 20) = 300$ minutes.
$300 \div 30 =$ **10 laps for Peter**.
$300 \div 25 =$ **12 laps for Paul**.
$300 \div 20 =$ **15 laps for Mary**.

17. **(a)** $\left(\frac{1}{3}\right)^{-1} = \left(\frac{3}{1}\right)^{1} = \mathbf{3}$.

(b) $\frac{a^{-3}}{a} = \frac{\left(\frac{1}{a}\right)^3}{a} = \frac{1}{a^3} \cdot \frac{1}{a} = \mathbf{\frac{1}{a^4}}$.

(c) $\frac{(a^{-4})^3}{a^{-4}} = \frac{a^{-12}}{a^{-4}} = a^{-12-(-4)} = a^{-8} = \mathbf{\frac{1}{a^8}}$.

(d) $\frac{a}{a^{-1}} = \frac{a}{\left(\frac{1}{a}\right)} = \frac{a}{1} \cdot \frac{a}{1} = \mathbf{a^2}$.

(e) $\frac{a^{-3}}{a^{-2}} = \frac{a^2}{a^3} = \mathbf{\frac{1}{a}}$.

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18. **(a)** $\left(\frac{1}{2}\right)^{10} \div \left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{10-2} = \left(\mathbf{\frac{1}{2}}\right)^{\mathbf{8}}$.

(b) $\left(\frac{2}{3}\right)^{5} \cdot \left(\frac{4}{9}\right)^{-2} = \left(\frac{2}{3}\right)^{5} \cdot \left[\left(\frac{2}{3}\right)^{2}\right]^{-2} = \left(\frac{2}{3}\right)^{5-4} = \mathbf{\frac{2}{3}}$.

(c) $\left(\frac{3}{5}\right)^{7} \div \left(\frac{5}{3}\right)^{4} = \left(\frac{3}{5}\right)^{7} \div \left(\frac{3}{5}\right)^{-4} = \left(\frac{3}{5}\right)^{7-{}^{-}4} = \left(\mathbf{\frac{3}{5}}\right)^{\mathbf{11}}$.

(d) $\left[\left(\frac{5}{6}\right)^{7}\right]^{3} = \left(\frac{5}{6}\right)^{7\cdot 3} = \left(\mathbf{\frac{5}{6}}\right)^{\mathbf{21}}$.

19. False counterexamples may vary:

(a) **False**. $\frac{2^3}{3^2} = \frac{8}{9} \neq \left(\frac{2}{3}\right)^{3-2} = \frac{2}{3}$.

(b) **True**.
$(ab)^{-m} = \frac{1}{(ab)^m} = \frac{1}{a^m b^m} = \frac{1}{a^m} \cdot \frac{1}{b^m}$.

(c) **False**. $\left(\frac{2}{2^{-1}+3^{-1}}\right)^{-1} = \frac{\left(\frac{1}{2}+\frac{1}{3}\right)}{2} = \frac{\left(\frac{5}{6}\right)}{2} = \frac{5}{12} \neq \frac{1}{2} \cdot \frac{1}{2+3} = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$.

(d) **True**. $2(a^{-1} + b^{-}1)^{-1} = 2 \cdot \frac{1}{a^{-1}+b^{-1}} = 2 \cdot \frac{1}{\frac{1}{a}+\frac{1}{b}} = 2 \cdot \frac{1}{\left(\frac{a+b}{ab}\right)} = \frac{2ab}{a+b}$.

(e) **False**. $3^{2\cdot 3} = 3^6 = 729 \neq 3^2 \cdot 3^3 = 243$.

(f) **True**. $\left(\frac{a}{b}\right)^{-1} = \frac{1}{\left(\frac{a}{b}\right)} = \frac{b}{a}$.

20. **(a)** $2^n = {}^{-}32 \Rightarrow$ **no integer solution** because 2^n will always be a positive number regardless of the value of n.

(b) $n^3 = {}^{-}\left(\frac{1}{27}\right) \Rightarrow n^3 = {}^{-}\left(\frac{1}{3}\right)^3$
$\Rightarrow \boldsymbol{n} = {}^{-}\left(\frac{1}{3}\right) \Rightarrow$ **no integer solution**.

(c) $2^n \cdot 2^7 = 1024 \Rightarrow 2^{n+7} = 2^{10}$
$\Rightarrow n + 7 = 10 \Rightarrow \boldsymbol{n = 3}$.

(d) $2^n \cdot 2^7 = 64 \Rightarrow 2^{n+7} = 2^6$
$\Rightarrow n + 7 = 6 \Rightarrow \boldsymbol{n = {}^{-}1}$.

(e) $(2 + n)^2 = 2^2 + n^2 \Rightarrow 2^2 + 4n + n^2 = 2^2 + n^2 \Rightarrow 4n = 0 \Rightarrow \boldsymbol{n = 0}$.

(f) $3^n = 27^5 \Rightarrow 3^n = (3^3)^5 \Rightarrow 3^n = 3^{15}$
$\Rightarrow \boldsymbol{n = 15}$.

21. **(a)** $3^x \geq 81 \Rightarrow 3^x \geq 3^4 \Rightarrow \boldsymbol{x \geq 4}$.

(b) $4^x \geq 8 \Rightarrow (2^2)^x \geq 2^3 \Rightarrow 2x \geq 3$
$\Rightarrow x \geq \frac{3}{2} \Rightarrow \boldsymbol{x \geq 2}$, where x is an integer.

(c) $3^{2x} \leq 27 \Rightarrow 3^{2x} \leq 3^3 \Rightarrow 2x \leq 3$
$\Rightarrow x \leq \frac{3}{2} \Rightarrow \boldsymbol{x \leq 1}$, where x is an integer.

(d) $2^x < 1 \Rightarrow 2^x < 2^0 \Rightarrow$
$\boldsymbol{x < 0 \Rightarrow x \leq {}^{-}1}$.

22. **(a)** $\left(\mathbf{\frac{4}{3}}\right)^{\mathbf{10}} \cdot \left(\frac{4}{3}\right)^{10} = \frac{4}{3} \cdot \frac{4}{3} \cdot \left(\frac{4}{3}\right)^{8}$; i.e., multiplying $\left(\frac{4}{3}\right)^{8}$ by a number greater than 1.

(b) $\left(\mathbf{\frac{4}{5}}\right)^{\mathbf{10}} \cdot \frac{4}{5} > \frac{3}{4} \Rightarrow \left(\frac{4}{5}\right)^{10} > \left(\frac{3}{4}\right)^{10}$.

(c) $\left(\mathbf{\frac{4}{3}}\right)^{\mathbf{10}} \cdot \frac{4}{3} > \frac{5}{4}$ (because $4 \cdot 4 > 3 \cdot 5$)
$\Rightarrow \left(\frac{4}{3}\right)^{10} > \left(\frac{5}{4}\right)^{10}$.

(d) $\left(\mathbf{\frac{3}{4}}\right)^{\mathbf{100}} \cdot \frac{3}{4} > \frac{3\cdot 9}{4\cdot 10} = \frac{9}{10} \cdot \frac{3}{4}$
$\Rightarrow \left(\frac{3}{4}\right)^{100} > \left(\frac{3\cdot 9}{4\cdot 10}\right)^{100}$.

23. **(a)** $\mathbf{32^{100}}$. $32^{100} = (2^5)^{100} = 2^{500}$, while $4^{200} = (2^2)^{200} = 2^{400}$.

(b) $\mathbf{({}^{-}3)^{-50}}$. $({}^{-}27)^{-15} = [({}^{-}3)^3]^{-15} = ({}^{-}3)^{-45} = \frac{1}{({}^{-}3)^{45}} = {}^{-}\left(\frac{1}{3^{45}}\right)$, while $({}^{-}3)^{-50} = \frac{1}{({}^{-}3)^{50}} = \left(\frac{1}{3^{50}}\right)$, a positive number.
So $({}^{-}3)^{-50} > ({}^{-}27)^{-15}$
$(\frac{1}{3^{45}} > \frac{1}{3^{50}}$, but the negative reverses the direction of the inequality).

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24. Let $x =$ the amount Brandy spent on the horse's keep. Counting the amount she paid for the horse, Brandy spent $270 + x$. Her profit or loss can be found by $270 + x - 540$. In the second sentence we are told Brandy lost $\frac{1}{2}(270) + \frac{1}{4}x$. Thus,

$$\begin{aligned} \tfrac{1}{2}(270) + \tfrac{1}{4}x &= 270 + x - 540 \\ 135 + \tfrac{1}{4}x &= x - 270 \\ 135 + 270 &= x - \tfrac{1}{4}x \\ 405 &= \tfrac{3}{4}x \\ x &= \tfrac{4}{3} \cdot 405 = 540 \text{ dollars.} \end{aligned}$$

Since x represents the money Brandy spent on keep, her loss is $270 + 540 - 540 =$ **270 dollars**.

25. $1 - \frac{8}{25} = \frac{25}{25} - \frac{8}{25} = \frac{17}{25}$ of the students were male.

26. **(a)** The hotel tax minus the property tax is approximately $210,000,000 - 180,000,000 = 40,000,000$ which is approximately $\frac{40,000,000}{670,000,000} \approx \frac{4}{67}$ of the total tax.

(b) The sales tax increased by $\frac{1}{11}$ would be approximately $\frac{1}{11}(187,000,000) \approx 17,000,000$ .That is approximately $\frac{17,000,000}{670,000,000} \approx \frac{20}{700} \approx \frac{2}{70}$.

27. **(a)** The fractional decrease in homicides from 2005 to 2012 was $\frac{196-88}{196} = \frac{108}{196} = \frac{27}{49}$.

(b) The fractional increase in homicides from 2012 to 2013 was $\frac{104-88}{88} = \frac{16}{88} = \frac{2}{11}$.

28. $1 - \frac{1}{33} - \frac{2}{7} = \frac{231}{231} - \frac{7}{231} - \frac{66}{231} = \frac{158}{231}$ of the deer population in the United States is not in Mississippi and Montana.

29. $\frac{2}{5}$ of $\frac{1}{4} = \frac{2}{20} = \frac{1}{10}$ is stocks and $\frac{3}{5}$ of $\frac{1}{4} = \frac{3}{20}$ is for bonds.

Mathematical Connections 6-3: Review Problems

18. **(a)** $\frac{-3}{16} + \frac{7}{4} = \frac{-3}{16} + \frac{28}{16} = \frac{25}{16} = \mathbf{1\frac{9}{16}}$.

(b) $\frac{1}{6} + \frac{-4}{9} + \frac{5}{3} = \frac{3}{18} + \frac{-8}{18} + \frac{30}{18} = \frac{25}{18} = \mathbf{1\frac{7}{18}}$.

(c) $\frac{-5}{2^3 \cdot 3^2} - \frac{-5}{2 \cdot 3^3} = \frac{-5 \cdot 3}{2^3 \cdot 3^3} - \frac{-5 \cdot 2^2}{2^3 \cdot 3^3} = \frac{-15}{216} + \frac{20}{216} = \mathbf{\frac{5}{216}}$.

(d) $3\frac{4}{5} + 4\frac{5}{6} = 3\frac{24}{30} + 4\frac{25}{30} = 7\frac{49}{30} = \mathbf{8\frac{19}{30}}$.

(e) $5\frac{1}{6} - 3\frac{5}{8} = 5\frac{4}{24} - 3\frac{15}{24} = 4\frac{28}{24} - 3\frac{15}{24} = \mathbf{1\frac{13}{24}}$.

(f) $-4\frac{1}{3} - 5\frac{5}{12} = \frac{-13}{3} - \frac{65}{12} = \frac{-52}{12} - \frac{65}{12} = \frac{-117}{12} = \frac{-39}{4} = \mathbf{-9\frac{3}{4}}$.

19. The portion of students that take Spanish, French, or German is $\frac{2}{3} + \frac{1}{9} + \frac{1}{18} = \frac{5}{6}$. The portion of students not taking one of these languages is $1 - \frac{5}{6} = \frac{1}{6}$. That number is $\frac{1}{6} \cdot 720 =$ **120 students**.

20. **(a)** $\frac{3x}{xy^2} + \frac{y}{x^2} = \frac{3x^2}{x^2y^2} + \frac{y^3}{x^2y^2} = \frac{3x^2 + y^3}{x^2y^2}$.

(b) $\frac{a}{xy^2} - \frac{b}{xyz} = \frac{az}{xy^2z} - \frac{by}{xy^2z} = \frac{az - by}{xy^2z}$.

(c)

$$\frac{a^2}{a^2 - b^2} - \frac{a-b}{a+b} = \frac{a^2}{(a-b)(a+b)} - \frac{(a-b)(a-b)}{(a-b)(a+b)}$$

$$= \frac{a^2 - (a^2 - ab - ab + b^2)}{(a-b)(a+b)} = \frac{2ab - b^2}{a^2 - b^2}.$$

21. **(a)** This is not correct. For example, $\frac{2 \cdot 3 + 5}{3} \neq 2 + 5$ because $\frac{11}{3} \neq 7$.

(b) This is not correct. For example, because $\frac{5}{7} \neq \frac{3}{5}$.

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(c) This is correct as long as a≠0. For example, $\frac{2\cdot 3+2\cdot 5}{2\cdot 5}=\frac{3+5}{5}$ because $\frac{6+10}{10}=\frac{3+5}{5}\Rightarrow\frac{16}{10}=\frac{8}{5}$.

Assessment 6-4A: Proportional Reasoning

1. **(a)** There are five vowels and 21 consonants. Their ratio is $\frac{5 \text{ vowels}}{21 \text{ consonants}}=\frac{5}{21}$, or **5:21**.

(b) $\frac{21 \text{ consonants}}{5 \text{ vowels}}=\frac{21}{5}$, or **21:5**.

(c) $\frac{21 \text{ consonants}}{26 \text{ alphabet letters}}=\frac{21}{26}$, or **21:26**.

(d) Answers may vary. "Break" (2 vowels, 3 consonants) or "minor" (2 vowels, 3 consonants) are two.

2. $\frac{a}{b}=\frac{c}{d}$ only if $ad=bc$:

(a) $\frac{12}{x}=\frac{18}{45}\Rightarrow 18x=12\cdot 45\Rightarrow x=\frac{12\cdot 45}{18}=$ **30**.

(b) $\frac{x}{7}=\frac{^{-}10}{21}\Rightarrow 21x=7\cdot {}^{-}10\Rightarrow x=\frac{7\cdot {}^{-}10}{21}=\frac{^{-}10}{3}=$ $\mathbf{^{-}3\frac{1}{3}}$.

(c) $\frac{5}{7}=\frac{3x}{98}\Rightarrow 7\cdot 3x=5\cdot 98\Rightarrow x=\frac{5\cdot 98}{7\cdot 3}=\frac{70}{3}=$ $\mathbf{23\frac{1}{3}}$.

(d) $3\frac{1}{2}$ is to 5 as x is to 15 $\Rightarrow \frac{3\frac{1}{2}}{5}=\frac{x}{15}\Rightarrow$ $5\cdot x=3\frac{1}{2}\cdot 15\Rightarrow x=\frac{3\frac{1}{2}\cdot 15}{5}=\frac{21}{2}=$ $\mathbf{10\frac{1}{2}}$.

3. **(a)** Because the ratio is 2:3, there are $2x$ boys and $3x$ girls. The ratio of boys to all students is then $\frac{2x}{2x+3x}=\frac{2x}{5x}=\frac{2}{5}=$ **2:5**.

(b) ***m*:(*m* + *n*)**. See part (a) above.

(c) Because the ratio of girls to all students is $\frac{3}{5}$, there are 3 girls to every 2 boys, or a ratio of girls to boys of **3:2**.

4. $\frac{2 \text{ pounds muscle}}{5 \text{ pounds body weight}}=\frac{x \text{ pounds muscle}}{90 \text{ pounds body weight}}\Rightarrow$ $5\cdot x=2\cdot 90\Rightarrow x=\frac{2\cdot 90}{5}=$ **36 pounds**.

5. $\frac{4 \text{ grapefruit}}{80¢}$ and $\frac{12 \text{ grapefruit}}{\$2.00}$ $\Rightarrow \frac{1 \text{ grapefruit}}{20¢}$ and $\frac{1 \text{ grapefruit}}{16.\overline{6}¢}$ so **$2.00 for 12** is a better buy.

6. $\frac{\frac{1}{3} \text{ inch}}{5 \text{ miles}}=\frac{18 \text{ inches}}{x \text{ miles}}\Rightarrow \frac{1}{3}\cdot x=5\cdot 18\Rightarrow x=\frac{5\cdot 18}{\frac{1}{3}}=$ **270 miles**.

7. $\frac{40 \text{ pages}}{50 \text{ minutes}}=\frac{x \text{ pages}}{80 \text{ minutes}}\Rightarrow 50\cdot x=40\cdot 80\Rightarrow$ $x=\frac{40\cdot 80}{50}=$ **64 pages**.

8. **(a)** Let u be a unit value. Then $3u+4u=98\Rightarrow 7u=98\Rightarrow u=14$. $3u=$ **42** and $4u=$ **56**.

(b) Let u be a unit value. $3u\cdot 4u=768\Rightarrow 12u^2=768\Rightarrow u^2=64\Rightarrow u=8$ or $u={}^{-}8$

[since $8^2=64$ and $({}^{-}8)^2=64$]. Then $3u=$ **24** and $4u=$ **32 or** $3u=$ **$^{-}$24** and $4u=$ **$^{-}$32**.

9. ***(i)*** There are a total of $2+3+5=10$ shares. Using a proportion, and letting G represent the amount of money Gary would receive, we write: $\frac{2}{10}=\frac{G}{82{,}000}\Rightarrow 10G=$ $2\cdot 82{,}000\Rightarrow G=$ **$16,400**.

Alternative thinking: Common Core State Standards suggests we consider using rates to solve this type of problem. Gary's rate is 2 to 10. Thus, $\frac{2}{10}\cdot 82{,}000=\frac{2\cdot 82{,}000}{10}=$ **$16,400**.

(ii) $\frac{3}{10}=\frac{\text{Bill's amount}}{82{,}000}\Rightarrow 10(\text{Bill's})=$ $3\cdot 82{,}000\Rightarrow$ Bill's amount = **$24,600**.

(iii) $\frac{5}{10}=\frac{\text{Carmella's amount}}{82{,}000}\Rightarrow$ $10\cdot \text{Carmella's}=5\cdot 82{,}000\Rightarrow$ Carmella's amount = **$41,000**.

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10. The ratio of Sheila's hours to Dora's is $3\frac{1}{2}$ to $4\frac{1}{2}$. Then $3\frac{1}{2}x + 4\frac{1}{2}x = \$176 \Rightarrow 8x = 176 \Rightarrow x = 22$. Thus **Sheila's** earnings are $3\frac{1}{2} \cdot 22 =$ **\$77** and **Dora's** are $4\frac{1}{2} \cdot 22 =$ **\$99**.

11. Success:failure = 5:4 $\Rightarrow \frac{5 \text{ successes}}{4 \text{ failures}} = \frac{75 \text{ successes}}{x \text{ failures}} \Rightarrow 5 \cdot x = 4 \cdot 75 \Rightarrow x = \frac{4 \cdot 75}{5} =$ 60 failures. 75 successes + 60 failures = **135 attempts**.

12. **(a)** $\frac{1}{6}{:}1 \Rightarrow (6)\frac{1}{6}{:}(6)1 \Rightarrow 1{:}6 = \mathbf{\frac{1}{6}}$.

(b) $\frac{1}{3}{:}\frac{1}{3} \Rightarrow (3)\frac{1}{3}{:}(3)\frac{1}{3} \Rightarrow 1{:}1 = \mathbf{\frac{1}{1}}$.

(c) $\frac{1}{6}{:}\frac{2}{7} \Rightarrow (42)\frac{1}{6}{:}(42)\frac{2}{7} \Rightarrow 7{:}12 = \mathbf{\frac{7}{12}}$.

13. The proportion implies $12\not{\text{¢}} \cdot 48\,\text{oz} = 16\not{\text{¢}} \cdot 36\,\text{oz}$. Other equivalent proportions are thus:

(i) $\frac{12\not{\text{¢}}}{16\not{\text{¢}}} = \frac{36 \text{ ounces}}{48 \text{ ounces}}$.

(ii) $\frac{48 \text{ ounces}}{16\not{\text{¢}}} = \frac{36 \text{ ounces}}{12\not{\text{¢}}}$.

(iii) $\frac{16\not{\text{¢}}}{12\not{\text{¢}}} = \frac{48\,\text{oz}}{36\,\text{oz}}$.

14. **(a)** $\frac{\text{Rise}}{\text{half-span}} = \frac{10}{14} = \frac{5}{7}$ or **5:7**.

(b) $\frac{\text{Rise}}{\text{half-span}} = \text{pitch}. \frac{\text{Rise}}{8} = \frac{3}{4} \Rightarrow \text{rise} \cdot 4 = 8 \cdot 3 \Rightarrow \text{rise} = \frac{8 \cdot 3}{4} =$ **6 feet**.

15. **(a)** $\frac{4 \text{ rpm on large gear}}{6 \text{ rpm on small gear}} = \frac{18 \text{ teeth on small gear}}{x \text{ teeth on large gear}} \Rightarrow 4 \cdot x = 6 \cdot 18 \Rightarrow x = \frac{6 \cdot 18}{4} =$ **27 teeth**.

(b) $\frac{200 \text{ rpm on large gear}}{600 \text{ rpm on small gear}} = \frac{x \text{ teeth on small gear}}{60 \text{ teeth on large gear}} \Rightarrow 600 \cdot x = 200 \cdot 60 \Rightarrow x = \frac{200 \cdot 60}{600} =$ **20 teeth**.

16. $\frac{230 \text{ feet length}}{195 \text{ feet wingspan}} = \frac{40 \text{ cm length}}{x \text{ cm wingspan}} \Rightarrow 230 \cdot x = 195 \cdot 40 \Rightarrow x = \frac{195 \cdot 40}{230} = 33\frac{21}{23}$, or **about 34 cm**.

17. **(a)** $\frac{3 \text{ cups tomato sauce}}{2 \text{ cups tomato sauce}} = \frac{3}{2}$, so:

(i) $\frac{1 \text{ tsp mustard seed}}{x \text{ tsp mustard seed}} = \frac{3}{2} \Rightarrow 3 \cdot x = 1 \cdot 2 \Rightarrow x = \mathbf{\frac{2}{3}}$ **tsp mustard seed**.

(ii) $\frac{1\frac{1}{2} \text{ cups scallions}}{x \text{ cups scallions}} = \frac{3}{2} \Rightarrow 3 \cdot x = 1\frac{1}{2} \cdot 2 \Rightarrow x = \frac{1\frac{1}{2} \cdot 2}{3} =$ **1 cup scallions**.

(iii) $\frac{3\frac{1}{4} \text{ cups beans}}{x \text{ cups beans}} = \frac{3}{2} \Rightarrow 3 \cdot x = 3\frac{1}{4} \cdot 2 \Rightarrow x = \frac{3\frac{1}{4} \cdot 2}{3} =$ $\mathbf{2\frac{1}{6}}$ **cups beans**.

(b) $\frac{1\frac{1}{2} \text{ cups sacallions}}{1 \text{ cup sacallions}} = \frac{3}{2}$, which is the same ratio as in (a) above, or $\mathbf{\frac{2}{3}}$ **tsp mustard seed**, **2 cups tomato sauce**, $\mathbf{2\frac{1}{6}}$ **cups beans**.

(c) $\frac{3\frac{1}{4} \text{ cups beans}}{1\frac{3}{4} \text{ cups beans}} = \frac{13}{7}$, so:

(i) $\frac{1 \text{ tsp mustard seed}}{x \text{ tsp mustard seed}} = \frac{13}{7} \Rightarrow 13 \cdot x = 1 \cdot 7 \Rightarrow x = \frac{1 \cdot 7}{13} =$ $\mathbf{\frac{7}{13}}$ **tsp mustard seed**.

(ii) $\frac{3 \text{ cups tomato sauce}}{x \text{ cups tomato sauce}} = \frac{13}{7} \Rightarrow 13 \cdot x = 3 \cdot 7 \Rightarrow x = \frac{3 \cdot 7}{13} =$ $\mathbf{1\frac{8}{13}}$ **cups tomato sauce**.

(iii) $\frac{1\frac{1}{2} \text{ cups scallions}}{x \text{ cups scallions}} = \frac{13}{7} \Rightarrow 13 \cdot x = 1\frac{1}{2} \cdot 7 \Rightarrow x = \frac{1\frac{1}{2} \cdot 7}{13} =$ $\mathbf{\frac{21}{26}}$ **cups scallions**.

18. $\frac{4 \text{ ohms}}{5 \text{ feet}} = \frac{x \text{ ohms}}{20 \text{ feet}} \Rightarrow 5 \cdot x = 4 \cdot 20 \Rightarrow x = \frac{4 \cdot 20}{5} = 16$ **ohms**.

19. $\frac{2 \text{ cm (daughter)}}{6 \text{ cm (father)}} = \frac{x \text{ cm (daughter)}}{183 \text{ cm (father)}} \Rightarrow 6x = 2 \cdot 183 \Rightarrow x = \frac{2 \cdot 183}{6} = 61\,\text{cm}$.

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20. The ratio between the mass of the gold in the ring to the mass of the ring is 18:24. If x is the number of ounces of pure gold in a ring that weighs 0.4 ounces, then $\frac{18 \text{ ounces of gold}}{24 \text{ ounces of ring}} = \frac{x \text{ ounces of gold}}{4 \text{ ounces of ring}} \Rightarrow 24x = 18 \cdot 4 \Rightarrow x = \frac{18 \cdot 4}{24} = 3$ ounces of gold in the ring.

3 ounces at \$1800 per ounce of $= \$5400$.

21. **(a)** $\frac{5 \text{ hours}}{\$40} = \frac{40 \text{ hours}}{\$x} \Rightarrow 5x = 40 \cdot 40 \Rightarrow x = \frac{40 \cdot 40}{5} \Rightarrow x =$ **\$320**. The same result could have been obtained by using any of the other ratios; e.g., \$16 for 2 hours.

(b) The constant of proportionality is **8**, representing \$8 dollars per hour.

22. **(a)** The total number of men in all three rooms is $1 + 2 + 5 = 8$; the total number of women in all three rooms is $2 + 4 + 10 = 16$. The ratio of men to women is $\frac{8}{16} = \frac{\mathbf{1}}{\mathbf{2}}$, or **1:2**.

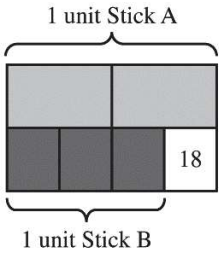
(b) Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = r$.

Then $a = br$;
$c = dr$;
$e = fr$.

So $a + c + e = br + dr + fr \Rightarrow$
$a + c + e = r(b + d + f) \Rightarrow$
$r = \frac{a+c+e}{b+d+f}$

Thus $r = \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$.

23. As seen in the drawing, $\frac{1}{2}$ of stick A is $\frac{2}{3}$ of stick B, and stick B is 18 cm shorter than stick A.



We have the following: Stick B is 3 • 18 unit sections whikle stick A is 4 • 18 unit sections. Thus the lenths of the sticks are A, 72 cm, and B, 54 cm.

24. $\frac{26 miles}{1 gallon}$ for the car and $\frac{250 miles}{14 gallons} \approx \frac{17.85 miles}{1 gallon}$ for the truck. The car gets the better gas mileage because it travels 26 miles on 1 gallon and the truck only travels 17.85 miles on 1 gallon.

25. $\frac{20 miles}{2 hours} = \frac{10 miles}{1 hour}$ for Susan and $\frac{32 miles}{3 hours} \approx \frac{10.\overline{6} miles}{1 hour}$ for Nick. So Nick travels faster.

26. The desired ratio of students to computers is 3:1. If there are 40 students then the ratio should be 40:13.3. So there would need to be 14 computers.

27. The ratio of oblong tables to round tables is 5:1. The ratio of oblong tables to total tables is then 5:6. If there are 102 total tables then the ratio of oblong tables to total tables is $\frac{5}{6} = \frac{85}{102}$. There are 85 oblong tables. 102- 85 = 17, so there are 17 round tables.

28. $1\frac{1}{2}$ cups for 2 dozen cookies

3 cups for 4 dozen cookies

$4\frac{1}{2}$ for 6 dozen cookies

4 cups would not be enough for 6 dozen cookies because it would take $4\frac{1}{2}$ cups for 6 dozen cookies.

Assessment 6-4B

1. **(a)** There are four vowels and seven consonants in "Mississippi." Their ratio is $\frac{4 \text{ vowels}}{7 \text{ consonants}} = \frac{4}{7}$, or **4:7**.

(b) $\frac{7 \text{ consonants}}{4 \text{ vowels}} = \frac{7}{4}$, or **7:4**.

(c) There are seven consonants in the eleven letters, so $\frac{7 \text{ consonants}}{11 \text{ letters}} = \frac{7}{11}$, or **7:11**.

2. $\frac{a}{b} = \frac{c}{d}$ only if $ad = bc$:

(a) $\frac{5}{x} = \frac{30}{42} \Rightarrow 30x = 5 \cdot 42 \Rightarrow x = \frac{5 \cdot 42}{30} = \frac{210}{30} =$ **7**.

(b) $\frac{x}{8} = \frac{-12}{32} \Rightarrow 32x = {}^{-}12 \cdot 8 \Rightarrow x = \frac{{}^{-}12 \cdot 8}{32} = {}^{-}$**3**.

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(c) $\frac{7}{8} = \frac{3x}{48} \Rightarrow 3x \cdot 8 = 7 \cdot 48 \Rightarrow x = \frac{7 \cdot 48}{3 \cdot 8} = \mathbf{14}$.

(d) $3\frac{1}{2}$ is to 8 as x is to 24 $\Rightarrow \frac{3\frac{1}{2}}{8} = \frac{x}{24} \Rightarrow 8x = 3\frac{1}{2} \cdot 24 \Rightarrow x = \frac{3\frac{1}{2} \cdot 24}{8} = \mathbf{10\frac{1}{2}}$.

3. $\frac{5 \text{ adults}}{1 \text{ teen}} = \frac{12{,}345 \text{ adults}}{x \text{ teens}} \Rightarrow 5 \cdot x = 1 \cdot 12{,}345 \Rightarrow x = \frac{1 \cdot 12{,}345}{5} = \mathbf{2469 \text{ teen drivers}}$.

4. The candle has burned 5 inches in 12 minutes. $\frac{5 \text{ inches}}{12 \text{ minutes}} = \frac{30 \text{ inches}}{x \text{ minutes}} \Rightarrow 5 \cdot x = 12 \cdot 30 \Rightarrow x = \frac{12 \cdot 30}{5} = \mathbf{72 \text{ minutes}}$ for a 30-inch candle.

5. Let $5x$ represent width and $9x$ represent length. The perimeter of the rectangle is $2 \cdot 5x + 2 \cdot 9x = 2800 \Rightarrow 28x = 2800 \Rightarrow x = 100$. Thus width $= 5 \cdot 100 =$ **500 feet** and length $= 9 \cdot 100 =$ **900 feet**.

6. If the proportion of jump:length is 20:1 $\Rightarrow \frac{\text{jump}}{6 \text{ feet}} = \frac{20}{1} \Rightarrow 1 \cdot \text{jump} = 6 \cdot 20 \Rightarrow \text{jump} =$ **120 feet**, assuming the length of a grasshopper corresponds to the height of a human.

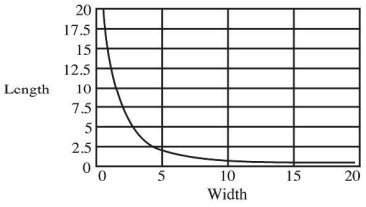
7. $\frac{9 \text{ months}}{6 \text{ vacation days}} = \frac{12 \text{ months}}{x \text{ vacation days}} \Rightarrow 9 \cdot x = 6 \cdot 12 \Rightarrow x = \frac{6 \cdot 12}{9} = \mathbf{8 \text{ days}}$.

8. $\frac{1 \text{ teacher}}{30 \text{ students}} = \frac{x \text{ teachers}}{1200 \text{ students}} \Rightarrow 30x = 1200 \Rightarrow x = 40$ teachers presently.

Then $\frac{1 \text{ teacher}}{20 \text{ students}} = \frac{40 + t \text{ additional teachers}}{1200 \text{ students}} \Rightarrow 20(40 + t) = 1200 \Rightarrow 800 + 20t = 1200 \Rightarrow 20t = 400 \Rightarrow t = \frac{400}{20} = \mathbf{20 \text{ additional teachers}}$.

9. $\frac{30 \text{ feet tall}}{12 \text{ foot shadow}} = \frac{x \text{ feet tall}}{14 \text{ foot shadow}} \Rightarrow 12 \cdot x = 30 \cdot 14 \Rightarrow x = \frac{30 \cdot 14}{12} = \mathbf{35 \text{ feet}}$.

10. **(a)** Plotting lengths against widths from the table:



(b) $\boldsymbol{WL = 10}$; or width $\times$ length $=$ area $= 10 \text{ ft}^2$.

11. Solutions may vary. The ratio of 4 to 20 is the same in each case; x (the number of tickets) and y (the cost) could therefore be $\frac{8 \text{ tickets}}{\$40}, \frac{12 \text{ tickets}}{\$60}, \frac{16 \text{ tickets}}{\$80}, \ldots$.

12. $\frac{\$850}{2 \text{ weeks}} = \frac{\$x}{7 \text{ weeks}} \Rightarrow 2x = 850 \cdot 7 \Rightarrow x = \frac{850 \cdot 7}{2} = 2975$, or the rent for seven weeks would be **\$2975**.

13. **(a)** $\frac{\text{Length of hand}}{\text{Length of big toe}} = \frac{14}{3} \Rightarrow \frac{14 \text{ cm}}{3 \text{ cm}} = \frac{x \text{ cm}}{6 \text{ cm}} \Rightarrow 3x = 14 \cdot 6 \Rightarrow x = \frac{14 \cdot 6}{3} = \mathbf{28 \text{ cm}}$.

(b) $\frac{\text{Length of hand}}{\text{Length of foot}} = \frac{7}{9} \Rightarrow \frac{7 \text{ cm}}{9 \text{ cm}} = \frac{21 \text{ cm}}{x \text{ cm}} \Rightarrow 7x = 21 \cdot 9 \Rightarrow x = \frac{21 \cdot 9}{7} = \mathbf{27 \text{ cm}}$.

14. The ratio of length to width of the park is $\frac{300 \text{ ft}}{200 \text{ ft}} = \frac{3}{2}$. Then $\frac{3 \text{ in}}{2 \text{ in}} = \frac{4 \text{ in}}{x \text{ in}} \Rightarrow 3x = 4 \bullet 2 \Rightarrow x = \frac{4(2)}{3} = \frac{8}{3} = 2\frac{2}{3} \text{ in}$

15. $\frac{240 \text{ mi}}{15 \text{ gallons}} = \frac{x \text{ mi}}{3 \text{ gallons}} \Rightarrow 15x = 240 \cdot 3 \Rightarrow x = \frac{240 \cdot 3}{15} = \mathbf{48 \text{ mi}}$.

16. $\frac{1 \text{ in}}{48 \text{ in}} = \frac{18 \text{ in}}{x \text{ in}} \Rightarrow 1x = 18 \cdot 48 \Rightarrow 864 \text{ in} =$ **72 ft**.

17. **(a)** There are 50 stars and 13 stripes in the flag, or a ratio of $\frac{50 \text{ stars}}{13 \text{ stripes}} =$ **50:13**.

(b) $\frac{13 \text{ stripes}}{50 \text{ stars}} =$ **13:50**.

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18. **(a)** Given $\frac{\text{length}}{\text{width}} = \frac{19}{10} \Rightarrow \frac{19 \text{ ft}}{10 \text{ ft}} = \frac{9\frac{1}{2} \text{ ft}}{x \text{ ft}} \Rightarrow$

$19x = 9\frac{1}{2} \cdot 10 \Rightarrow x = \frac{9\frac{1}{2} \cdot 10}{19} = \mathbf{5\ ft}$.

(b) **No**. $\frac{5 \text{ ft}}{3 \text{ ft}} = \frac{5}{3} \neq \frac{19}{10}$ because $5 \cdot 10 \neq 19 \cdot 3$.

19. $\frac{x}{y} = \frac{a}{b} \Rightarrow xb = ya \Rightarrow \frac{b}{a} = \frac{y}{x}$, $\frac{a}{x} = \frac{b}{y}$, and $\frac{x}{a} = \frac{y}{b}$ $(x, y, a, b \neq 0)$.

20. Let x represent the amount of milk needed in the recipe if only 1 cup of flour is used.

$\frac{1\frac{1}{2} \text{ cups flour}}{4 \text{ cups milk}} = \frac{1 \text{ cup flour}}{x \text{ cups milk}} \Rightarrow 1\frac{1}{2}x = 1 \cdot 4 \Rightarrow$

$\frac{3}{2}x = 4 \Rightarrow x = 4 \cdot \frac{2}{3} = \frac{8}{3} = \mathbf{2\frac{2}{3}\ cups}$.

21. Set up a proportion:

$\frac{21 \text{ tagged fish}}{68 \text{ fish}} = \frac{173 \text{ tagged fish}}{x \text{ fish}} \Rightarrow 21x =$

$68 \cdot 173 \Rightarrow x = \frac{68 \cdot 173}{21} \approx \mathbf{560\ fish}$.

22. $\frac{36 miles}{1 gallon}$ for the car and $\frac{200 miles}{14 gallons} \approx \frac{14.3 miles}{1 gallon}$ for the truck. The car gets the better gas mileage because it travels 36 miles on 1 gallon and the truck only travels 14.3 miles on 1 gallon.

23. $\frac{18 miles}{2 hours} = \frac{9 miles}{1 hour}$ for Susan and $\frac{30 miles}{3 hours} \approx \frac{10 miles}{1 hour}$ for Nick. So Nick travels faster.

24. The desired ratio of students to computers is 3:1. If there are 32 students then the ratio should be 32:10.6. So there would need to be 11 computers.

25. The ratio of oblong tables to round tables is 6:1. The ratio of oblong tables to total tables is then 6:7. If there are 112 total tables then the ratio of oblong tables to total tables is $\frac{6}{7} = \frac{96}{112}$. There are 96 oblong tables. 112- 96 = 16, so there are 16 round tables.

26. $1\frac{1}{2}$ cups for 3 dozen cookies

3 cups for 6 dozen cookies

4 cups would be enough for 6 dozen cookies because it would take 3 cups for 6 dozen cookies.

27. $\frac{160 lb}{416 lb} = \frac{40 lb}{104 lb} = \frac{120 lb}{312 lb}$ so Amy would weigh 312 lb on Jupiter if she weighs 120lb on Earth.

Mathematical Connections 6-4: Review Problems

18. If the numerator is 6 times the denominator then $n = 6d$. If the numerator is also 5 more than the denominator then $n = d + 5$. Solving for d, $6d = d + 5 \Rightarrow 5d = 5 \Rightarrow d = 1$. Solving for n, $n = 6d \Rightarrow n = 6 \bullet 1 \Rightarrow n = 6$. The numerator is 6 and the denominator is 1.

19. $\frac{3}{4}$ is a proper fraction because the numerator and denominator are both positive with the numerator less than the denominator.

20. Any integer can be written as a rational number with the integer as the numerator and the denominator 1.

21. The statement is not true in general; for example, $\frac{25}{35} \neq \frac{2}{3}$ because $75 \neq 70$.

22. At 6:00, the hands of the clock form a straight line approximately every 1hr, 5 min, and 27 sec. So after 6:00, the minute and hour hands form a straight line at 7:05:27, 8:10:55, 9:16:22, 10:21:49, 11:27:16, 12:32:44, 1:38:11, 2:43:38, 3:49:05, and 4:54:33.

23. No, $\frac{2}{3}$ of a class is not equivalent to $\frac{4}{5}$ of the school being absent because $2 \bullet 5 \neq 3 \bullet 4$, $10 \neq 12$.

24. $99 \bullet 96 < 98 \bullet 97$, $9504 < 9506$, so $\frac{99}{98} < \frac{97}{96}$.

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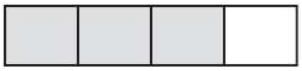
25. **(a)** $\frac{1}{\frac{1}{2}}+\frac{1}{\frac{1}{4}}=\frac{1}{t}\Rightarrow 2+4=\frac{1}{t}\Rightarrow 6=\frac{1}{t}\Rightarrow t=\frac{1}{6}$ of a day.

(b) $\frac{ab}{a+b}$ hours. In 1 hr Amal can finish $\frac{1}{a}$ of the job, while Sharif can finish $\frac{1}{b}$ of the job in 1 hr. If they work together, they can finish $\frac{1}{a}+\frac{1}{b}$ of the job in 1 hr. If it takes them x hr to finish the job working together, then $x(\frac{1}{a}+\frac{1}{b})=1$ or $(\frac{a+b}{ab})x=1$. Hence, $x=\frac{ab}{a+b}$.

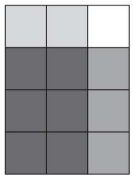
Chapter 6 Review

1. **(a)** Answers may vary. Shade three of four parts:



(b) Two of three blocks shaded:

(c) Three of four horizontal bars are shaded, meshed with two of three vertical bars. Six of the twelve bars are dark-shaded; i.e., $\frac{3}{4}\cdot\frac{2}{3}=\frac{6}{12}$:



2. Answers may vary; three are $\frac{10}{12}$, $\frac{15}{18}$, and $\frac{50}{60}$.

3. **(a)** $\frac{24}{28}=\frac{6\cdot4}{7\cdot4}=\mathbf{\frac{6}{7}}$.

(b) $\frac{ax^2}{bx}=\frac{ax\cdot x}{b\cdot x}=\mathbf{\frac{ax}{b}}$.

(c) $\frac{0}{17}=\frac{0\cdot17}{1\cdot17}=\mathbf{\frac{0}{1}}=0$.

(d) $\frac{45}{81}=\frac{5\cdot9}{9\cdot9}=\mathbf{\frac{5}{9}}$.

(e) $\frac{bx^2+bx}{b+x}=\frac{b\cdot(b+x)}{1\cdot(b+x)}=\mathbf{\frac{b}{1}}=b$.

(f) $\frac{16}{216}=\frac{2\cdot8}{27\cdot8}=\mathbf{\frac{2}{27}}$.

(g) $\frac{x+a}{x-a}$ **cannot be further reduced**. There are no factors common to both numerator and denominator (x and a are terms).

(h) $\frac{xa}{x+a}$ **cannot be further reduced**.

4. **(a)** $=.\ \frac{6\cdot20}{10\cdot20}=\frac{120}{200}$.

(b) $>.\ \frac{^-3}{4}=\frac{^-18}{24}>\frac{^-5}{6}=\frac{^-20}{24}$.

(c) $>.\ \left(\frac{4}{5}\right)^{10}>\left(\frac{4}{5}\right)^{20}$.

(d) $<.\ \left(1+\frac{1}{3}\right)^2=\left(\frac{4}{3}\right)^2<\left(1+\frac{1}{3}\right)^3=\left(\frac{4}{3}\right)^3$.

5. Additive inverse: $n+$ inverse $=0$.
Multiplicative inverse: $n\cdot$ inverse $=1$.

	n	Additive Inverse	Multiplicative Inverse
(a)	3	$^-3$	$\frac{1}{3}$
(b)	$3\frac{1}{7}=\frac{22}{7}$	$^-3\frac{1}{7}$	$\frac{7}{22}$
(c)	$\frac{5}{6}$	$\frac{^-5}{6}$	$\frac{6}{5}$
(d)	$\frac{^-3}{4}$	$\frac{3}{4}$	$\frac{^-4}{3}$

6. $^-2\frac{1}{3}<^-1\frac{7}{8}<0<\left(\frac{71}{140}\right)^{300}<\frac{69}{140}<\frac{1}{2}<\frac{71}{140}<\left(\frac{74}{73}\right)^{300}$.

7. **Yes**. Apply the laws of multiplication and the commutative and associative laws of multiplication to find: $\frac{4}{5}\cdot\frac{7}{8}\cdot\frac{5}{14}=\frac{4\cdot7\cdot5}{5\cdot8\cdot14}=\frac{4\cdot7\cdot5}{8\cdot14\cdot5}=\frac{4}{8}.\frac{7}{14}\cdot\frac{5}{5}$.

8. Methods may vary:

(a) $\frac{1}{3}\cdot(8\cdot9)=\left(\frac{1}{3}\cdot9\right)\cdot8=3\cdot8=\mathbf{24}$.

(b) $36\cdot1\frac{5}{6}=36\cdot\frac{11}{6}=6\cdot11=\mathbf{66}$.

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9. **(*a*)** Assuming no waste, $54\frac{1}{4} \div 3\frac{1}{12} = 17\frac{22}{37}$, so **17 pieces** can be cut.

(b) $\frac{22}{37} \cdot 3\frac{1}{12} = \mathbf{\frac{11}{6} = 1\frac{5}{6}}$ **yards left over**.

10. **(a)** **15**.
$\frac{30\frac{3}{8}}{4\frac{1}{9}} \cdot \frac{8\frac{1}{3}}{3\frac{8}{9}} \approx \frac{30\frac{2}{8}}{4\frac{1}{8}} \cdot \frac{8}{4} = \frac{30}{4} \cdot 2 = 7\frac{1}{2} \cdot 2.$

(b) **15**. $\left(\frac{3}{800} + \frac{4}{5000} + \frac{15}{6}\right) \cdot 6 \approx \frac{15}{6} \cdot 6.$

(c) **4**. $\frac{1}{407} \div \frac{1}{1609} \approx \frac{1}{400} \div \frac{1}{1600} = \frac{1}{400} \cdot \frac{1600}{1}.$

11. Answers may vary; for instance: A carpenter has a board measuring $4\frac{5}{8}$ feet. How many 6-inch pieces can be cut from it (disregarding the width of the cuts)? The solution is then $4\frac{5}{8} \div \frac{1}{2}$; a diagram would show four foot-long pieces plus $\frac{5}{8}$ of another, with $\frac{1}{2}$-foot size pieces shaded.

12. Answers may vary. $\frac{3}{4} = \frac{60}{80}$ and $\frac{4}{5} = \frac{64}{80}$ so $\frac{61}{80}$ and $\frac{62}{80}$ are between $\frac{3}{4}$ and $\frac{4}{5}$.

13. Think of $504792 \div 23$ as $504792 \cdot \frac{1}{23}$ and enter:

5	0	4	7	9	2	×	2	3	$\frac{1}{x}$	=

14. Jim ate $\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$ pizza. $\frac{1}{6} \cdot 2000 = \mathbf{333\frac{1}{3}}$ **calories**.

15. Let t be the number of times the coin was flipped. Then $\frac{1}{2}t = 376 \Rightarrow t = 376 \div \frac{1}{2} =$ **752 times**.

16. $\frac{240 \text{ heads}}{1000 \text{ flips}} = \frac{6 \cdot 40}{25 \cdot 40} = \mathbf{\frac{6}{25}}$ **of the time**.

17. **Not reasonable**. The statement implies that the University won $\frac{3}{4} + \frac{5}{8} = \frac{11}{8}$ of its games, or that it won more games than it played.

18. We could convert to the same units (for example inches to feet); however, this is not necessary when using proportional reasoning provide corresponding measures are in the same units.

Let $x =$ the distance between pupils on the carring of George Washington's head.

$\frac{2\frac{1}{2} \text{ in}}{9 \text{ in}} = \frac{x \text{ ft}}{60 \text{ ft}} \Rightarrow 9x = 2\frac{1}{2} \cdot 60 \Rightarrow x =$
$\frac{2\frac{1}{2} \cdot 60}{9} = \frac{\frac{5}{2} \cdot 60}{9} = \frac{150}{9} = \frac{50}{3} = \mathbf{16\frac{2}{3}}$ **ft**.

19. $\frac{\left(\frac{2}{3}\right)}{\left(\frac{3}{4}\right)} = \frac{\frac{2}{3} \cdot \frac{4}{3}}{\frac{3}{4} \cdot \frac{4}{3}} = \frac{\left(\frac{8}{9}\right)}{1} = \frac{8}{9}$, which is the quotient of two integers.

20. 12 acres $\cdot\, 9\frac{1}{3}$ bags per acre $= 108 + 4 =$ **112 bags**.

21. $\frac{2}{3}$ female $\cdot\, \frac{2}{5}$ blond $= \mathbf{\frac{4}{15}}$ **blond females**; i.e., a fraction of a fraction of the student population.

22. $\frac{-12}{10}$ is greater than $\frac{-11}{9}$ because $-12 \cdot 9 > -11 \cdot 10$. Alternatively, $\frac{-12}{10} - \frac{-11}{9} = \frac{-108}{90} - \frac{-110}{90} = \frac{2}{90}$, which is positive; therefore $-\frac{-12}{10} > \frac{-11}{9}$.

23. **(a)** $7^x = 343 \Rightarrow 7^x = 7^3 \Rightarrow \boldsymbol{x = 3}$.

(b) $2^{-3x} = \frac{1}{512} \Rightarrow 2^{-3x} = 2^{-9}$
$\Rightarrow -3x = -9 \Rightarrow \boldsymbol{x = 3}$.

(c) $2x - \frac{5}{3} = \frac{5}{6} \Rightarrow 2x = \frac{5}{6} + \frac{5}{3} = \frac{15}{6} \Rightarrow$
$x = \frac{15}{6} \div 2 = \frac{15}{6} \cdot \frac{1}{2} \Rightarrow \boldsymbol{x = \frac{5}{4}}$.

(d) $x + 2\frac{1}{2} = 5\frac{2}{3} \Rightarrow x = 5\frac{2}{3} - 2\frac{1}{2} \Rightarrow$
$\boldsymbol{x = 3\frac{1}{6}}$.

(e) $\frac{20+x}{x} = \frac{4}{5} \Rightarrow 5(20 + x) = 4x \Rightarrow 100 +$
$5x = 4x \Rightarrow 100 = -x \Rightarrow \boldsymbol{x = -100}$.

(f) $2x + 4 = 3x - \frac{1}{3} \Rightarrow 4 + \frac{1}{3} =$
$3x - 2x \Rightarrow \boldsymbol{x = 4\frac{1}{3}}$.

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24. **(a)** $\dfrac{(x^3a^{-1})^{-2}}{xa^{-1}} = \dfrac{x^{-6}a^2}{xa^{-1}} = \dfrac{\left(\frac{a^2}{x^6}\right)}{\left(\frac{x}{a}\right)} = \boldsymbol{\dfrac{a^3}{x^7}}$.

(b) $\left(\dfrac{x^2y^{-2}}{x^{-3}y^2}\right)^{-2} = \left(\dfrac{x^{-4}y^4}{x^6y^{-4}}\right) = \dfrac{\left(\frac{y^4}{x^4}\right)}{\left(\frac{x^6}{y^4}\right)} = \boldsymbol{\dfrac{y^8}{x^{10}}}$.

25. **(a)** $\frac{3a}{xy^2} + \frac{b}{x^2y^2} = \frac{3a}{xy^2} \cdot \frac{x}{x} + \frac{b}{x^2y^2} = \boldsymbol{\frac{3ax+b}{x^2y^2}}$.

(b) $\frac{5}{xy^2} - \frac{2}{3x} = \frac{5}{xy^2} \cdot \frac{3}{3} - \frac{2}{3x} \cdot \frac{y^2}{y^2} = \boldsymbol{\frac{15-2y^2}{3xy^2}}$.

(c) $\frac{a}{x^3y^2z} - \frac{b}{xyz} = \frac{a}{x^3y^2z} - \frac{b}{xyz} \cdot \frac{x^2y}{x^2y} = \boldsymbol{\frac{a-bx^2y}{x^3y^2z}}$.

(d) $\frac{7}{2^3 3^2} + \frac{5}{2^2 3^3} = \frac{7}{2^3 3^2} \cdot \frac{3}{3} + \frac{5}{2^2 3^3} \cdot \frac{2}{2} = \frac{21+10}{2^3 3^3} = \boldsymbol{\frac{31}{216}}$.

26. Answers may vary. The problem is to find how many $\frac{1}{2}$-yard pieces can be cut from a $1\frac{3}{4}$-yard ribbon. The method is to divide $1\frac{3}{4}$ by $\frac{1}{2} \Rightarrow \frac{7}{4} \div \frac{1}{2} = \frac{14}{4}$ or 3 pieces $\left(1\frac{1}{2} \text{ yards}\right)$ with $1\frac{3}{4} - 1\frac{1}{2} = \frac{1}{4}$ yard left over. This agrees with his answer obtained by his drawing. His algorithm concludes that he will have $3\frac{1}{2}$ half-yard pieces, and $\frac{1}{2}$ of a half-yard piece would be $\frac{1}{4}$ yard. He mistook a half-piece for a half-yard.

27. **(a)** $\frac{\text{Number of heads}}{\text{Total tosses}} = \frac{17}{30} = \mathbf{17:30}$.

(b) If 17 heads were recorded, there were 13 tails $\Rightarrow \frac{\text{Number of heads}}{\text{Number of tails}} = \frac{17}{13} = \mathbf{17:13}$.

(c) $\frac{\text{Number of tails}}{\text{Number of heads}} = \frac{13}{17} = \mathbf{13:17}$.

28. ***(i)*** 48 fl oz for \$3 $\Rightarrow \frac{300¢}{48 \text{ fl oz}} \approx$ 6.25¢ per fl oz;

(ii) 64 fl oz for \$4 $\Rightarrow \frac{400¢}{64 \text{ fl oz}} \approx$ 6.25¢ per fl oz;

Based on cost per ounce, neither is a better buy because they are the same price.

29. $\frac{18 \text{ parts gold}}{6 \text{ parts other metals}} \neq \frac{12 \text{ parts gold}}{3 \text{ parts other metals}}$ because $18 \cdot 3 \neq 12 \cdot 6$. Her ring may be more or less, but it is **not 18 karat gold**.

30. 4 people = 3 oranges

1 person = $\frac{3}{4}$ orange

11 people = $11 \cdot \frac{3}{4}$ oranges

$= \frac{33}{4} = 8\frac{1}{4}$ orange.

4 people = 16 grapes

And 1 person = $\frac{16}{4}$ = 4 grapes

11 people = $11 \cdot 4$ grapes

= 44 grapes.

31. $\frac{1 \text{ cm}}{2.5 \text{ m}} = \frac{3 \text{ cm}}{x \text{ m}} \Rightarrow x = 3 \cdot 2.5 = \mathbf{7.5\,m}$.

32. If the ratio of O weight to H_2 weight is 8:1 then $8x + x = 16$(ounces). $9x = 16 \Rightarrow x = \frac{16}{9}$ and $1x = \frac{16}{9} = \mathbf{1\frac{7}{9}}$ **ounces**.

33. The ratio **cannot be determined exactly**, but it will always be between 12:100 and 15:100 as a ratio of defective to non-defective chips. 12:100 is $\frac{12}{112} = 10\frac{5}{7}\%$ defective; 15:100 is $\frac{15}{115} = 13\frac{1}{23}\%$ defective. If the observed defective percentage is closer to $13\frac{1}{23}\%$ then more chips came from the first plant. If the percentage is closer to $10\frac{5}{7}\%$, then more chips came from the second.

34. The area of a square is the square of the length of its side. If the ratio of the sides is 1:r, then the ratio of their areas is 1^2:r^2, or $\mathbf{1:r^2}$.

35. **(a)** Games won to games lost was $\frac{18}{7}$ or **18:7**.

(b) 25 games were played. Games won to games played was $\frac{18}{25}$ or **18:25**.

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36. **(a)** $\frac{1}{5}:1 \Rightarrow 5\left(\frac{1}{5}\right):5(1) = \mathbf{1:5}$.

(b) $\frac{2}{5}:\frac{3}{4} \Rightarrow 20\left(\frac{2}{5}\right):20\left(\frac{3}{4}\right) = \mathbf{8:15}$.

37. $\frac{\text{boys}}{\text{girls}} = \frac{3}{5} = \frac{x \text{ boys}}{15 \text{ girls}} \Rightarrow 5 \cdot x = 3 \cdot 15 \Rightarrow$
$x = \frac{3 \cdot 15}{5} = \mathbf{9 \text{ boys}}$.

38. If the ratio of states of the United States using the *Common Core* Standards to those not using it is 9:1, so 1 out of 10 are not using it. With 50 states, there are 45:5 states of the United States using the *Common Core* Standards to those not using it, so 5 states are not using the *Standards*.

39. $1-\frac{1}{20}-\frac{1}{3}-\frac{1}{4}-\frac{1}{4}-\frac{1}{20}$

$=\frac{60}{60}-\frac{3}{60}-\frac{20}{60}-\frac{15}{60}-\frac{15}{60}-\frac{3}{60}=\frac{4}{60}=\frac{1}{15}$
identified themselves with none of the labels.

40. The ratio of professors of other political persuasions to liberals is 1:7 to 1:9.

41. The ratio of perimeters is 6 + 6 + 6 = 18 to 10 + 10 + 10 = 30. The ration 18:30 is equal to 3:5.

42. The cup that has 1 oz cream because $1\text{oz} > \frac{9}{10}\text{oz}$.

43. $\frac{45}{99} \bullet 121 = \frac{5}{11} \bullet 121 = 5 \bullet 11 = 55$ bulbs are expected to bloom and 121 – 55 = 66 are not expected to bloom.

44. No, the will is impossible to follow because the fractions of cats to be shared do not add to 1. If the woman had x cats then the number of cats her will directs to be distributed is $\frac{1}{2}x+\frac{1}{3}x+\frac{1}{9}x=\frac{17}{18}x$, but the sum should be $1x$, or $(\frac{18}{18})x$.

45. If each guard demanded half the bags plus one more bag the problem can be worked backwards. Add a bag and double the amount. Starting with the 1 bag left, add one bag and double to get 4 bags (fourth bridge). Plus 1 and double is 10 bags (third bridge). Add one bag and double is 22 bags (second bridge). Add one bag and double is 46 bags (first bridge). Prince Juan stared with 46 bags of gold.

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CHAPTER 7

RATIONAL NUMBERS AS DECIMALS AND PERCENT

Assessment 7-1A: Introduction to Finite Decimals

1. (a) $0.023 = \mathbf{0 \cdot 10^0 + 0 \cdot 10^{-1} + 2 \cdot 10^{-2} + 3 \cdot 10^{-3}}$.

(b) $206.06 = \mathbf{2 \cdot 10^2 + 0 \cdot 10^1 + 6 \cdot 10^0 + 0 \cdot 10^{-1} + 6 \cdot 10^{-2}}$.

(c) $312.0103 = \mathbf{3 \cdot 10^2 + 1 \cdot 10^1 + 2 \cdot 10^0 + 0 \cdot 10^{-1} + 1 \cdot 10^{-2} + 0 \cdot 10^{-3} + 3 \cdot 10^{-4}}$.

(d) $0.000132 = \mathbf{0 \cdot 10^0 + 0 \cdot 10^{-1} + 0 \cdot 10^{-2} + 0 \cdot 10^{-3} + 1 \cdot 10^{-4} + 3 \cdot 10^{-5} + 2 \cdot 10^{-6}}$.

2. (a) $4000 + 300 + 50 + 6 + 0.7 + 0.08 =$ **4356.78**.

(b) $4000 + 0.6 + 0.008 =$ **4000.608**.

(c) $40{,}000 + 0.03 =$ **40,000.03**.

(d) $0.2 + 0.0004 + 0.0000007 =$ **0.2004007**.

3. (a) 536.0076

(b) 3.008

(c) 0.000436

(d) 5,000,000.2

4. (a) $0.34 =$ **thirty-four hundredths**.

(b) $20.34 =$ **twenty and thirty-four hundredths**.

(c) $2.034 =$ **two and thirty-four thousandths**.

(d) $0.000034 =$ **thirty-four mllionths; i.e.**, $34 \cdot 10^{-6}$.

5. (a) $0.436 = \frac{436}{1000} = \frac{109 \cdot 4}{250 \cdot 4} = \mathbf{\frac{109}{250}}$.

(b) $25.16 = 25\frac{16}{100} = 25 + \frac{16}{100} = \frac{2500}{100} + \frac{16}{100} = \frac{2516}{100} = \mathbf{\frac{629}{25}}$.

(c) $^{-}316.027 = {}^{-}316\frac{27}{1000} = \mathbf{\frac{^{-}316{,}027}{1000}}$.

(d) $28.1902 = 28\frac{1902}{10{,}000} = \frac{281{,}902}{10{,}000} = \mathbf{\frac{140{,}951}{5000}}$.

(e) $^{-}4.3 = {}^{-}4\frac{3}{10} = \mathbf{\frac{^{-}43}{10}}$.

(f) $^{-}62.01 = {}^{-}62\frac{1}{100} = \mathbf{\frac{^{-}6201}{100}}$.

6. A rational number in simplest form can be written as a terminating decimal if and only if the prime factorization of the denominator contains no primes other than 2 or 5:

(a) Terminating. The denominator contains only 5 as a prime factor.

(b) Terminating. The denominator contains no prime factors other than 2 and 5.

(c) Terminating. The denominator in reduced form contains only 2 as a prime factor.

(d) Terminating. The denominator contains only 2 as a prime factor.

(e) Not terminating. The denominator contains 2 and 17 as prime factors.

(f) Terminating. The denominator contains only 5 as a prime factor.

7. (a) $\frac{4}{5} = \frac{4}{5} \cdot \frac{2}{2} = \frac{8}{10} =$ **0.8**.

(b) $\frac{61}{2^2 \cdot 5} = \frac{61}{2^2 \cdot 5} \cdot \frac{5}{5} = \frac{305}{2^2 \cdot 5^2} = \frac{305}{100} =$ **3.05**.

(c) $\frac{3}{6} = \frac{1}{2} = \frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10} =$ **0.5**.

(d) $\frac{1}{2^5} = \frac{1}{2^5} \cdot \frac{5^5}{5^5} = \frac{3125}{10^5} = \frac{3125}{100{,}000} =$ **0.03125**.

(e) Not a terminating decimal.

(f) $\frac{133}{625} = \frac{133}{5^4} \cdot \frac{2^4}{2^4} = \frac{2128}{10^4} = \frac{2128}{10{,}000} =$ **0.2128**.

8. One hour is 60 minutes, so 7 minutes would be $\frac{7}{60}$ of an hour. This is **nonterminating** decimal because $\frac{7}{60}$ is in simplest form and 3 is a factor of the denominator.

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9. Answers may vary. Many values between 0 and 100 are composed of whole-number powers of 2 and 5, thus dividing 100 and being capable of being expressed as a two-digit decimal. These numbers include the coin designations of 1, 5, 10, 25, and 50 cents, but there are others which could have been used, such as $2¢$ or $20¢$.

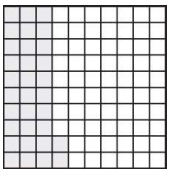
10. **(a)** Line up by place value:
13.49190
13.49200
13.49183
13.49199
From greatest to least is:
13.492 > 13.49199 > 13.4919 > 13.49183.

(b) Line up by place value:
$^{-}$1.45300
$^{-}$1.45000
$^{-}$1.40530
$^{-}$1.49300
From greatest (i.e., closest to 0) to least is:
$^{-}$1.4053 > $^{-}$1.45 > $^{-}$1.453 > $^{-}$1.493.

11. **(a)** Fourteen thousandths inch $= \frac{14}{1000}$ inch $=$ **0.014 inch**.

(b) Twenty-four hundredths $= \frac{24}{100} = 0.24$ days. The rotational period is thus **365.24 days**.

12. There are 32 of 100 squares shaded, representing $\frac{32}{100}$ of the whole grid $= 0.32$ of the grid.



13. The largest number is furthest to the right on the number line. Thus $0.804 < 0.8399 <$ **0.84**.

14. Answers may vary. A decimal carried to the ten-thousandths place would have four digits to the right of the decimal point, and a number between 8.3400 and 8.3410 might be 8.3401 or 8.3405.

15. **(a)** Answers may vary. One method could be to find the difference, not matter how slight, between the two decimal numbers and add some fraction of that to the smaller.

(b) Part (a) is a recursive process; no matter how small the difference between the terminating decimals, that differcence can be divided.

16. A meaning could be: $\mathbf{3 \cdot 6^0 + 1 \cdot 6^{-1} + 4 \cdot 6^{-2} + 5 \cdot 6^{-3}}$ in base ten. In base 6, 10_{six} is used instead of 6, so the numeral would be $3 \cdot (10_{six})^0 + 1 \cdot (10_{six})^{-1} + 4 \cdot (10_{six})^{-2} + 5 \cdot (10_{six})^{-3}$.

17. **Rhonda**, **Martha**, **Kathy**, **Molly**, **Emily**, because 63.54 (Rhonda) $<$ 63.59 (Martha) $<$ 64.02 (Kathy) $<$ 64.46 (Molly) $<$ 64.54 (Emily).

18. Using our knowledge of mixed number from the previous chapters, we can write: $\frac{1\frac{1}{2}}{16} = \frac{1.5}{16} = \frac{1.5 \cdot 2}{16 \cdot 2} = \frac{3}{32} \cdot \frac{3}{32} = \frac{3 \cdot 5^5}{2^5 \cdot 5^5} =$ **0.09375**.

Assessment 7-1B

1. **(a)** $0.045 = \mathbf{0 \cdot 10^0 + 0 \cdot 10^{-1} + 4 \cdot 10^{-2} + 5 \cdot 10^{-3}}$.

(b) $103.03 = \mathbf{1 \cdot 10^2 + 0 \cdot 10^1 + 3 \cdot 10^0 + 0 \cdot 10^{-1} + 3 \cdot 10^{-2}}$.

(c) $245.6701 = \mathbf{2 \cdot 10^2 + 4 \cdot 10^1 + 5 \cdot 10^0 + 6 \cdot 10^{-1} + 7 \cdot 10^{-2} + 0 \cdot 10^{-3} + 1 \cdot 10^{-4}}$.

(d) $0.00034 = \mathbf{0 \cdot 10^0 + 0 \cdot 10^{-1} + 0 \cdot 10^{-2} + 0 \cdot 10^{-3} + 3 \cdot 10^{-4} + 4 \cdot 10^{-5}}$.

2. **(a)** $5000 + 200 + 0.4 =$ **5200.4**.

(b) $0.004 + 20{,}000 =$ **20,000.004**.

(c) $200 + 30{,}000 =$ **30,200**.

(d) $10^{-3} + 10^{-5} = 0.001 + 0.00001 = 0.00101$.

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3. **(a)** **2.027**

(b) **2000.027**

(c) **2020.007**

(d) **0.00004**

4. **(a)** $0.45 =$ **forty-five hundredths**.

(b) $2.035 =$ **two and thirty-five thousandths**.

(c) $45.0006 =$ **forty-five and six ten-thousandths**. Six ten-thousandths $= 6 \cdot 10^{-4}$.

(d) $0.0000445 =$ **four hundred forty-five ten-millionths**. Four hundred forty five ten-millionths $= 4 \cdot 10^{-5} + 4 \cdot 10^{-6} + 5 \cdot 10^{-7}$.

5. **(a)** $28.32 = 28 + \frac{32}{100} = \frac{2800}{100} + \frac{32}{100} = \frac{2832}{100} = \frac{708 \cdot 4}{25 \cdot 4} = \mathbf{\frac{708}{25}}$.

(b) $34.1736 = 34 + \frac{1736}{10{,}000} = \frac{340{,}000}{10{,}000} + \frac{1736}{10{,}000} = \frac{341{,}736}{10{,}000} = \frac{42{,}717 \cdot 8}{1250 \cdot 8} = \mathbf{\frac{42{,}717}{1250}}$.

(c) $^{-}27.32 = {}^{-}\left(27 + \frac{32}{100}\right) = {}^{-}\left(\frac{2700}{100} + \frac{32}{100}\right) = {}^{-}\left(\frac{2732}{100}\right) = {}^{-}\left(\mathbf{\frac{683}{25}}\right)$.

6. A rational number in simplest form can be written as a terminating decimal if and only if the prime factorization of the denominator contains no primes other than 2 or 5:

(a) **Terminating**. The denominator in reduced form contains only 2 as a prime factor.

(b) **Terminating**. The denominator contains only 2 as a prime factor.

(c) **Terminating**. The denominator contains only 5 as a prime factor.

(d) **Nonterminating**. The denominator contains 17 as a prime factor.

(e) **Terminating**. The denominator contains only 5 as a prime factor.

(f) **Terminating**. $\frac{14}{35}$ reduces to $\frac{2}{5}$. Since the reduced fraction has only a five in its denominator, the fraction terminates.

7. **(a)** $\frac{4}{8} = \frac{1}{2} = \frac{1}{2} \cdot \frac{5}{5} = \frac{5}{10} = \mathbf{0.5}$.

(b) $\frac{1}{2^6} = \frac{1}{2^6} \cdot \frac{5^6}{5^6} = \frac{15{,}625}{10^6} = \frac{15{,}625}{1{,}000{,}000} =$ **0.015625**.

(c) $\frac{137}{625} = \frac{137}{5^4} = \frac{137}{5^4} \cdot \frac{2^4}{2^4} = \frac{2192}{10^4} = \frac{2192}{10{,}000} = \mathbf{0.2192}$.

(d) Nonterminating.

(e) $\frac{3}{25} = \frac{3}{5^2} = \frac{3}{5^2} \cdot \frac{2^2}{2^2} = \frac{12}{10^2} = \frac{12}{100} = \mathbf{0.12}$.

(f) $\frac{14}{35} = \frac{2}{5} = \frac{4}{10} = \mathbf{0.4}$.

8. All parts of an hour having a reduced fraction without 3 as a factor in the denominator; i.e., having a numerator divisible by 3: **0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57**.

9. **(a)** If a block is represented by $\frac{1}{10}$, there are 1000 cubes in a base-ten block, so each cube would have a value of $\frac{1}{10} \div 1000 = \frac{1}{10{,}000}$, or **one ten-thousandth**.

(b) If each cube has a value of $\frac{1}{10{,}000}$, and there are 100 cubes in a flat, each flat would have a value of $100 \cdot \frac{1}{10{,}000} = \frac{1}{100}$, or **one hundredth**.

(c) One long is composed of 10 cubes, thus a long would have a value of $10 \cdot \frac{1}{10{,}000} = \frac{1}{1000}$, or **one thousandth**.

(d) Three blocks, 1 long, and 4 cubes $= 3 \cdot \frac{1}{10} + 1 \cdot \frac{1}{1000} + 4 \cdot \frac{1}{10{,}000} = 0.3 + 0.001 + 0.0004 =$ **0.3014**, or **three thousand fourteen ten-thousandths**.

10. In each part, insert following zeros to make each value have the same number of digits.

(a) Line up by place value:
24.94190
24.94200
24.94189
24.94199

From least to greatest is:
24.94189 < 24.9419 < 24.94199 < 24.942.

(b) Line up by place value:
$^{-}34.2500$
$^{-}34.2510$
$^{-}34.2050$
$^{-}34.2519$

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From least to greatest is:

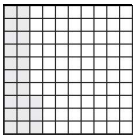
$\mathbf{^{-}34.2519 < {}^{-}34.251 < {}^{-}34.25 < {}^{-}34.205}$.

(The least numbers are those furthest from 0.)

11. **(a)** $\frac{1}{16} = \frac{1}{2^4} = \frac{5^4}{2^4 \cdot 5^4} = \frac{625}{10{,}000} = \mathbf{0.0625}$.

(b) $224 + \frac{7006}{10{,}000} = 224.7006$.

12. There are 23 of 100 squares shaded, representing $\frac{23}{100}$ of the whole grid $= 0.23$ of the grid.



13. $0.8114 = \frac{8114}{10{,}000}$, $0.8119 = \frac{8119}{10{,}000}$, and $0.82 = \frac{8200}{10{,}000}$. Since $8114 < 8119 < 8200$, $\frac{8200}{10{,}000} =$ **0.82** is furthest to the right. Alternetively, .82 has two hundredths while the other two fractions have one hundredth.

14. Answers may vary. Write the decimals as 8.3450 and 8.3456; a decimal between them could be **8.34553**.

15. Answers may vary, but for example just as there are infinitely many rational numbers of the form $\frac{a}{b}$ where $a, b \in I$ and $b \neq 0$, there are infinitely many terminating decimals. E.g., just as between 0.0625 and 0.125. (representing $\frac{1}{16}$ and $\frac{1}{8}$, respectively), $\frac{1}{16} = \frac{2}{32}$ and $\frac{1}{8} = \frac{2}{16} = \frac{4}{32}$, so $\frac{3}{32} = 0.09375$ is between 0.0625 and 0.125. This process can continue indefinitely.

16. $0.00334_{seven} = \mathbf{0 \cdot 7^{1} + 0 \cdot 7^{-1} + 0 \cdot 7^{-2} + 3 \cdot 7^{-3} + 3 \cdot 7^{-4} + 4 \cdot 7^{-5}}$. This is the number in base ten. To remain in base-seven, replace 7 with 10_{seven}.

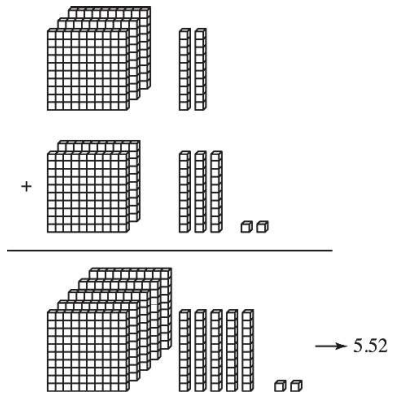
17. Sort first by the tens values, then by the ones values, then by the tenths, and finally by the hundredths. **Ricky**, **Michael**, **Karl**, **Marius**, **Eddie**.

18. Change each sixteenth to a decimal by dividing the numerators, 1 through 16, by the denominator, 16. (Each will be a terminating decimal.)

Rule Mark	Decimal Equivalent
0	0
$\frac{1}{16}$	0.0625
$\frac{2}{16}\left(=\frac{1}{8}\right)$	0.125
$\frac{3}{16}$	0.1875
$\frac{4}{16}\left(=\frac{1}{4}\right)$	0.25
$\frac{5}{16}$	0.3125
$\frac{6}{16}\left(=\frac{3}{8}\right)$	0.375
$\frac{7}{16}$	0.4375
$\frac{8}{16}\left(=\frac{1}{2}\right)$	0.5
$\frac{9}{16}$	0.5625
$\frac{10}{16}\left(=\frac{5}{8}\right)$	0.625
$\frac{11}{16}$	0.6875
$\frac{12}{16}\left(=\frac{3}{4}\right)$	0.75
$\frac{13}{16}$	0.8125
$\frac{14}{16}\left(=\frac{7}{8}\right)$	0.875
$\frac{15}{16}$	0.9375
$\frac{16}{16}$	1.0

Assessment 7-2A: Operations on Decimals

1.



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2. **(a)** $\frac{86}{10}+\frac{231}{10}+\frac{92}{100}=\frac{860}{100}+\frac{2310}{100}+\frac{92}{100}=\frac{3262}{100}$

$= 32.62.$

(b) $\frac{232}{100}+\frac{21008}{1000}=\frac{2320}{1000}+\frac{21008}{1000}$.

$=\frac{23328}{1000}=23.328$

3. **(a)** $0.5\bullet 0.6=\frac{5}{10}\bullet\frac{6}{10}=\frac{30}{100}=0.30.$

(b) $203\bullet 0.03=203\bullet\frac{3}{100}=\frac{609}{100}=6.09.$

(c) $0.003\bullet 0.006=\frac{3}{1000}\bullet\frac{6}{1000}=\frac{18}{1,000,000}$

$= 0.000018.$

4. Answers vary.

(a) 934.23 has 2 decimal places, and 3.2 has 1 decimal place so we move 2 + 1 = 3 decimal places: 2989.536.

(b) The product is 900 • 3 = 2700, so the decimal point is placed as 2989.436.

5. **(a)** $6\bullet 854.14(\approx 6\bullet 850\approx 5100)=5124.84$

(b) $81.6\bullet 212.34(\approx 80\bullet 200\approx 16,000)$

$=17,326.944$

(c) $\frac{137.025}{1.75}(\approx\frac{140}{2}\approx 70)=78.3$.

6. Dividing each number by 0.01 is equal to multiplying each number by $\frac{1}{100}$ which is equal to dividing each number by 100.

(a) $\frac{2586}{100}=25.86$

(b) $\frac{34.79}{100}=0.3479$

(c) $\frac{0.24}{100}=0.0024$

(d) $\frac{0.0037}{100}=0.000037$.

7. **(a)** When you multiply a natural number by a decimal the result is less than the natural number if the decimal is less than 1. For example $5\bullet 0.5=2.5$ and $2.5<5$.

(b) When you multiply a natural number by a decimal the result is greater than the natural number when the decimal is greater than 1. For example $5\bullet 1.5=7.5$ and $7.5>5$.

8. She bought a total of:

$$\begin{array}{r}\$17.95\\13.59\\14.86\\179.98\\2.43\\2.43\\\hline\textbf{\$231.24}\end{array}$$

in her shopping (excluding sales tax)

9. **(a)** Sum along the diagonal, yielding 16.5. In rows and columns with two figures subtract their sum from 16.5 to obtain the missing element:

8.2	**1.9**	**6.4**
3.7	5.5	**7.3**
4.6	9.1	2.8

(b) **Yes**.

(c) Each row, column, and diagonal will have $3\cdot 0.85=2.55$ added to it, for a sum of $16.5+2.55=$ **19.05**.

10. Let $x=$ the price per pound of the other type of fruit. The total cost of the fruit is $25\cdot 4+15\cdot 2+10x=130+10x$ dollars. The total weight of the fruit is $25+15+10=50$ lbs.

To find the average cost per pound divide the total cost by the total weight. This will equal \$3.50 per lb.

$\frac{130+10x}{50}=3.50\Rightarrow\frac{130+10x}{50}=\frac{350}{100}$

$\Rightarrow 130+10x=\frac{350}{100}\cdot 50\Rightarrow 130+10x=175$

$\Rightarrow 10x=45\Rightarrow x=$ **4.5 lbs**.

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11. The ratio $\frac{2.54\,\text{cm}}{1\,\text{in}}$ is equal to 1. Thus,

$$\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right)^3 = 1 \cdot \left(\frac{2.54\,\text{cm}}{\text{in}}\right)^3$$

$$= \frac{254}{100} \cdot \frac{254}{100} \cdot \frac{254}{100} \frac{\text{cm}^3}{\text{in}^3}$$

$$= \frac{16{,}387{,}064}{1{,}000{,}000} \frac{\text{cm}^3}{\text{in}^3}$$

$$= 16.387064 \frac{\text{cm}^3}{\text{in}^3}$$

$$390\,\text{in}^3 \cdot 16.387064 \frac{\text{cm}^3}{\text{in}^3} \approx \mathbf{6{,}391\,cm^3}.$$

12. Let p be the price of the stock. Then $p + 0.24 = 73.245 \Rightarrow p = 73.245 - 0.24$, or

$$\begin{array}{r} 73.245 \\ -0.240 \\ \hline \mathbf{73.005} \end{array}$$

13. If one U.S. dollar is valued at 1.046 Canadian, then \$27.32 U.S. would be valued at $27.32 \times 1.046 = \$28.57672$. which rounds to **\$28.58** Canadian.

14. **(a)** $\frac{3\text{ heaters}}{1} \cdot \frac{1200\text{ w}}{1\text{ heater}} \cdot \frac{24\text{ hrs}}{1\text{ day}} \cdot \frac{1\text{kw}}{1000\text{ w}} \cdot \frac{\$0.06715}{\text{kw-hr}} = \$5.80176$ which rounds to **\$5.80**. (The actual bill would be lumped with other appliance usage.)

(b) 75 watts $= 0.075$ kilowatts. 0.075 kw $\times$ 1 hour $\times$ \$0.06715 per kw-hr $= \$0.00503625$ to operate one bulb for one hour. $\$1.00 \div \$0.00503625 =$ **199 hours** (rounded to the nearest hour).

15. If one liter is 4.224 cups, then the number in 36.5 cups is $36.5 \div 4.224 \approx$ **8.64 liters**.

16. **(a)** There is a difference of 0.9 between each element of the sequence, thus it is arithmetic. The next three elements are: $4.5 + 0.9 =$ **5.4**, $5.4 + 0.9 =$ **6.3**, $6.3 + 0.9 =$ **7.2**.

(b) There is a difference of 0.2 between each element of the sequence, thus it is arithmetic. The next three elements are: $1.1 + 0.2 =$ **1.3**, $1.3 + 0.2 =$ **1.5**, $1.5 + 0.2 =$ **1.7**.

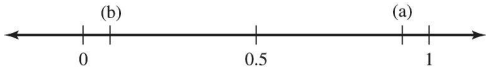
17. If: $a_1 = 0.9$;
$a_2 = 0.9 \cdot 0.2 = 0.18$;
$a_3 = 0.18 \cdot 0.2 = 0.036$;
$a_4 = 0.036 \cdot 0.2 = 0.0072$; and
$a_5 = 0.0072 \cdot 0.2 = 0.00144$:
Their sum is **1.12464**.

18. A finite geometric sequence is one with a constant ratio between terms and a finite number of terms. 0.2222 can be expressed as a sum: $\frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10{,}000}$; note that the denominators are powers of ten. Thus to make it a geometric sequence with constant ratio $\frac{1}{10}$, it could be written as $\frac{2}{10} + \left(\frac{2}{10}\right)\left(\frac{1}{10}\right) + \left(\frac{2}{10}\right)\left(\frac{1}{10}\right)^2 + \left(\frac{2}{10}\right)\left(\frac{1}{10}\right)^3$, or $\mathbf{0.2 + (0.2)(0.1) + (0.2)(0.1)^2 + (0.2)(0.1)^3}$.

19. **(a)** $0.3 \div 0.31$ is close to 1.

(b) $0.3 \cdot 0.31$ is close to 0.09.

Thus on the numbers line:



20. The bank is **not correct**. The total of outstanding checks is:

$$\begin{array}{r} \$3.21 \\ 14.56 \\ 12.44 \\ 6.98 \\ 9.51 \\ 7.49 \\ \hline \$54.19 \end{array}$$

Adding the total of outstanding checks to the checkbook balance gives $\$54.19 + 21.69 = \$75.88$, which differs from the bank statement by \$7.74. The bank balance is too high.

21. $1776\,ft \cdot 0.005 = 8.88\,ft$ The height of the model will be 8.88 ft.

22. $102.3\,cm \cdot 1.5 = 153.45\,cm$ If Allie is 102.3 cm now she will be 153.45 cm when she is twelve years old.

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23. **(a)** $\$150 \bullet 99.925 = 14988.75 yen$. If \$1 is worth 99.925 yen then \$150 is worth 14,988.75 yen.

(b) If \$1 is worth 99.925 yen then 1 yen is worth \$0.010007506 and 3500 yen is worth approximately \$35.03.

24. If there are 2.54 cm in 1 inch then there are 0.393700787 inches in 1 cm.

(a) 35 cm would roughly equal 14 inches.

(b) 35 cm • 0.393700787 = 13.78 inches.

(c) 154.5 cm • 0.393700787 = 60.83 inches.

25. If 1 km is approximately 0.62 miles then 1 mile is approximately 1.612903226 km.

(a) 30 km • 0.62 = 18.6 miles.

(b) 90 km/hr • 0.62 = 55.8 mi/hr.

(c) 75 mi/hr • 1.612903226 = 120.97 km/hr.

26. **(a)** $3.2 \div 10^9 = \mathbf{0.0000000032}$; i.e., move the decimal point 9 places to the left.

(b) $3.2 \cdot 10^9 = \mathbf{3,200,000,000}$; i.e., move the decimal point 9 places to the right.

(c) $4.2 \div 10^1 = \mathbf{0.42}$; i.e., move the decimal point 1 place to the left.

(d) $6.2 \cdot 10^5 = \mathbf{620,000}$; i.e., move the decimal point 5 places to the right.

27. **(a)** Move the decimal point 7 places to the left to obtain the product of a number between 1 and 10 and an integer power of 10. I.e., 1.27 multiplied by 10^7, or about $\mathbf{1.27 \cdot 10^7 m}$.

(b) Move the decimal point 9 places to the left and multiply the result by 10^9 to obtain about $\mathbf{4.486 \cdot 10^9 km}$.

(c) Move the decimal point 7 places to the left and multiply the result by 10^7 to obtain about $\mathbf{5 \cdot 10^7 cans}$.

28. **(a)** $8.56 = 3 - 2x \Rightarrow 8.56 - 3 = 2x \Rightarrow$

$$5.56 = {}^{-}2x \Rightarrow x = \frac{5.56}{{}^{-}2} = \frac{{}^{-}556}{100} \cdot \frac{1}{2} =$$

$$\frac{{}^{-}278}{100} = {}^{-}\mathbf{2.78}.$$

(b) $2.3x - 2 = x + 2.55 \Rightarrow 2.3x - x$

$$= 2 + 2.55 \Rightarrow 1.3x = 4.55 \Rightarrow x$$

$$= \frac{4.55}{1.3} = \frac{455}{130} = 3.5.$$

29. **(a)** Move the decimal point ten places to the left to obtain **0.000000000753 g**.

(b) Move the decimal point five places to the right to obtain **298,000 km per sec**.

(c) Move the decimal point eight places to the right to obtain **778,570,000 km**.

30. **(a)** $(8 \cdot 10^{12}) \cdot (6 \cdot 10^{15}) = 8 \cdot 6 \cdot 10^{12} \cdot 10^{15} =$

$48 \cdot 10^{27} = \mathbf{4.8 \cdot 10^{28}}$.

(b) $(16 \cdot 10^{12}) \div (4 \cdot 10^5) = \frac{16}{4} \cdot \frac{10^{12}}{10^5} =$

$4 \cdot 10^{12} \cdot 10^{-5} = \mathbf{4 \cdot 10^7}$.

(c) $(5 \cdot 10^8) \cdot (6 \cdot 10^9) \div (15 \cdot 10^{15}) =$

$\frac{5 \cdot 6}{15} \cdot 10^8 \cdot 10^9 \cdot 10^{-15} = \mathbf{2 \cdot 10^2}$.

31. **(a)** 203.651 is between 200 and 300 and is closer to 200, so round down to **200**.

(b) 203.651 is between 200 and 210 and is closer to 200, so round down to **200**.

(c) 203.651 is between 203 and 204 and is closer to 204, so round up to **204**.

(d) 203.651 is between 203.6 and 203.7 and is closer to 203.7, so round up to **203.7**.

(e) 203.651 is between 203.65 and 203.66 and is closer to 203.65, so round down to **203.65**.

32. $\frac{243 \text{ miles}}{12 \text{ gallons}} = 20.25$ **mpg**.

33. Camera rounds to \$55, film rounds to \$5, and case rounds to \$18. Total estimated cost is $55 + 5 + 18 = \mathbf{\$78}$.

34. Answers may vary:

(a) ***(i)*** 65.84 rounds to 66;
24.29 rounds to 24;
12.18 rounds to 12;
19.75 rounds to 20;
Rounded estimate $= 66 + 24 + 12 + 20 = \mathbf{122}$.

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(ii) Lead digits sum to $60 + 20 + 10 + 10 = 100$; adjustments are about $6 + 4 + 2 + 10 = 22$. Front-end estimate $= 100 + 22 =$ **122**.

(iii) Actual sum = **122.06**.

(b) ***(i)*** 89.47 rounds to 89;
32.16 rounds to 32.
Rounded estimate $= 89 - 32 =$ **57**.

(ii) Lead digit difference is $80 - 30 = 50$; adjustments are about $9 - 2 = 7$. Front-end estimate $= 50 + 7 =$ **57**.

(iii) Actual difference = **57.31**.

(c) ***(i)*** 5.85 rounds to 6;
6.13 rounds to 6;
9.10 rounds to 9;
4.32 rounds to 4.
Rounded estimate $= 6 + 6 + 9 + 4 =$ **25**.

(ii) Lead digits sum to $5 + 6 + 9 + 4 = 24$; adjustments are about $1 + 0 + 0 + 0 = 1$.
Front-end estimate $= 24 + 1 =$ **25**.

(iii) Actual sum = **25.40**.

(d) ***(i)*** 223.75 rounds to 224;
87.60 rounds to 88.
Rounded estimate $= 224 - 88 =$ **136**.

(ii) Lead digit difference $= 200 - 80 = 120$; adjustments are about $24 - 8 = 16$. Front-end estimate $= 120 + 16 =$ **136**.

(iii) Actual difference = **136.15**.

35. To find the greatest products use the largest digits in the highest place values:

(i) Least: $\boxed{2}.\boxed{3} \times \boxed{1} =$ **2.3**.

(ii) Greatest: $\boxed{8}.\boxed{7} \times \boxed{9} =$ **78.3**.

36. To find the greatest possible number use the largest digits in the highest place values:

$$4\boxed{9}\boxed{7}3\boxed{6} \cdot \boxed{5}\boxed{2}8\boxed{1}.$$

37. Answers may vary. E.g., $40 \cdot \$8 + 40 \cdot \$\left(\frac{1}{4}\right) = \$320 + \$10 =$ **$330**.

38. **(a)** $8.4 \cdot 6 = 4.2 \cdot \underline{\mathbf{12}} \left[\frac{8.4}{\cancel{2}} \cdot (6 \cdot \cancel{2}) = 8.4 \cdot 6\right]$.

(b) $10.2 \div 0.3 = 20.4 \div \underline{\mathbf{0.6}}\left(\frac{10.2 \cdot \cancel{2}}{0.3 \cdot \cancel{2}} = \frac{10.2}{0.3}\right)$.

(c) $a \cdot b = \frac{a}{2} \cdot \underline{\mathbf{2b}}\left(\frac{a}{\cancel{2}} \cdot \frac{\cancel{2}b}{1} = a \cdot b\right)$.

(d) $a \div b = 2a \div \underline{\mathbf{(2b)}}\left(\frac{\cancel{2}a}{\cancel{2}b} = \frac{a}{b}\right)$.

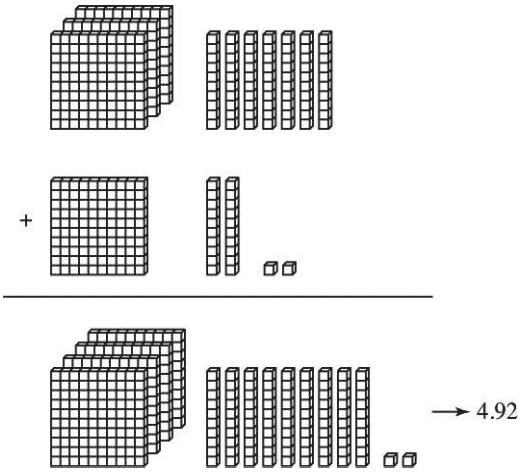
39. $\frac{7}{0.25} = \frac{70}{2.5} = \frac{700}{25}$. Thus, ***a, b,* and *d*** have equal quotients.

40. **(a)** $2 \cdot 1 + 0.25 = 2.25 = 1.5^2$
$3 \cdot 2 + 0.25 = 6.25 = 2.5^2$
A conjecture is that the next two are:
$4 \cdot 3 + 0.25 = (3.5)^2$ and $5 \cdot 4 + 0.25 = (4.5)^2$.

(b) Since $1.5 = 2 - .5$, $2.5 = 3 - .5$, $3.5 = 4 - .5$, and $4.5 = 5 - .5$,
We conjecture that for each natural numbers, $n(n - 1) + 0.25 = (n - .5)^2$.
This is true since
$(n - .5)^2 = (n - .5)(n - .5)$
$= n^2 - .5n - .5n + .25 = n^2 - n + .25 = n(n - 1) + .25$.

Assessment 7-2B

1.



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2. **(a)** $\frac{53}{10}+\frac{132}{10}+\frac{86}{100}=\frac{530}{100}+\frac{1320}{100}+\frac{86}{100}=\frac{1936}{100}$

$=19.36.$

(b) $\frac{542}{100}+\frac{32005}{1000}=\frac{5420}{1000}+\frac{32005}{1000}$.

$=\frac{37425}{1000}=37.425$

3. **(a)** $0.3\bullet 0.8=\frac{3}{10}\bullet\frac{8}{10}=\frac{24}{100}=0.24.$

(b) $502\bullet 0.04=502\bullet\frac{4}{100}=\frac{2008}{100}=20.08.$

(c) $0.004\bullet 0.06=\frac{4}{1000}\bullet\frac{6}{100}=\frac{24}{100,000}$

$=0.00024.$

4. Answers vary.

(a) 564.23 has 2 decimal places, and 6.7 has 1 decimal place so we move 2 + 1 = 3 decimal places: 3780.341.

(b) The product is 600 • 7 = 4200, so the decimal point is placed as 3780.341.

5. **(a)** $5\bullet 254.15(\approx 5\bullet 250\approx 1250)=1270.75$

(b) $31.6\bullet 122.34(\approx 30\bullet 120\approx 3600)=3865.944$

(c) $\frac{813.45}{4.25}(\approx\frac{800}{4}\approx 200)=191.4$.

6. Dividing each number by 0.001 is equal to multiplying each number by $\frac{1}{1000}$ which is equal to dividing each number by 1000.

(a) $\frac{5280}{1000}=5.280$

(b) $\frac{54.56}{1000}=0.05456$

(c) $\frac{0.275}{1000}=0.000275$

(d) $\frac{0.78}{1000}=0.00078$ **.**

7. When you multiply a positive decimal less than 1 by a decimal greater than 1 the results is not always greater than 1. For example $0.5\bullet 1.5=0.75$ and $0.75<1$.

8. He bought a total of:

$$\begin{array}{r}\$4.99\\0.79\\49.99\\1.49\\\hline\$57.26\end{array}$$ in his shopping (excluding sales tex).

9. **(a)** Sum along the diagonal, yielding 14.4. In rows and columns with two figures subtract their sum from 14.4 to obtain the missing element:

7.5	**1.2**	**5.7**
3.0	4.8	**6.6**
3.9	8.4	2.1

(b) **Yes**, the square is still magic. For example, the sum of the first row is $.5\cdot 7.5+.5\cdot 1.2+.5\cdot 5.7=.5(7.5+1.2+5.7)=.5(14.4)=$ **7.2**.

This key idea example demonstrates that if we multiply all the cells by .5 the sum of any row, column, or diagonal is halved.

10. There would be a total of $30+20+10=$ 60 pounds of nuts; at an average price per pound of \$4.50 he would pay a total of $60\cdot 4.50=\$270.$ He has already paid $\$3.00\cdot 30+\$5.00\cdot 20=$ $\$190.00,$ so he has $\$270.00-190.00=$ **\$80.00** to pay for 10 pounds.

11. $3000\text{ cm}^3\div 16.387064\text{ cm}^3\text{ per in}^3\approx$ **183 in.**^{3} (to the nearest in^3).

(***i***) Kluft versus Sokolova: $\begin{array}{r}48.89\\-47.86\\\hline\textbf{1.03 m}\end{array}$

(***ii***) Kluft versus Burrell: $\begin{array}{r}48.89\\-47.69\\\hline\textbf{1.20 m}\end{array}$

12. $\$63.28$

$-\underline{27.45}$

\$35.83 was the loss.

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13. The ratio of Canadian dollars to U.S. dollars is $\frac{1.046}{1}$ Thus. $28.43\,\text{U.S. dollars} = 28.43 \cdot \frac{1.046}{1}$ $= 29.74$ **Canadian dollars**.

14. $18.5 \text{ cups} = 18.5 \text{ cups} \cdot \frac{1 \text{ quarts}}{4 \text{ cups}} = \frac{185}{40} \text{ quarts} =$ $\frac{37}{8}$ quarts $=$ **4.625 quarts**.

15. **(a)** Each element of the sequence is 0.5 times the previous element, thus it is geometric. The next three elements are: $0.125 \cdot 0.5 =$ **0.0625**, $0.0625 \cdot 0.5 =$ **0.03125**, $0.03125 \cdot 0.5 =$ **0.015625**.

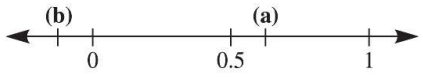
(b) There is a difference of 1.3 between each element of the sequence, thus it is arithmetic. The next three elements are: $5.4 + 1.3 =$ **6.7**, $6.7 + 1.3 =$ **8.0**, $8.0 + 1.3 =$ **9.3**.

16. If $a_1 = 0.4$;
$a_2 = 0.4 \cdot 0.3 = 0.12$;
$a_3 = 0.12 \cdot 0.3 = 0.036$;
$a_4 = 0.036 \cdot 0.3 = 0.0108$;
$a_5 = 0.0108 \cdot 0.3 = 0.00324$;
Their sum is **0.57004**.

17. $0.3333333 = 0.3 + 0.03 + 0.003 + \cdots +$ 0.0000003 or $\frac{3}{10} + \frac{3}{10}\left(\frac{1}{10}\right) + \frac{3}{10}\left(\frac{1}{10}\right)^2 + \cdots +$ $\frac{3}{10}\left(\frac{1}{10}\right)^6$. This is the finite sum of a geometric sequence with $a_1 = 0.3$, $n = 7$, $a_7 = 0.0000003$, and $r = \frac{0.03}{0.3} = \frac{1}{10} = 0.1$.

18. **(a)** $0.3 + 0.31$ is close to 0.6.

(b) $0.3 - 0.31$ is close to 0. Thus on the number line:



19. $18 \text{ shares} \cdot \$61.48 = \$1106.64$ net return on first transaction.
$350 \text{ shares} \cdot \$85.35 - \$495$ commission $=$ $\$29,377.50$ net return on second transaction.
Total net return $= \$1106.64 + 29377.50 =$ $\$30,484.14$.
Total cost $= \$964.00 + 27,422.50 =$ $\$28,386.50$.
Profit $= \$30,484.14 - 28,386.50 = \$2097.64$, which rounds to **$2098**.

20. $1451 ft \bullet 0.005 = 7.255 ft$ The height of the model will be 7.255 ft.

21. $105.2 cm \bullet 1.5 = 157.8 cm$ If Allie is 105.2 cm now she will be 157.8 cm when she is twelve years old.

22. **(a)** $\$200 \bullet 99.925 = 19,985 yen$. If \$1 is worth 99.925 yen then \$200 is worth 19.985 yen.

(b) If \$1 is worth 99.925 yen then 1 yen is worth \$0.010007506 and 7000 yen is worth approximately \$70.05.

23. If there are 2.54 cm in 1 inch then there are 0.393700787 inches in 1 cm.

(a) 160 cm would roughly equal 60 inches.

(b) 160 cm • 0.393700787 = 62.99 inches.

(c) 182 cm • 0.393700787 = 71.65 inches.

24. If 1 km is approximately 0.62 miles then 1 mile is approximately 1.612903226 km.

(a) 50 km • 0.62 = 31 miles.

(b) 120 km/hr • 0.62 = 74.4 mi/hr.

(c) 50 mi/hr • 1.612903226 = 80.65 km/hr.

25. **(a)** $3.5 \cdot 10^7 =$ **35,000,000**; i.e., move the decimal point seven places to the right.

(b) $3.5 \cdot 10^{-7} =$ **0.00000035**; i.e., move the decimal point seven places to the left.

(c) $-2.4 \cdot 10^{-3} = -$**0.0024**; i.e., move the decimal point three places to the left.

26. **(a)** Move the decimal point five places to the left to obtain the product of a number between l and 10 and an integer power of 10, i.e., 9.98199 multiplied by 10^5, or $9.98199 \bullet 10^5$ people.

(b) Move the decimal point seven places to the left and multiply the result by 10^7 to obtain $\mathbf{2.449 \cdot 10^7\ mi^2}$.

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27. **(a)** $2x + 1.3 = 4.1 \Rightarrow 2x = 2.8 \Rightarrow$ $\boldsymbol{x = 1.4}$.

(b) $4.2 - 3x = 10.2 \Rightarrow {}^{-}3x = 6.0 \Rightarrow$ $\boldsymbol{x = {}^{-}2.0}$.

28. **(a)** Move the decimal point 6 places to the left to obtain **0.0000044 sec**.

(b) Move the decimal point 4 places to the right to obtain about **19,900 km**.

(c) Move the decimal point 9 places to the right to obtain approximately **3,000,000,000 years**.

29. **(a)** $(5 \cdot 10^7)(7 \cdot 10^{12}) = 5 \cdot 7 \cdot 10^7 \cdot 10^{12}$ $= 35 \cdot 10^{19} = \boldsymbol{3.5 \cdot 10^{20}}$.

(b) $({}^{-}13 \cdot 10^4) \div 65 = \frac{{}^{-}13}{65} \cdot 10^4 =$ $\frac{{}^{-}1}{5} \cdot 10^4 = {}^{-}0.2 \cdot 10^4 = \boldsymbol{{}^{-}2.0 \cdot 10^3}$.

(c) $(3 \cdot 10^7)(4 \cdot 10^5) \div (6 \cdot 10^{-7}) =$ $\frac{3 \cdot 4}{6} \cdot 10^7 \cdot 10^5 \cdot 10^7 = \boldsymbol{2.0 \cdot 10^{19}}$.

30. **(a)** $\underline{7}15.04$ is between 700 and 800 and is closer to 700, so round down to **700**.

(b) $715.\underline{0}4$ is between 715.0 and 715.1 and is closer to 715.0, so round down to **715.0**.

(c) $71\underline{5}.04$ is between 715 and 716 and is closer to 715, so round down to **715**.

(d) $71\underline{5}.04$ is between 710 and 720 and is closer to 720, so round up to **720**.

(e) $\underline{0}715.04$ is between 0 and 1000, and is closer to 1000, so round up to **1000**.

31. Distance = speed × time = 55.5 mph × 0.75 hr = **41.625 miles**.

32. Answers may vary. E.g., **10.3** because $9 \cdot 10 = 90$ and $9 \cdot 0.3$ is greater than 2 and less than 4.

33. **(a)** ***(i)*** 47.62 rounds to 48;
27.99 rounds to 28;
13.14 rounds to 13;
7.61 rounds to 8;
Rounded estimate = $48 + 28 + 13 + 8 =$ **97**.

(ii) Lead digits sum to $40 + 20 + 10 + 0 = 70$; adjustments are about $8 + 8 + 3 + 8 = 27$.
Front-end estimate = $70 + 27 =$ **97**.

(iii) Actual sum = **96.36**.

(b) ***(i)*** 79.86 rounds to 80;
27.37 rounds to 27.
Rounded estimate = $80 - 27 =$ **53**.

(ii) Lead digit difference is $70 - 20 = 50$; adjustments are about $10 - 7 = 3$.
Front-end estimate = $50 + 3 =$ **53**.

(iii) Actual difference = **52.49**.

(c) ***(i)*** 5.85 rounds to 6;
6.17 rounds to 6;
9.10 rounds to 9;
4.23 rounds to 4.
Rounded estimate = $6 + 6 + 9 + 4 =$ **25**.

(ii) Lead digits sum to $5 + 6 + 9 + 4 = 24$;
adjustments are about $1 + 0 + 0 + 0 = 1$.
Front-end estimate = $24 + 1 =$ **25**.

(iii) Actual sum = **25.35**.

(d) ***(i)*** 232.65 rounds to 230;
78.92. rounds to 80.
Rounded estimate = $230 - 80 =$ **150**.

(ii) Lead digit difference = $200 - 70 = 130$;
adjustments are about $33 - 9 = 24$.
Front-end estimate = $130 + 24 =$ **154**.

(iii) Actual difference = **153.73**.

34. To find the least products use the smallest digits in the highest place values. To find the greatest products use the largest digits in the highest place values:

(i) Least: $\boxed{1}.\boxed{3} \times \boxed{2}.\boxed{4} =$ **3.12**.

(ii) Greatest: $\boxed{8}.\boxed{7} \times \boxed{9}.\boxed{6} =$ **83.52**.

35. To find the least possible number use the smallest digits in the highest place values:

$4\boxed{1}\boxed{2}3\boxed{5}.\boxed{6}\boxed{7}8\boxed{9}$.

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36. Answers may vary. E.g., $40 \cdot \$6 + 40 \cdot \$\left(\frac{1}{4}\right) = \$240 + \$10 = \mathbf{\$250}$.

37. **(a)** $12.4 \cdot 7 = 6.2 \cdot 2 \cdot 7 = 6.2 \cdot \mathbf{14}$.

(b) $12.4 \div 0.2 = \dfrac{12.4}{2 \cdot 0.1} = \dfrac{6.2}{0.1} = \mathbf{6.2} \div 0.1$.

(c) $ab = a \cdot 10^{-1} \cdot \mathbf{10} \cdot \boldsymbol{b}$

(d) $12.3 = \dfrac{12.3}{1} = \dfrac{1230}{10^2} = 10^{-2} \cdot \mathbf{1230}$.

38. $9 \div 0.35 = \dfrac{9}{0.35} = \dfrac{90}{3.5} = \dfrac{900}{35} = \dfrac{.9 \cdot 1000}{.035 \cdot 1000} = \dfrac{.9}{.035}$.

Thus, *i*, *ii*, *iii* and *iv* have the same quotient.

39. **(a)** ***(i)*** $1 \cdot 2 + 0.25 = \mathbf{(1.5)^2}$;

(ii) $2 \cdot 3 + 0.25 = \mathbf{(2.5)^2}$;

(iii) The next two equations might be $3 \cdot 4 + 0.25 = (3.5)^2$ and $4 \cdot 5 + 0.25 = (4.5)^2$.

(b) The above equations are correct.

(c) $n(n + 1) + 0.25 = \mathbf{(n + 0.5)^2}$.

Mathematical Connections 7-2: Review Problems

19. $14.0479 = 1 \cdot 10^1 + 4 \cdot 10^0 + 0 \cdot 10^{-1} + 4 \cdot 10^{-2} + 7 \cdot 10^{-3} + 9 \cdot 10^{-4}$.

20. **(a)** **Nonterminating**. The reduced denominator has only 3 as a factor.

(b) **Terminating**. The simplified fraction has a denominator with only 2 as a factor.

21. **Yes**, if the numerator is a multiple of 13. E.g., $\frac{13}{26} = \frac{1}{2}$ in simplest form, and $\frac{1}{2}$ can be written as a terminating decimal.

22. $\frac{35}{56} = \frac{5}{8}$ in simplest form, and 8 in the denominator may be written as 2^3.

Assessment 7-3A: Repeating Decimals

1. The fraction is in simplest form and if the prime factorization of the denominator contains only 2 and/or 5's then the fraction will be a terminating decimal. Since the denominator ends on 9 it is not divisible by 2 or by 5 it will be a non-terminating decimal.

2. **(a)** $0.45\overline{777}, 0.45\overline{77}$, and $0.45\overline{7}$ are all the same number. The only difference is that the repeating block of 7's is indicated in different ways.

(b) The preferred part is the shortest repeating part: $0.45\overline{7}$.

(c) The shortest period in $0.45\overline{7}$ is 1.

3. **(a)** The first ten decimal places of $0.\overline{123} = 0.1231231231$

(b) The first ten decimal places of $0.1\overline{23} = 0.1232323232$

(c) The first ten decimal places of $0.12\overline{3} = 0.1233333333$

(d) The first ten decimal places of $0.123\overline{43} = 0.1234343434$

4. We can write out the first ten decimal to compare:

$0.36\overline{25} = 0.3625252525$

$0.3\overline{625} = 0.3625625625$

and in the fifth decimal place 6>2 so $0.36\overline{25}$ is greater.

5. **(a)** $0.4444... = 0.\overline{4}$ and the period is 1.

(b) $0.36454545... = 0.36\overline{45}$ and the period is 2.

(c) $0.18273273273... = 0.18\overline{273}$ and the period is 3.

6. In each of the following, divide the numerator by the denominator either with a calculator or by use of the division algorithm. The overline in the quotient indicates that the block of digits underneath is repeated an infinite number of times.

(a) $\frac{4}{9} = 4 \div 9 = \mathbf{0.\overline{4}}$.

(b) $\frac{2}{7} = 2 \div 7 = \mathbf{0.\overline{285714}}$.

(c) $\frac{3}{11} = 3 \div 11 = \mathbf{0.\overline{27}}$.

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(d) $\frac{1}{15} = 1 \div 15 = \mathbf{0.0\overline{6}}$.

(e) $\frac{2}{75} = 2 \div 75 = \mathbf{0.02\overline{6}}$.

(f) $\frac{1}{99} = 1 \div 99 = \mathbf{0.\overline{01}}$.

(g) $\frac{5}{6} = 5 \div 6 = \mathbf{0.8\overline{3}}$.

(h) $\frac{1}{13} = 1 \div 13 = \mathbf{0.\overline{076923}}$.

(i) $\frac{1}{21} = 1 \div 21 = \mathbf{0.\overline{047619}}$.

(j) $\frac{3}{19} = 3 \div 19 = \mathbf{0.\overline{157894736842105263}}$.

The repetend has more digits than most calculator's display capability. Use the division algorithm, assuming an 8-digit display:

$$
\begin{array}{r}
0.157894736842105263 \\
19\overline{)\,3.0000000000000000000} \\
\underline{2.999998} \\
1400000 \\
\underline{1399999} \\
20000 \\
\underline{19999} \\
3
\end{array}
$$

where the last remainder, 3, again begins the repeating pattern. Note that since the denominator is 19, the most digits that can be in the repetend is 18.

7. **(a)** $n = 0.\overline{4}$; a one-digit repetend:

$$\begin{array}{rcl} 10n &=& 4.4444\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}0.4444\ldots} \\ 9n &=& 4 \end{array}$$

$\Rightarrow n = \mathbf{\frac{4}{9}}$.

(b) $n = 0.\overline{61}$; a two-digit repetend:

$$\begin{array}{rcl} 100n &=& 61.6161\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}0.6161\ldots} \\ 99n &=& 61 \end{array}$$

$\Rightarrow n = \mathbf{\frac{61}{99}}$.

(c) $n = 1.3\overline{96}$; a two-digit repetend:

$$\begin{array}{rcl} 1000n &=& 1396.9696\ldots \\ \underline{{}^{-}10n} &=& \underline{{}^{-}13.9696\ldots} \\ 990n &=& 1383 \end{array}$$

$\Rightarrow n = \frac{1383}{990} = \mathbf{\frac{461}{330}}$.

(d) $n = 0.\overline{55} = 0.\overline{5}$; a one-digit repetend:

$$\begin{array}{rcl} 10n &=& 5.5555\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}0.5555\ldots} \\ 9n &=& 5 \end{array}$$

$\Rightarrow n = \mathbf{\frac{5}{9}}$.

(e) $n = {}^{-}2.3\overline{4}$; $10n = {}^{-}23.\overline{4}$; now a one-digit repetend:

$$\begin{array}{rcl} 100n &=& {}^{-}234.4444\ldots \\ \underline{{}^{-}10n} &=& \underline{23.4444\ldots} \\ 90n &=& {}^{-}211 \end{array}$$

$\Rightarrow n = \mathbf{\frac{{}^{-}211}{90}}$.

(f) $n = {}^{-}0.0\overline{2}$; a one-digit repetend:

$$\begin{array}{rcl} 100n &=& {}^{-}2.2222\ldots \\ \underline{{}^{-}10n} &=& \underline{0.2222\ldots} \\ 90n &=& {}^{-}2 \end{array}$$

$\Rightarrow n = \frac{{}^{-}2}{90} = \mathbf{\frac{{}^{-}1}{45}}$.

8. 1 minute $= \frac{1}{60}$ hour, and $1 \div 60 = \mathbf{0.01\overline{6}\ hour}$.

9. Line up the decimal points:

$${}^{-}1.4\overline{54} = {}^{-}1.4545454\ldots$$
$${}^{-}1.\overline{454} = {}^{-}1.4544544\ldots$$
$${}^{-}1.\overline{45} = {}^{-}1.4545454\ldots$$
$${}^{-}1.45\overline{4} = {}^{-}1.4544444\ldots$$
$${}^{-}1.454 = {}^{-}1.4540000\ldots$$

Ordering from greatest (i.e., closest to 0) yields:
$\mathbf{{}^{-}1.454 > {}^{-}1.45\overline{4} > {}^{-}1.\overline{454} > {}^{-}1.4\overline{54} = {}^{-}1.\overline{45}}$.

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10. Each element could be a decimal representation of $\frac{n}{n+1}$, beginning at $n = 0$. If so, then the next three elements would be:

$$\frac{6}{7} = \mathbf{0.\overline{857142}},$$
$$\frac{7}{8} = \mathbf{0.875}, \text{ and}$$
$$\frac{8}{9} = \mathbf{0.\overline{8}}.$$

11. $0.454545\ldots$ may be written as $0.45 + (0.45)(0.01) + (0.45)(0.01)(0.01) + \cdots$. This is a geometric sequence with common ratio $\boldsymbol{r = 0.01}$.

Mulitply $.\overline{5} \cdot .\overline{5}$ using the decimal expansion to obtain a precise answer. Instead, write $.\overline{5}$ in the form $\frac{a}{b}$.

$$\begin{aligned} 10(.\overline{5}) &= 5.\overline{5} \\ -\ .\overline{5} &= -\ .\overline{5} \\ \hline 9(.\overline{5}) &= 5 \Rightarrow .\overline{5} = \frac{5}{9}. \end{aligned}$$

Thus, $(.\overline{5})^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81} = \overline{.308641975}$

12. The repeating decimal part is determined by the 7 in the denominator of $3\frac{1}{7} = \frac{22}{7}$. Seven has no prime factors of 2 or 5 and so is a repeating decimal.

13. ***(i)***
$$\begin{aligned} a + b = \quad & 0.323232323232\ldots \\ & +\ 0.123123123123\ldots \\ \hline & \mathbf{0.446355446355\ldots} \end{aligned}$$

(ii) there are six digits in the repetend.

14. **Yes**. Zeros after the last non-zero digit of the decimal can be repeated.

15. Answers may vary; e.g.,

(a) Write $3.\overline{2}$ as $3.222\ldots$ and write 3.22 as 3.220. $3.220 < 3.221 < 3.2211 < 3.22111 < 3.222\ldots$

(b) Write $462.\overline{24}$ as $464.2424\ldots$ and write 462.243 as 462.2430. $462.2424\ldots < 462.2425 < 462.2426 < 462.2427 < 462.2430$.

16. To find a number halfway between any two others, find their average; i.e., add the original numbers and divide by 2.

$(0.5 + 0.\overline{4}) \div 2 = (0.9444\ldots) \div 2 =$ $\mathbf{0.47222\cdots = 0.47\overline{2}}$.

17. Answers may vary; e.g.,

(a) Write $\frac{3}{4}$ as 0.750 and write $0.\overline{75}$ as $0.757\ldots$; $\frac{3}{4} < 0.751 < 0.752 < 0.753 < 0.\overline{75}$.

(b) Write $\frac{1}{3}$ as $0.333\ldots$ and write $0.\overline{34}$ as $0.343\ldots$; $\frac{1}{3} < 0.334 < 0.335 < 0.336 < 0.\overline{34}$.

18. **(a)** $\frac{3}{7} = 0.\overline{428571}$, a six-digit repetend. 21 when divided by 6 has a remainder of 3. Thus the 21st digit is the third in the repetend $=$ **8**.

(b) $17^{-1} = 0.\overline{0588235294117647}$, a sixteen-digit repetend. 5280 when divided by 16 has a remainder of 0. Thus the 5280th digit is the sixteenth in the repetend $=$ **7**.

19. **(a)** ***(i)*** Use the repeating block strategy:

$n = 0.\overline{1}$, a one-digit repetend.

$$\begin{aligned} 10n &= 1.1111\ldots \\ -n &= -0.1111\ldots \\ \hline 9n &= 1 \end{aligned}$$

$\Rightarrow n = \mathbf{\frac{1}{9}}$.

Observe that when the repetend is immediately to the right of the decimal point in the repeating decimal, a in the fraction $\frac{a}{b}$ will be the digit(s) in the repetend and b will be one or more 9's. The number of 9's will be the same as the number of digits in the repetend.

(ii) $0.\overline{01} = \mathbf{\frac{1}{99}}$.

(iii) $0.\overline{001} = \mathbf{\frac{1}{999}}$.

(b) $0.\overline{0001} = \mathbf{\frac{1}{9999}}$.

(c) $0.\overline{1} = \frac{1}{9}$ is true, so $(0.\overline{1})\left(\frac{1}{10}\right) = \frac{1}{9}\left(\frac{1}{10}\right) \Rightarrow$ $\mathbf{(0.0\overline{1})} = \frac{1}{90}$.

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20. **(a)** Since $0.\overline{9} = 1, 0.0\overline{9} = \frac{1}{10} \cdot (0.\overline{9}) = \frac{1}{10}(1) = \mathbf{\frac{1}{10}} = \mathbf{0.1}$.

(b) $0.3\overline{9} = 0.3 + 0.0\overline{9} = 0.3 + 0.1 = \mathbf{0.4}$.

(c) $9.\overline{9} = 9 + 0.\overline{9} = 9 + 1 = \mathbf{10}$.

21. See problem 14(a)(*i*) above and observe that when the repetend is immediately to the right of the decimal point in the repeating decimal, a in the fraction $\frac{a}{b}$ will be the digit(s) in the repetend and b will be one or more 9's. The number of 9's will be the same as the number of digits in the repetend.

(a) $0.\overline{2} = \mathbf{\frac{2}{9}}$.

(b) $0.\overline{3} = \frac{3}{9} = \mathbf{\frac{1}{3}}$.

(c) $0.\overline{5} = \mathbf{\frac{5}{9}}$.

(d) $2.\overline{7} = 2\frac{7}{9} = \mathbf{\frac{25}{9}}$.

(e) $9.\overline{9} = 9\frac{9}{9} = \mathbf{10}$.

22. See problem 14(a)(*i*) above:

(a) $0.\overline{05} = \mathbf{\frac{5}{99}}$. In the fraction $\frac{a}{b}, a = 5$ and b is two 9's.

(b) $0.\overline{003} = \frac{3}{999} = \mathbf{\frac{1}{333}}$. In the fraction $\frac{a}{b}$, $a = 3$ and b is three 9's.

23. The sum is $0.4 + 0.4(0.5) + 0.4(0.5)^2 + 0.4(0.5)^3 + 0.4(0.5)^4 = 0.4 + 0.2 + 0.1 + 0.05 + 0.025 = \mathbf{0.775}$.

Another method by which to attack this problem would be to find the sum of a finite geometric sequence:

$$S_n = a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}$$
$$rS_n = a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^n$$
$$rS_n - S_n = a_1 r^n - a_1$$
$$(r-1)S_n = a_1(r^n - 1)$$
$$S_n = \frac{a_1(r^n - 1)}{r - 1} = \frac{a_1(1 - r^n)}{1 - r}$$

Thus if $a_1 = 0.4, r = 0.5$, and $n = 5$,

$S_5 = \frac{0.4[(1-0.5^5)]}{1-0.5} = \frac{0.4 \cdot 0.96875}{0.5} = \mathbf{0.775}$.

24. To find the sum of an infinite geometric series with ratio $0 < r < 1$ (if r were greater than 1, the series could not have a finite sum; if $r = 0$ the series would be just the first term):

$$S_\infty = a + ar + ar^2 + ar^3 + \cdots$$
$$^-rS_\infty = {}^-(ar + ar^2 + ar^3 + \cdots)$$
$$S_\infty - rS_\infty = a$$
$$S_\infty(1 - r) = a$$
$$S_\infty = \frac{a}{1 - r}$$

(a) $0.2\overline{9} = 0.29999\ldots = 0.2 + 0.9999\ldots = 0.2 + 0.09 + 0.09(0.1) + 0.09(0.01) + \cdots$.

Thus $0.2\overline{9}$ is 0.2 plus an infinite geometric sequence where $a_1 = 0.09$ and $r = 0.1$.

$S_\infty = 0.2 + \frac{0.09}{1-0.1} = 0.2 + \frac{0.09}{0.9} = 0.2 + 0.1 = \frac{3}{10}$.

(b) $2.0\overline{29} = 2 + 0.029 + 0.00029 + \cdots$.

Thus $2.0\overline{29}$ is 2 plus an infinite geometric sequence where $a_1 = 0.029$ and $r = 0.01$.

$S_\infty = 2 + \frac{0.029}{1-0.01} = 2\frac{29}{990} = \mathbf{\frac{2009}{990}}$.

25. If we sum $.\overline{235}$ and $.\overline{2356}$, the digits in each place value will not realign until we reach the least common multiple of the length of each number's repetend. Thus, we expect the length of the repetend to be $LCM(3,4) = 12$. To see this, consider the follwing.

$$0.235\,235\,235\,235\,235\ldots$$
$$+\,0.235\,623\,562\,356\,2356\ldots$$

At the 10^{-13} place the digits realign.

26. **(a)** $1 - 3x = 8 \Rightarrow -3x = 7 \Rightarrow x = \frac{^-7}{3} = {}^-2.\overline{3}$.

(b) $1 = 3x + 8 \Rightarrow {}^-7 = 3x \Rightarrow 3x = {}^-7 \Rightarrow x = \frac{^-7}{3} = {}^-2.\overline{3}$.

(c) $1 = 8 - 3x \Rightarrow {}^-7 = -3x \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3} = 2.\overline{3}$.

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27. $n = 0.\overline{abc}$; a three-digit repetend:

$$\begin{aligned} 1000n &= abc.abcabc\ldots \\ \bar{}n &= \bar{}0.\text{abcabc}\ldots \\ \hline 999n &= abc \end{aligned}$$

$\Rightarrow n = \frac{abc}{999}$.

$0.\overline{abc}$ written as a fraction is $\frac{abc}{999}$.

28. $\frac{1}{333,333} = 0.000003$ so the repetend is 0.000003.

29. If $\frac{9}{23} = 0.\overline{391304347826086956521 7}$ and there are 22 repetends, then the digits repeat after 22, 44, 66, and 88 digits. The 100th digit in decimal form is the 12th digit of the reptend (88 + 12 = 100) which is 6.

Assessment 7-3B

1. The fraction is in simplest form because $3 \nmid 1234567$and if the prime factorization of the denominator contains only 2 and/or 5's then the fraction will be a terminating decimal. Since the denominator ends on 7 it is not divisible by 2 or by 5 it will be a non-terminating decimal.

2. **(a)** $0.34\overline{666}, 0.34\overline{66}$, and $0.34\overline{6}$ are all the same number. The only difference is that the repeating block of 6's is indicated in different ways.

(b) The preferred part is the shortest repeating part: $0.34\overline{6}$.

(c) The shortest period in $0.34\overline{6}$ is 1.

3. **(a)** The first ten decimal places of $0.\overline{246} = 0.2462462462$

(b) The first ten decimal places of $0.2\overline{46} = 0.2464646464$

(c) The first ten decimal places of $0.24\overline{6} = 0.2466666666$

(d) The first ten decimal places of $0.24\overline{68} = 0.2468686868$

4. We can write out the first ten decimal to compare:

$0.57\overline{89} = 0.5789898989$

$0.\overline{5789} = 0.5789789789$

and in the fifth decimal place 8>7 so $0.57\overline{89}$ is greater.

5. **(a)** $0.9999... = 0.\overline{9}$ and the period is 1.

(b) $0.567323232... = 0.56\overline{732}$ and the period is 2.

(c) $0.1827482748274... = 0.1\overline{82748}$ and the period is 4.

6. In each of the following, divide the numerator by the denominator either with a calculator or by use of the division algorithm. The overline in the quotient indicates that the block of digits underneath is repeated an infinite number of times.

(a) $\frac{2}{3} = 2 \div 3 = \mathbf{0.\overline{6}}$.

(b) $\frac{7}{9} = 7 \div 9 = \mathbf{0.\overline{7}}$.

(c) $\frac{1}{24} = 1 \div 24 = \mathbf{0.041\overline{6}}$.

(d) $\frac{3}{60} = \frac{1}{20} = 1 \div 20 = \mathbf{0.05}$.

(e) $\frac{2}{99} = 2 \div 99 = \mathbf{0.\overline{02}}$.

(f) $\frac{7}{6} = 7 \div 6 = \mathbf{1.1\overline{6}}$.

(g) $\frac{2}{21} = 2 \div 21 = \mathbf{0.\overline{095238}}$.

(h) $\frac{4}{19} = 4 \div 19 = \mathbf{0.\overline{210526315789473684}}$.

The repetend has more digits than most calculators' display capability. Use the division algorithm, assuming an 8-digit display:

$$\begin{array}{r} 0.\,2\,1\,0\,5\,2\,6\,3\,1\,5\,7\,8\,9\,4\,7\,3\,6\,8\,4 \\ 19\overline{)\,4.0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0} \\ \underline{3.9\,9\,9\,9\,9\,9\,4} \\ 6\,0\,0\,0\,0\,0\,0 \\ \underline{5\,9\,9\,9\,9\,9\,1} \\ 9\,0\,0\,0\,0\,0\,0 \\ \underline{8\,9\,9\,9\,9\,9\,6} \\ 4 \end{array}$$

where the last remainder, 4, again begins the repeating pattern. Note that since the denominator is 19, the most digits that can be in the repetend is 18.

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7. **(a)** $n = 0.\overline{7}$; a one-digit repetend:

$$\begin{array}{rcl} 10n &=& 7.7777\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}0.7777\ldots} \\ 9n &=& 7 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{7}{9}}.$$

(b) $n = 0.\overline{46}$; a two-digit repetend:

$$\begin{array}{rcl} 100n &=& 46.4646\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}0.4646\ldots} \\ 99n &=& 46 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{46}{99}}.$$

(c) $n = 2.3\overline{7}$; a two-digit repetend:

$$\begin{array}{rcl} 100n &=& 237.3737\ldots \\ \underline{{}^{-}n} &=& \underline{{}^{-}2.3737\ldots} \\ 99n &=& 235 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{235}{99}}.$$

(d) $n = 2.3\overline{4}$; a one-digit repetend:

$$\begin{array}{rcl} 100n &=& 234.4444\ldots \\ \underline{{}^{-}10n} &=& \underline{{}^{-}23.4444\ldots} \\ 90n &=& 211 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{211}{90}}.$$

(e) $n = {}^{-}4.3\overline{4}$; a one-digit repetend:

$$\begin{array}{rcl} 100n &=& {}^{-}434.4444\ldots \\ \underline{{}^{-}10n} &=& \underline{43.4444\ldots} \\ 90n &=& {}^{-}391 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{{}^{-}391}{90}}.$$

(f) $n = {}^{-}0.0\overline{3}$; a one-digit repetend:

$$\begin{array}{rcl} 100n &=& {}^{-}3.3333\ldots \\ \underline{{}^{-}10n} &=& \underline{0.3333\ldots} \\ 90n &=& {}^{-}3 \end{array}$$

$$\Rightarrow \boldsymbol{n = \frac{{}^{-}3}{90} = \frac{{}^{-}1}{30}}.$$

8. One second $= \frac{1}{3600}$ hour; $1 \div 3600 =$ $\boldsymbol{0.0002\overline{7}\ \text{hour}}$.

9. Line up the decimal points:

$${}^{-}4.340000\ldots$$
$${}^{-}4.343434\ldots$$
$${}^{-}4.344444\ldots$$
$${}^{-}4.343434\ldots$$
$${}^{-}4.434343\ldots$$

Ordering from least (i.e., furthest from 0) yields:
${}^{-}4.4\overline{34} < {}^{-}4.3\overline{4} < {}^{-}4.3\overline{43} = {}^{-}4.\overline{34} < {}^{-}4.34$.

10. If this is an arithmetic sequence, then add 0.3 to each element to obtain the following element. The next three elements thus could be:

$$1.\overline{3} + 0.\overline{3} = \boldsymbol{1.\overline{6}};$$
$$1.\overline{6} + 0.\overline{3} = \boldsymbol{1.\overline{9}} = \boldsymbol{2.0};$$
$$2.0 + 0.\overline{3} = \boldsymbol{2.\overline{3}}.$$

11. We can write the decimals in the product as a fraction and perform the operation. $0.4\overline{9} \cdot 0.\overline{62} = 0.\overline{5} \cdot 0.\overline{62} = \frac{5}{9} \cdot \frac{62}{99} = \frac{310}{891} =$ $\boldsymbol{0.\overline{347923681257014590}}$.

12. $\frac{2}{26} = \frac{1}{13}$. Since the denominator has a prime factor other than 2 or 5, the decimal will not terminate. Because $\frac{2}{26}$ is equal to the reduced fraction $\frac{1}{13}$ and the number of digits in the repetend for a proper fraction cannot exceed the denominator minus one, the maximum possible length of the repetend is **12**.

13.

$$\begin{array}{rcl} a &=& 1.23\,43\,43\,43\,43\ldots \\ \underline{+\ b} &=& \underline{+\,0.12\,34\,12\,34\,12\ldots} \\ a + b &=& \boldsymbol{1.35\,77\ \ 55\,77\,55}\ldots \end{array}$$

Since $a + b$ is a repeating decimal, it is a rational number. Alternatively, a and b are rational numbers and the sum of two rational numbers is also rational. In this case, the length of the repetend is four.

14. **Yes**. If, for example, the repeating digits are zeros, then the repeating decimal could be written as a terminating decimal.

15. Answers may vary; e.g.,

(a) Write $4.\overline{3}$ as $4.3333\ldots$ and 4.3 as $4.3000\ldots$.
Then $4.3 < 4.310 < 4.320 < 4.330 < 4.\overline{3}$.

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(b) Write $203.\overline{76}$ as $203.7676\ldots$ and $203.\overline{7}$ as $203.7777\ldots$. Then $203.\overline{76} < 203.7680 < 203.7690 < 203.7691 < 203.\overline{7}$.

16. To find a number halfway between any two others, find their average; i.e., add the original numbers and divide by 2. (Note that $0.\overline{9} = 1.0$.)

$(0.\overline{9} + 1.1) \div 2 = (2.1) \div 2 = \mathbf{1.05}$.

17. Answers may vary; e.g.,

(a) Write $\frac{2}{3}$ as $0.6666\ldots$ and 0.67 as 0.6700. Then $\frac{2}{3} < 0.6671 < 0.06672 < 0.06673 < 0.67$.

(b) Write $\frac{2}{3}$ as $0.6666\ldots$ and $0.6\overline{7}$ as $0.6777\ldots$. Then $\frac{2}{3} < 0.6667 < 0.6668 < 0.6669 < 0.6\overline{7}$.

18. $17^{-1} = 0.\overline{0588235294117647}$, a sixteen-digit repetend. 23 when divided by 16 has a remainder of 7, so the 23rd digit is the 7th in the repetend, or **5**.

19. **(a)** ***(i)*** Use the repeating block strategy:

$n = 0.\overline{2}$, a one-digit repetend.

$$\begin{aligned} 10n &= 2.2222\ldots \\ \underline{{}^{-}n} &= \underline{{}^{-}0.2222\ldots} \\ 9n &= 2 \end{aligned}$$

$\Rightarrow n = \mathbf{\frac{2}{9}}$.

(ii) $n = 0.0\overline{2}$, a one-digit repetend.

$$\begin{aligned} 100n &= 2.2222\ldots \\ \underline{{}^{-}10n} &= \underline{0.2222\ldots} \\ 90n &= 2 \end{aligned}$$

$\Rightarrow n = \mathbf{\frac{2}{90}}$.

(iii) $n = 0.00\overline{2}$, a one-digit repetend.

$$\begin{aligned} 1000n &= 2.2222\ldots \\ \underline{{}^{-}100n} &= \underline{0.2222\ldots} \\ 900n &= 2 \end{aligned}$$

$\Rightarrow n = \mathbf{\frac{2}{900}}$.

(b) Based on (a) (*i*) and (*ii*), $0.000\overline{2} = \mathbf{\frac{2}{9000}}$.

(c) $\frac{4}{90} = 2\left(\frac{2}{90}\right) = 2(0.0\overline{2}) = \mathbf{0.0\overline{4}}$.

20. **(a)** $1.\overline{9} = 1 + 0.\overline{9} = 1 + 1 = 2$.

(b) $0.00\overline{9} = \frac{1}{100} \cdot 0.\overline{9} = \frac{1}{100} \cdot 1 = \mathbf{0.01}$.

(c) $0.3\overline{9} = 0.3 + 0.0\overline{9} = 0.3 + \frac{1}{10} \cdot 0.\overline{9}$
$= 0.3 + 0.1 = \mathbf{0.4}$.

21. Observe that when the repetend is immediatly to the right of the decimal point in the repeating decimal, a in the fraction $\frac{a}{b}$ will be the digit(s) in the repetend and b will be one or more 9's. The number of 9's will be the same as the number of digits in the repetend.

(a) $0.\overline{4} = \mathbf{\frac{4}{9}}$.

(b) $0.\overline{12} = \frac{12}{99} = \mathbf{\frac{4}{33}}$.

(c) $0.\overline{111} = \frac{111}{999} = \mathbf{\frac{1}{9}}$.

22. **(a)** First observe that $25(0.010101\ldots) = 0.252525\ldots$. Therefore, $3.\overline{25} = 3 + 25(0.\overline{01}) = 3 + \frac{25}{99} = \mathbf{\frac{322}{99}}$.

(b) $3.\overline{125} = 3 + 125(0.\overline{001}) = 3 + \frac{125}{999} = \mathbf{\frac{3122}{999}}$.

23. The sum is $0.1 + 0.1(0.3) + 0.1(0.3)^2 + 0.1(0.3)^3$
$= 0.1 + 0.03 + 0.009 + 0.0027 = \mathbf{0.1417}$.
Another method by which to attack this problem would be to find the sum of a finite geometric sequence:

$$\begin{aligned} S_n &= a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1} \\ rS_n &= a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^n \\ rS_n - S_n &= a_1 r^n - a_1 \\ (r-1)S_n &= a_1(r^n - 1) \\ S_n &= \frac{a_1(r^n - 1)}{r - 1} = \frac{a_1(1 - r^n)}{1 - r} \end{aligned}$$

Thus if $a_1 = 0.1$, $r = 0.3$, and $n = 4$,

$S_4 = \frac{0.1[(1-0.3^4)]}{1-0.3} = \frac{0.1 \cdot 0.9919}{0.7} = \mathbf{0.1417}$.

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24. To find the sum of an infinite geometric series with ratio $0 < r < 1$ (if r were greater than l, the series could not have a finite sum; if $r = 0$ the series would be just the first term):

$$\begin{aligned} S\infty &= a + ar + ar^2 + ar^3 + \cdots \\ {}^{-}rS_{\infty} &= {}^{-}(ar + ar^2 + ar^3 + \cdots) \\ S_{\infty} - rS\infty &= a \\ S_{\infty}(1 - r) &= a \\ S\infty &= \frac{a}{1 - r} \end{aligned}$$

(a) $0.\overline{29} = 0.29 + 0.0029 + \cdots = 0.29 + 0.29(0.01) + 0.29(0.0001) + \cdots$.
Thus $a_1 = 0.29$ and $r = 0.01$.
$S_{\infty} = \frac{0.29}{1-0.01} = \frac{0.29}{0.99} = \mathbf{\frac{29}{99}}$.

(b) $0.00\overline{29} = 0.00029 + 0.0000029 + \cdots$.
Thus $0.00\overline{29}$ is an infinite geometric sequence where $a_1 = 0.00029$ and $r = 0.01$.
$S = \frac{0.00029}{1-0.01} = \frac{0.00029}{0.99} = \mathbf{\frac{29}{99{,}000}}$.

25.
$$\begin{aligned} 0.\overline{23} &= 0.2323232323\ldots \\ +\,0.\overline{235} &= \underline{0.2352352352\ldots} \\ &= 0.4675584675\ldots \end{aligned}$$
So there are six places in the repetend of the sum.

26. **(a)** $3x = 8 \Rightarrow x = \frac{8}{3} = \mathbf{2\frac{2}{3}} = \mathbf{2.\overline{6}}$.

(b) $3x + 1 = 8 \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3} = \mathbf{2\frac{1}{3}} = \mathbf{2.\overline{3}}$.

(c) $3x - 1 = 8 \Rightarrow 3x = 9 \Rightarrow x = \mathbf{3.0}$.

27. $n = 0.\overline{ab}$; a two-digit repetend:

$$\begin{aligned} 100n &= ab.abab\ldots \\ {}^{-}n &= {}^{-}0.\text{abab}\ldots \\ \hline 99n &= ab \end{aligned}$$
$\Rightarrow n = \frac{ab}{99}$.

$0.\overline{ab}$ written as a fraction is $\dfrac{ab}{99}$.

28. $\dfrac{1}{333{,}333{,}333} = 0.\overline{000000003}$ so the repetend is 0.000000003.

29. If $\dfrac{9}{23} = 0.\overline{391304347826086956521 7}$ and there are 22 repetends, then the digits repeat after 22, 44, 66, 88, etc. digits. The 900th digit in decimal form is the 20th digit of the repetend (880 + 20 = 900) which is 2.

Mathematical Connections 7-3: Review Problems

16. Total deductions were $\$1520.63 + \$723.30 + \$2843.62 = \$5087.55$. Gross pay less deductions was $\$27{,}849.50 - \$5087.55 = \mathbf{\$22{,}761.95}$.

17. Distance at the speed of light would be $1.86 \cdot 10^5$ mi/second $\cdot\ 3.1536 \cdot 10^7$ sec/yr $\cdot$ 4 yrs $\approx 23.5 \cdot 10^{12} = \mathbf{2.35 \cdot 10^{13}}$ **miles**.

18. The rule states that decimal point placement should be four places, or 0.0770. Because 0.077 as found on the calculator equals 0.0770; i.e., trailing zeros add no value; the rule still applies.

19. Answers may vary

(a) $\mathbf{3.024} + {}^{-}0.023 = 3.001$, which is between 3 and 4.

(b) ${}^{-}0.023 - {}^{-}\mathbf{3.024} = 3.001$, which is between 3 and 4.

(c) Pick 3.5 as a number between 3 and 4.
$0.023 \cdot n = 3.5 \Rightarrow n = \frac{3.5}{0.023} \approx \mathbf{152}$.

(d) Pick 3.5 as a number between 3 and 4.
$\frac{0.023}{n} = 3.5 \Rightarrow n = \frac{0.023}{3.5} \approx \mathbf{0.00657}$.

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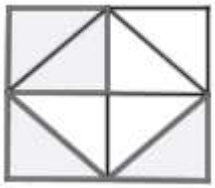
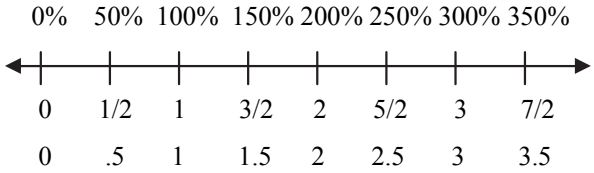
Assessment 7-4A: Percents and Interest

1. Cut the shape into equal size pieces. If squares are created and then triangles inside the squares, there are 8 triangles and 4 are shaded. $\frac{4}{8} = 50\%$.



2.



3. **(a)** $7.89 = 100\left[\frac{7.89}{100}\right] = \frac{789}{100} = \mathbf{789\%}$.

(b) $193.1 = 100\left[\frac{193.1}{100}\right] = \frac{19310}{100} = \mathbf{19310\%}$.

(c) $\frac{5}{6} = 100\left[\frac{\frac{5}{6}}{100}\right] = \frac{\mathbf{100}\left[\frac{5}{6}\right]}{100}$.

$= \frac{\frac{50}{1}\cdot\frac{5}{3}}{100} = \frac{\frac{250}{3}}{100} = \frac{83\frac{1}{3}}{100} = \mathbf{83\frac{1}{3}\%}$.

(d) $\frac{1}{8} = 100\left[\frac{\left(\frac{1}{8}\right)}{100}\right] = \frac{\left(\frac{100}{8}\right)}{100} = \frac{12.5}{100} = \mathbf{12.5\%}$.

(e) $\frac{5}{8} = 100\left[\frac{\left(\frac{5}{8}\right)}{100}\right] = \frac{\left(\frac{500}{8}\right)}{100} = \frac{62.5}{100} = \mathbf{62.5\%}$.

(f) $\frac{4}{5} = 100\left[\frac{\left(\frac{4}{5}\right)}{100}\right] = \frac{\left(\frac{400}{5}\right)}{100} = \frac{80}{100} = \mathbf{80\%}$.

4. **(a)** $16\% = \frac{16}{100} = \mathbf{0.16}$.

(b) $\frac{1}{5}\% = 0.2\% = \frac{0.2}{100} = \frac{2}{1000} = \mathbf{0.002}$.

(c) $13\frac{2}{3}\% = 13.\overline{6}\% = \frac{13.\overline{6}}{100} = \mathbf{0.13\overline{6}}$.

(d) $\frac{1}{3}\% = 0.\overline{3}\% = \frac{0.\overline{3}}{100} = \mathbf{0.00\overline{3}}$.

5. **(a)** $\underline{\mathbf{4}}$ for every 100. $4\% = \frac{4}{100}$.

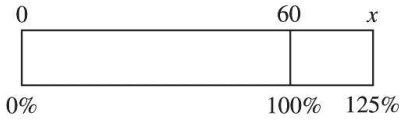
(b) $\underline{\mathbf{2}}$ for every 50. $4\% = \frac{4}{100} = \frac{2}{50}$.

(c) 1 for every $\underline{\mathbf{25}}$. $4\% = \frac{4}{100} = \frac{1}{25}$.

(d) 8 for every $\underline{\mathbf{200}}$. $4\% = \frac{4}{100} = \frac{8}{200}$.

(e) 0.5 for every $\underline{\mathbf{12.5}}$. $4\% = \frac{4}{100} = \frac{0.5}{12.5}$.

6.



$\frac{60}{100} = \frac{x}{125} \Rightarrow 100x = 6000 \Rightarrow x = 75$ **.**

7. **(a)** 6% of 34 $\Rightarrow \frac{6}{100} \cdot 34 = \frac{204}{100} = \mathbf{2.04}$.

(b) 17 is n% of 34 $\Rightarrow 17 = \frac{n}{100} \cdot 34$

$\Rightarrow 34n = 1700 \Rightarrow n = \frac{1700}{34} =$

50, or 17 is **50%** of 34.

(c) 18 is 30% of $n \Rightarrow 18 = \frac{30}{100} \cdot n$

$\Rightarrow 30n = 1800 \Rightarrow n = \frac{1800}{30} = \mathbf{60}$.

(d) 7% of 49 $\Rightarrow \frac{7}{100} \cdot 49 = \frac{343}{100} = \mathbf{3.43}$.

8. **(a)** 5% of $x \Rightarrow \frac{5}{100}x$, or $\frac{5x}{100} = \frac{x}{20}$.

(b) 10% of amount $= a \Rightarrow \frac{10}{100} \cdot$ amount $= a$

$\Rightarrow$ amount $= \mathbf{10}a$.

9. 75% of 84 boxes $\Rightarrow \frac{75}{100} \cdot 84 = \mathbf{63\ boxes}$.

10. Let s be her last salary. 7% of s + 100% of s = $\$27,285 \Rightarrow 107\% \cdot s = \$27,285 \Rightarrow 1.07 \cdot s =$ 27,285. Thus $s = \frac{27,285}{1.07} = \mathbf{\$25,500}$.

11. **(a)** **Bill**. Joe sold 180 papers; Bill sold $\frac{85}{100} \cdot 260 = 221$ papers; Ron sold 212 papers.

(b) **Joe**. Joe sold $100\left[\frac{\left(\frac{180}{200}\right)}{100}\right] = 90\%$ of his papers; Bill sold 85% of his papers; Ron sold 80% of his papers.

(c) **Ron**. Joe started with 200 papers; Bill started with 260 papers; Ron started with $\frac{212}{0.80} = 265$ papers.

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12. The amount of the discount is $\$35 - \$28 = \$7$. The percent of the discount is \$7 as percent of $\$35 \Rightarrow \frac{7}{35} = 0.2 = 100\left(\frac{0.2}{100}\right) = \frac{20}{100} =$ **20%**.

13. The amount of decrease in value was $\$359{,}000 - \$195{,}000 = \$164{,}000$. The decrease as a percentage of the original value was $\frac{164{,}000}{359{,}000} \approx 0.46 = 100\left(\frac{0.46}{100}\right) = \frac{46}{100}$, or **about 46%.**

14. Sale price was regular price – 20% of regular price. $\$28.00 - 20\%$ of $\$28.00 = \$28 - (0.2 \cdot \$28) = \$28.00 - \$5.60 =$ **\$22.40**.

15. The amount of the tax is 5% of \$320, or $0.05 \cdot 320 = \$16$. The total cost is $\$320 + \$16 =$ **\$336**.

16. Bill answered $80 - 52 = 28$ questions incorrectly. 28 as percentage of 80 is $\frac{28}{80} = 0.35 = 100\left(\frac{0.35}{100}\right) = \frac{35}{100} =$ **35%**.

17. $66\frac{2}{3}\%$ of 1800 employees means $\frac{66\frac{2}{3}}{100} = \frac{\left(\frac{200}{3}\right)}{100} = \frac{200}{300} = \frac{2}{3}$ of 1800. $\frac{2}{3} \cdot 1800 =$ **1200 employees**.

18. $\frac{325}{500} \cdot \frac{325}{500} = \frac{650}{1000} = \frac{65}{100} = 65\%$; $\frac{600}{1000} = \frac{60}{100} = 60\%$.

19. **(a)** ***(i)*** $8 \cdot \$9.50 =$ **\$76.00** for 8 items.

(ii) 10 items at \$9.50 each $= \$95$.
$\$95 - 20\%$ of $\$95 = 95 - 0.2 \cdot 95$
$= \$95.00 - \$19.00 =$ **\$76** for 10 items.

(b) **10 items**. You get two more items for the same price.

20. The amount of John's 20% profit is $0.20 \cdot \$330 = \$66$. Sale price of the bike after a 10% discount must be $\$330 + \$66 = \$396$.

Let p be the list price; then $p - 10\%$ of p is the sale price. $p - 10\%$ of $p = \$396 \Rightarrow 0.9p = 396 \Rightarrow p = \frac{396}{0.9} = \$440$. Thus if John prices the bike at **\$440** he can offer a 10% discount of \$44 and still realize his \$66 profit.

21. Techniques may vary:

(a) 10% of $\$22 = \$2.20$. 5% of \$22 is half 10%, or \$1.10. $\$2.20 + \$1.10 =$ **\$3.30**.

(b) 10% of $\$120 = \$12$. 20% is twice 10%, or twice $\$12 =$ **\$24**.

(c) 10% of $\$38 = \$3.80$. 5% is half of 10%, or half of $\$3.80 =$ **\$1.90**.

(d) 25% is $\frac{1}{4}$; $\frac{1}{4}$ of $\$98 = \$98 \div 4$, or **\$24.50**. (To divide 98 by 4, think of it as $100 \div 4 = 25$. Then subtract $2 \div 4 = 0.50$.)

22. A journeyman makes 200% of an apprentice's pay and a master makes 150% of a journeyman's pay. 150% of $200\% = 1.5 \cdot 2 = 3 = 300\%$ of an apprentice's pay.

The \$4200 must have $1 + 2 + 3 = 6$ shares, or $\frac{\$4200}{6} = \$700$ per share. The **apprentice earns \$700**. The **journeyman** earns 200% of $\$700 = 2 \cdot \$700 =$ **\$1400**. The **master** earns 300% of $\$700 = 3 \cdot \$700 =$ **\$2100**.

23. **(a)** $\frac{20 \text{ math majors}}{500 \text{ incoming students}} = \frac{4}{100} =$ **4%** math majors.

(b) ***(i)*** There were 480 non-math-major students originally. 5% of $480 = 0.05 \cdot 480 = 24$ who switched. 20 original + 24 switches = **44** math majors now.

(ii) $\frac{44 \text{ math majors}}{500 \text{ students}} = 0.088 = 100\left(\frac{0.088}{100}\right) = \frac{8.8}{100} =$ **8.8%** math majors.

24. Let s be the salary of the previous year. 100% of s + 10% of s = 110% of $s = 1.1 \cdot s$ is current salary. $1.1s = \$100{,}000$ (this year), so $s = \frac{\$100.000}{1.1} \approx \$90{,}909.09$ (last year).

$1.1s = \$90{,}909.09$ (last year), so $s = \frac{\$90{,}909{,}09}{1.1}$, or about **\$82,644.63** (two years ago).

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25. **(a)** There are $815 \cdot 365 \cdot 24 \cdot 60^2 \approx$ 25.7 billion seconds in 815 years. \$100 per second means about \$2.57 trillion, so the report is essentially **true**.

(b) $100\left[\frac{\left(\frac{1}{815}\right)}{100}\right] \approx \mathbf{0.12\%}$ of the money each year.

26. **No**. Since 56% is more than double 25%, \$950 should be more than double \$500, but it is not.

27. **(a)** Since there is no quorum requirement stated, **3** members; i.e., the number of members required to obtain a majority vote; can make a change.

(b) The vote of two non-chair members, or $100\left[\frac{\left(\frac{2}{100,000}\right)}{100}\right] = \mathbf{0.002\%}$, can change the bylaws, even though 3 members including one chair, or 0.003%, must attend the meeting.

28. **(a)** 15% of $\$30 = 0.15 \cdot 30 = \mathbf{\$4.50}$.

(b) $100\left[\frac{\left(\frac{1}{2}\right)}{100}\right] = \mathbf{50\%}$.

(c) $100\left(\frac{1}{100}\right) = \mathbf{100\%}$.

29. According to the percent bar, 374 students represent 68% of the total. Let t be the total number of students. Then 68% of $t = 374 \Rightarrow 0.68t = 374 \Rightarrow t = \frac{374}{0.68} = \mathbf{550\ students}$.

30. **(a)** There are 9 single digit numbers in the numbers 1 through 50: 1, 2, 3, 4, 5, 6, 7, 8, and, 9. This is $\frac{9}{50} = 0.18$ or 18%. There are 100% - 18% = 82% two-digit numbers.

(b) The multiples of 9 are 9, 18, 27, 36, and 45. That is 5 out of 50 or 10%.

(c) Half of the numbers are even. That is 25 out of 50 or 50%.

(d) The prime numbers between 1 and 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. That is 15 out of 50 or 30%.

(e) The numbers that are even and prime is only 2. That is 1 out of 50 or 2%.

31. **(a)** $\frac{14}{15} = 0.9\overline{3} = 93\frac{1}{3}\%$.

(b) $80\% = \frac{80}{100} = \frac{4}{5}$ of the sun's UV rays are blocked.

(c) The SPF is 5.

(d) **Yes.** A sunblock with SPF 30 blocks $\frac{29}{30} = 0.96\overline{6} = 96\frac{2}{3}\%$. The label is correct.

32. In the table below:

(i) Interest rate per period is found by dividing the annual rate by the number of compounding periods per year.

(ii) Number of periods: Semiannually is twice per year, quarterly is four times per year, monthly is twelve times per year, daily is 365 times per year.

(iii) Amount of interest paid: Where

A = compound amount;
P = principal (\$1000 in this example);
i = interest rate per period expressed as a decimal;
n = number of periods per year;
I = compound interest;

then $A = P(1 + i)^n$ and $I = A - P$.

	Interest Rate per Period (i)	Number of Periods (n)	Amount of Interest Paid (I)	Compound Amount (A)
(a)	$\frac{6\%}{2} = 3\% = 0.03$	4	\$125.51	\$1125.51
(b)	$\frac{8\%}{4} = 2\% = 0.02$	12	\$268.24	\$1268.24
(c)	$\frac{10\%}{12} = 0.8\overline{3}\% = 0.008\overline{3}$	60	\$645.31	\$1645.31
(d)	$\frac{12\%}{365} = \frac{0.12}{365}$	1460	\$615.95	\$1615.95

33. Where I is interest, P is principal, r is rate expressed as a decimal, and t is time in years: $I = Prt = \$42,000 \cdot 0.0875 \cdot 1 = \mathbf{\$3675.00}$.

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34. Look for the principal to be invested.
If $A = P(1+i)^n$ then $P = \frac{A}{(1+i)^n}$, where $A = \$50,000$, $i = \frac{3\%}{4} = 0.75\% = 0.0075$, and $n = 5 \text{ years} \cdot 4 \text{ periods per year} = 20$.
$P = \frac{\$50,000}{(1+0.0075)^{20}} \approx \mathbf{\$43,059.49}$ rounded to the nearest penny.

35. $A = P(1+i)^n$ where $P = \$4000$, $i = \frac{5.9\%}{4} = 1.475\% = 0.01475$, $n = 20 \cdot 4 = 80$ periods.
$A = 4000(1+0.01475)^{80} \approx \mathbf{\$12,905.80}$, the value of the investment after 20 years.

36. \$300 compounded monthly at $1.1\% = 0.011$ per month for 11 months will yield:
$A = P(1+i)^n = \$300(1+0.011)^{11} \approx \$338.36$.
Interest earned is $A - P = \$338.36 - \$300 = \$38.36$. The *annual* rate, though, is based on annual investment, or $300(1+0.011)^{12} \approx \$342.09 \Rightarrow \$42.09$ interest earned. Interest as a percentage of investment would be $\frac{\$12.09}{\$300} = 0.1403 = \mathbf{14.03\%}$.

37. First three years: $P = \$3000$, $i = \frac{2\%}{4} = 0.005$, and $n = 4 \cdot 3 = 12$. $A = \$3000(1+0.005)^{12} \approx \$3185.03$.
Second three years: $P = \$3185.03$, $i = \frac{3\%}{4} = 0.0075$, and $n = 4 \cdot 3 = 12$.
$A = \$3185.03(1+0.0075)^{12} \approx \mathbf{\$3483.81}$ after six years.

38. Suppose \$1 is invested in each bank for 1 year.

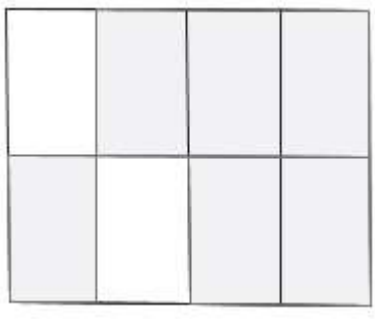
(i) At New Age, $i = \frac{0.04}{365}$ and $n = 365$.
$A = \$1\left(1 + \frac{0.04}{365}\right)^{365} \approx \$1.041$.
Subtracting the \$1 invested, interest is $4.1¢$ which corresponds to an effective annual rate of $\frac{4.1¢}{100¢} = 4.1\%$.

(ii) At Pay More, $i = \frac{0.052}{1}$ and $n = 1 \cdot 1 = 1$. $A = \$1(1+0.052)^1 = \$1.052$. Subtracting the \$1 invested, interest is $5.2¢$ which corresponds to an effective annual rate of 5.2%. The **Pay More bank offers the better rate**.

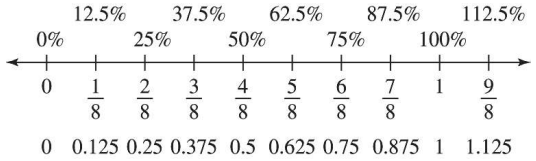
39. Observe that Amy's monthly interest is $12\% \div 12 = 1\%$ and that over two years, she has $12 \cdot 2 = 24$ compoundings. Thus, she owes $2000(1+0.01)^{24} = 2000(1.01)^{24} = \$2539.47$. The answer is **no**, the amount she owes has not doubled.

Assessment 7-4B: Percents and Interests

1. Cut the shape into equal size pieces. There are 8 rectangles and 6 are shaded. $\frac{6}{8} = 75\%$.



2.



3. **(a)** $0.032 = 100\left[\frac{0.032}{100}\right] = \frac{3.2}{100} = \mathbf{3.2\%}$.

(b) $0.2 = 100\left[\frac{0.2}{100}\right] = \frac{20}{100} = \mathbf{20\%}$.

(c) $\frac{3}{20} = 100\left[\frac{\left(\frac{3}{20}\right)}{100}\right] = \frac{\left(\frac{300}{20}\right)}{100} = \frac{15}{100} = \mathbf{15\%}$.

(d) $\frac{13}{8} = 100\left[\frac{\left(\frac{13}{8}\right)}{100}\right] = \frac{\left(\frac{1300}{8}\right)}{100} = \frac{162.5}{100} = \mathbf{162.5\%}$.

(e) $\frac{1}{6} = 100\left[\frac{\left(\frac{1}{6}\right)}{100}\right] = \frac{\left(\frac{100}{6}\right)}{100} = \frac{16.\overline{6}}{100} = \mathbf{16.6\%} = \mathbf{16\frac{2}{3}\%}$.

(f) $\frac{1}{40} = 100\left[\frac{\left(\frac{1}{40}\right)}{100}\right] = \frac{\left(\frac{100}{40}\right)}{100} = \frac{2.5}{100} = \mathbf{2.5\%}$.

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4. **(a)** $4\frac{1}{2}\% = 4.5\% = \frac{4.5}{100} = \frac{45}{1000} = \mathbf{0.045}$.

(b) $\frac{2}{7}\% = 0.\overline{285714}\% = \frac{0.\overline{285714}}{100}$
$= \mathbf{0.00\overline{285714}}$.

(c) $125\% = \frac{125}{100} = \mathbf{1.25}$.

(d) $\frac{1}{4}\% = 0.25\% = \frac{0.25}{100} = \mathbf{0.0025}$.

5. **(a)** $\underline{\mathbf{5}}$ for every 100. $5\% = \frac{5}{100}$.

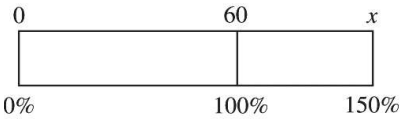
(b) $\underline{\mathbf{2.5}}$ for every 50. $5\% = \frac{5}{100} = \frac{2.5}{50}$.

(c) 1 for every $\underline{\mathbf{20}}$. $5\% = \frac{5}{100} = \frac{1}{20}$.

(d) 8 for every $\underline{\mathbf{160}}$. $5\% = \frac{5}{100} \cdot \frac{1.6}{1.6} = \frac{8}{160}$.

(e) 0.5 for every $\underline{\mathbf{10}}$. $5\% = \frac{5}{100} = \frac{0.5}{10}$.

6.



$$\frac{60}{100} = \frac{x}{150} \Rightarrow 100x = 9000 \Rightarrow x = 90$$

7. **(a)** 63 is 30% of $n \Rightarrow 63 = \frac{30}{100} \cdot n \Rightarrow 30n$
$= 6300 \Rightarrow n = \frac{6300}{30} = \mathbf{210}$.

(b) 7% of 150 $\Rightarrow \frac{7}{100} \cdot 150 = \frac{1050}{100} = \mathbf{10.5}$.

(c) 61.5 is n% of 20.5 $\Rightarrow 61.5 = \frac{n}{100} \cdot 20.5$
$\Rightarrow 20.5n = 6150 \Rightarrow n = \frac{6150}{20.5} = 300$,
or 61.5 is **300%** of 20.5.

(d) 16 is 40% of $n \Rightarrow 16 = \frac{40}{100} \cdot n \Rightarrow 40n =$
$1600 \Rightarrow n = \frac{1600}{40} = \mathbf{40}$.

8. **(a)** $.005x = \frac{5}{1000}x = \frac{x}{\mathbf{200}}$.

(b) One percent of x (an amount) is $0.001x$. If this equals a, then $0.001x = a \Rightarrow x = \frac{a}{0.001} \Rightarrow x = \mathbf{1000}\boldsymbol{a}$.

9. The amount of the discount is 25% of \$6.80, or $0.25 \cdot \$6.80 = \$1.70$. The sale price is $\$6.80 - \$1.70 = \mathbf{\$5.10}$.

10. Let x represent her previous salary.
$x + .1x = 60{,}000 \Rightarrow 1.1x = 60{,}000 \Rightarrow$
$x = \frac{60{,}000}{1.1} \approx \$54{,}545.45$.

11. A line segment 3 in long would be 50%, or half of, a 6 in segment.

(a) 100% of $6 = \frac{100}{100} \cdot 6 = \mathbf{6\ in}$.

(b) 25% of $6 = \frac{25}{100} \cdot 6 = \mathbf{1\frac{1}{2}\ in}$.

(c) 150% of $6 = \frac{150}{100} \cdot 6 = \mathbf{9\ in}$.

12. The amount of depreciation was $\$1700 - \$1400 = \$300$. The depreciation as a percentage of \$1700 is given by $\frac{300}{1700} \approx 0.1765 =$
$100\left[\frac{0.1765}{100}\right] = \frac{17.65}{100} = \mathbf{17.65\%}$.

13. The number of eagles by which the population decreased was $728 - 594 = 134$. The decrease as a percentage of the original population was $\frac{134}{728} \approx 0.184 = 100\left(\frac{0.184}{100}\right) = \frac{18.4}{100}$, or **about 18.4%**.

14. Let t be the total income for the month. Then $t = \$900 + 4\%$ of $\$1800 = 900 + \frac{4}{100} \cdot 1800$
$= \mathbf{\$972}$.

15. Let t be the total bill. Then $t = \$380 + 9\%$ of $\$380 \Rightarrow t = 380 + \frac{9}{100} \cdot 380 = \mathbf{\$414.20}$.

16. $\frac{4500 \text{ mammals}}{1{,}700{,}000 \text{ species}} \approx 0.0026 = 100\left(\frac{0.0026}{100}\right)$
$= \frac{0.26}{100}$, or **about 0.26%**.

17. Shirt a cost $\$40 - 20\%$ of $\$40 = 40 - \frac{20}{100} \cdot 40$
$= \$32$. Shirt b cost $\$40 - 25\%$ of $\$40 =$
$40 - \frac{25}{100} \cdot 40 - \$30$. Sales tax was 4.5% of the total, or $\frac{4.5}{100}(32 + 30) = \$2.79$. Jim's total amount spent was thus $\$32.00 + \$30.00 +$
$\$2.79 = \mathbf{\$64.79}$.

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18. **0.625%** is greater. $0.625\% = \frac{62.5}{100}$. On the other hand $(0.625\%)^2 = \frac{62.5}{100} \cdot \frac{62.5}{100} = \frac{62.5 \cdot \frac{62.5}{100}}{100} = \frac{.625(62.5)}{100}$. Notice that 62.5 is multiplied by a number less than one and thus, it decreases.

19. **(a)** 30% of $2400 = \frac{30}{100} \cdot 2400 = 720$. No more than **720 calories** should be from fat.

(b) **Yes**. Three cookies would have 210 calories from fat, $720 - 210 = 510$ calories less than the recommended maximum.

20. The reduced price of the suit is $\mathbf{200 - .25(200) = 150}$. Let x be the percent increase. Then $150 + x(150) = 200 \Rightarrow x = \frac{50}{150} \Rightarrow x = \frac{1}{3} = \mathbf{33\frac{1}{3}\%}$.

21. Techniques may vary:

(a) 10% of \$42 = \$4.20. 5% of \$42 is half of 10%, or \$2.10. \$4.20 + \$2.10 = **\$6.30**.

(b) 10% of \$280 = \$28. 20% is twice 10%, or twice \$28 = **\$56**.

(c) 10% of \$28 = \$2.80. 5% is half of 10%, or half of \$2.80 = **\$1.40**.

(d) 25% is $\frac{1}{4}$; $\frac{1}{4}$ of \$84 = 84 ÷ 4, or **\$21**.

22. Set up a proportion: $\frac{\frac{1}{4}\text{cup}}{x\text{ cups}} = \frac{0.5\%}{100\%} \Rightarrow \frac{0.25\text{ cup}}{x\text{ cups}} = \frac{0.005}{1} \Rightarrow x \cdot 0.005 = 0.25 \cdot 1 \Rightarrow x = \frac{0.25}{0.005}$. $x = \mathbf{50\text{ cups}}$.

23. 12.13% of price = \$1116.88, or price = $\frac{1116.88}{0.1213}$, or **about \$9207.58** or \$9208 to the nearest dollar.

24. Let n be the original number and let N be 20% more than $n \Rightarrow 120\%$ of $n = 1.2n$. Then n as a percentage of N is $\frac{n}{N} = \frac{n}{1.2n} = \frac{1}{1.2} = 0.83\frac{1}{3}$; i.e., $n = 83\frac{1}{3}\%$ of N. 100% of $N - 83\frac{1}{3}\%$ of $N = \mathbf{16\frac{2}{3}\%}$ of N.

25. **(a)** Only the four corner blocks would have four faces painted, so there would be $\frac{4}{100} = \mathbf{4\%}$ painted.

(b) The blocks along each edge, exclusive of the corner blocks, would have three faces painted. There are eight of these along each edge, or 32 blocks. Thus there would be $\frac{32}{100} = \mathbf{32\%}$ painted.

(c) Only the interior blocks would have two faces painted. There are $100 - 4 - 32 = 64$ of these, so there would be $\frac{64}{100} = \mathbf{64\%}$ painted. Alternatively, the interior can be seen as an 8 by 8 square, which has 64 blocks.

26. If 2% milk has 5g of fat per cup and whole milk has 8g of fat per cup then the reduction is 3 g of fat. $\frac{3}{8} = .375 = 37.5\%$ The reduction is 37.5%.

27. 20% of 80 coins = $\frac{20}{100} \cdot 80 = 16$ quarters, worth $16 \cdot 25 = 400¢$. If the remaining $80 - 16 = 64$ coins are all pennies (the least possible amount), there would be $400¢ + 64¢ = 464¢$, or **\$4.64** in the piggy bank.

28. According to the percent bar, 100% of the population is 302 million. 46% of 302 million = $\frac{46}{100} \cdot 302$ million = 138.92 million ≈ **139 million**.

29. **(a)** There are 9 single digit numbers in the numbers 1 through 50: 1, 2, 3, 4, 5, 6, 7, 8, and, 9. This is $\frac{9}{50} = 0.18$ or 18%. So 82% of the numbers are greater than 9.

(b) The factors of 12 are 1, 2, 3, 4, 6, and 12. That is 6 out of 50 or 12%.

(c) The numbers with even digits are 2, 4, 6, 8, 20, 22, 24, 26, 28, 40, 42, 44, 46, and 48. That is 14 out of 50 or 28%.

(d) The numbers with odd digits are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 31, 33, 35, 37, and 39. That is 15 out of 50 or 30%.

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30. **(a)** 20% of \$30 = .20(30) = \$6 . The clock increased in value by \$6.

(b) The clock is now worth \$30 + \$ 6 = \$36.

(c) \$30 + (.20)\$30 = (1.20)\$30 = \$36 . The original price times 1.20 would be the way to calculate part (b) in one step.

31. In the table below:

(i) Interest rate per period is found by dividing the annual rate by the number of compounding periods per year.

(ii) Number of periods: Semiannually is twice per year, quarterly is four times per year, monthly is twelve times per year, daily is 365 times per year.

(iii) Amount of interest paid: Where

A = compound amount;
P = principal (\$1000 in this example);
i = interest rate per period expressed as a decimal;
n = number of periods per year;
I = compound interest;

then $A = P(1 + i)^n$ and $I = A - P$.

	Interest Rate per Period (i)	Number of Periods (n)	Amount of Interest Paid (I)	Compound Amount (A)
(a)	$\frac{4\%}{2} = 2\% = 0.02$	4	\$82.43	\$1082.43
(b)	$\frac{6\%}{4} = 1.5\% = 0.015$	12	\$195.62	\$1195.62
(c)	$\frac{18\%}{12} = 1.5\% = 0.015$	60	\$1443.22	\$2443.22
(d)	$\frac{18\%}{365} = \frac{0.18}{365}$	1460	\$1054.07	\$2054.07

32. Interest charged was $\$28,500 - \$25,000 = \$3500$. If $I = Prt$ then $r = \frac{I}{Pt} = \frac{\$3500}{\$25{,}000.4} = 0.035 =$ **3.5%**.

33. An interest rate of 3% annually is a rate of $\frac{3}{4}\%$ each quarter. Over 5 years, there will be $5 \cdot 4 = 20$ compoundings.

$100{,}000 = P(1 + .0075)^{20} \Rightarrow 100{,}000 = P(1.161184142) \Rightarrow P \approx$ **\$86,119**.

34. $A = P(1 + i)^n \Rightarrow P = \frac{A}{(1+i)^n}$, where $A = \$4650, i = \frac{3.5\%}{4} = 0.875\% = 0.00875,$ and $n = 4 \cdot 4 = 16$ periods. $P = \frac{\$4650}{(1+0.00875)^{16}} \approx$ **\$4044.98**.

35. \$10,000 compounded daily at 2% annual interest for 15 years is calculated as $A = \$10{,}000\left(1 + \frac{0.02}{365}\right)^{365 \cdot 15} \approx$ **\$13,498.48**.

36. $(1 + 0.005)^{12} - 1 \approx$ **43% per year**.

37. The expression for compound growth, where the growth rate is 10% compounded annually, the end amount is 1000, the beginning amount is 500, and the number of editions is n gives:

$1000 = 500(1 + 0.10)^n$

$\frac{1000}{500} = 2 = 1.1^n$

By trial and error, 1.1^8 is slightly more than 2, so only **7 editions** could be published if the limit is 1000 pages. $500(1 + 0.10)^7 =$ would result in about 974 pages.

38. Al's yearly return is given by $P\left(1 + \frac{.06}{365}\right)^{365}$. Betty's is given by $1000(1 + .07)$.

	Al's Investment	Betty's Investment
Year 1	1061.83	1070
Year 2	1127.49	1140
Year 3	1197.20	1210
Year 4	1271.22	1280
Year 5	1349.83	1350
Year 6	1433.29	1420

39. To develop a formula for compound decay, let A be the amount remaining after one period, r be the rate of decay, and B be the beginning amount. $A = B -$ decay $= B - rB = B(1 - r)$. After n periods, $A = B(1 - r)^n$.

In the rain forest, where $B = 2.34 \cdot 10^9$, $r = 0.5\% = 0.005$, and $n = 12 \cdot 20 = 240$, $A = 2.34 \cdot 10^9(1 - 0.005)^{240} \approx$ **$7.03 \cdot 10^8$ trees** remaining after 20 years.

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Mathematical Connections 7-4: Review Problems

31. **(a)** $0.18(120) = \mathbf{21.6\ lb}$.

(b) $0.4(120) = \mathbf{48\ lb}$.

32. **(a)** $16.72 = \frac{16.72 \cdot 100}{100} = \frac{1672}{100} = \mathbf{\frac{418}{25}}$.

(b) $0.003 = \mathbf{\frac{3}{1000}}$.

(c) $^{-}5.07 = \mathbf{\frac{^{-}507}{100}}$.

(d) $0.123 = \frac{.123 \cdot 1000}{1000} = \mathbf{\frac{123}{1000}}$.

33. **(a)** $5 = 4 + 1 = 4 + .\overline{9} = \mathbf{4.\overline{9}}$.

(b) $5.1 = 5 + .1 = 5 + \frac{1}{10}.\overline{9} = \mathbf{5.0\overline{9}}$.

(c) $\frac{1}{2} = .5 = 0.\mathbf{4\overline{9}}$.

34. $0.00024 = \frac{0.00024 \cdot 100{,}000}{100{,}000} = \frac{24}{100{,}000}$

$= \frac{2^3 \cdot 3}{10^5} = \frac{2^3 \cdot 3}{2^5 \cdot 5^5} = \frac{3}{2^2 5^5} = \mathbf{\frac{3}{12500}}$.

35. $0.\overline{24} = \frac{24}{99} = \frac{8.3}{3.33} = \mathbf{\frac{8}{33}}$.

36. **(a)** $2.08 \cdot 10^5 = \mathbf{208{,}000}$.

(b) $3.8 \cdot 10^{-4} = \mathbf{0.00038}$.

Chapter 7 Review

1. Each division on the number line corresponds to 0.01.

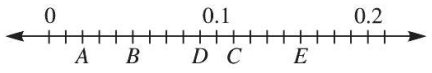
(a) ***(i)*** Point A is two divisions to the right of point 0, or **0.02**.

(ii) Point B is five divisions to the right of point 0, or **0.05**.

(iii) Point C is one division to the right of point 0.1, or **0.11**.

(b) ***(i)*** Point D (0.09) is one division to the left of point 0.1; i.e., $0.1 - 0.01 = 0.09$.

(ii) Point E (0.15) is five divisions to the right of point 0.1; i.e., $0.1 + 0.05 = 0.15$.



2. **(a)** $32.012 = 32\frac{12}{1000} = \frac{32.012}{1000} = \mathbf{\frac{8003}{250}}$.

(b) $0.00103 = \mathbf{\frac{103}{100{,}000}}$.

3. A fraction in simplest from, $\frac{a}{b}$, can be written as a terminating decimal if and only if the prime factorization of the denominator contains no primes other than 2 or 5. If only 2's and 5's are present, then a power of 10 can be obtained using the Fundamental Law of Fractions (which shows that the fraction can be written directly as a terminating decimal to the same number of decimal places as the smallest power of 10 in the denominator).

4. 442.4 cm ÷ 55.3 cm = **8 shelves**.

5. **(a)** $\frac{4}{7} = 4 \div 7 = \mathbf{0.\overline{571428}}$.

(b) $\frac{1}{8} = 1 \div 8 = \mathbf{0.125}$.

(c) $\frac{2}{3} = 2 \div 3 = \mathbf{0.\overline{6}}$.

(d) $\frac{5}{8} = 5 \div 8 = \mathbf{0.625}$.

6. **(a)** $0.28 = \frac{28}{100} = \mathbf{\frac{7}{25}}$.

(b) $^{-}6.07 = ^{-}6\frac{7}{100} = \mathbf{\frac{^{-}607}{100}}$.

(c) $0.\overline{3} = \frac{3}{9} = \mathbf{\frac{1}{3}}$.

(d) $2.0\overline{8} = 2\frac{80}{900} = 2\frac{4}{45} = \mathbf{\frac{94}{45}}$.

7. **(a)** **307.63**. $307.6\underline{2}5$ is between 307.62 and 307.63; when the number to be rounded is at the mid-point, by convention it is commonly rounded up.

(b) **307.6**. $307.\underline{6}25$ is between 307.6 and 307.7 and is closer to 307.6.

(c) **308**. $30\underline{7}.625$ is between 307 and 308 and is closer to 308.

(d) **300**. $\underline{3}07.625$ is between 300 and 400 and is closer to 300.

8. **(a)** Move the decimal point five units to the left and multiply by 10^5, or $\mathbf{4.26 \cdot 10^5}$.

(b) $324 \cdot 10^{-6} = 3.24 \cdot 10^2 \cdot 10^{-6} =$ $\mathbf{3.24 \cdot 10^{-4}}$.

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(c) Move the decimal point six units to the right and multiply by 10^{-6}, or $0.00000237 = \mathbf{2.37 \cdot 10^{-6}}$.

(d) $^{-}0.325 = \mathbf{^{-}3.25 \cdot 10^{-1}}$.

9. Line up the decimal points:

$$1.4\overline{519} = 1.4519519519...$$
$$1.451\overline{9} = 1.4519999999...$$
$$1.4519 = 1.4519000000...$$
$$1.45\overline{19} = 1.4519191919...$$
$$^{-}0.134 = {}^{-}0.1340000000...$$
$$^{-}0.13\overline{401} = {}^{-}0.1340140140...$$
$$0.13\overline{401} = 0.1340140140...$$

Ordering from greatest to least:

$\mathbf{1.451\overline{9} > 1.4\overline{519} > 1.45\overline{19} > 1.4519 > 0.13\overline{401} > {}^{-}0.134 > {}^{-}0.13\overline{401}}$.

10. **(a)** Move the decimal point six units to the right and multiply by 10^6, or $\mathbf{1.78341156 \cdot 10^6}$.

(b) $\frac{347}{10^8} = 347 \cdot 10^{-8} = 3.47 \cdot 10^2 \cdot 10^{-8} = \mathbf{3.47 \cdot 10^{-6}}$.

(c) $49.3 \cdot 10^8 = 4.93 \cdot 10^1 \cdot 10^8 = \mathbf{4.93 \cdot 10^9}$.

(d) $29.4 \cdot \frac{10^{12}}{10^1} = 2.94 \cdot 10^1 \cdot 10^{12} \cdot 10^4 = \mathbf{2.94 \cdot 10^{17}}$.

(e) $0.47 \cdot 1000^{12} = 4.7 \cdot 10^{-1} \cdot (10^3)^{12} = 4.7 \cdot 10^{-1} \cdot 10^{36} = \mathbf{4.7 \cdot 10^{35}}$.

(f) $\frac{3}{5^9} = \frac{3 \cdot 2^9}{5^9 \cdot 2^9} = \frac{1536}{10^9} = 1536 \cdot 10^{-9} = 1.536 \cdot 10^3 \cdot 10^{-9} = \mathbf{1.536 \cdot 10^{-6}}$.

11. **(a)** Answers may vary. E.g., $0.11 > 0.105 > 0.104 > 0.103 > 0.102 > 0.101 > 0.1$.

(b) Begin with 0.1 and find half of each succeeding element, or $0 < 0.00625 < 0.0125 < 0.025 < 0.05 < 0.1$.

(c) $0.1 < 0.15 < 0.175 < 0.1875 < 0.19375 < 0.2$.

12. **(a)** $6 = n\%$ of $24 \Rightarrow n\% = \frac{6}{24}$ $\Rightarrow \frac{n}{100} = \frac{1}{4} \Rightarrow 4n = 100 \Rightarrow n = 25$, or $6 = \mathbf{25\%}$ of 24.

(b) $n = 320\%$ of $60 \Rightarrow n = \frac{320}{100} \cdot 60 = \mathbf{192}$.

(c) $17 = 30\%$ of $n \Rightarrow n = \frac{17}{\left(\frac{30}{100}\right)} = \frac{1700}{30} = \mathbf{56.\overline{6}}$.

(d) $0.2 = n\%$ of $1 \Rightarrow \frac{n}{100} = \frac{0.2}{1} \Rightarrow n = 20$, or $0.2 = \mathbf{20\%}$ of 1.

13. **(a)** $\frac{1}{8} = 0.125 = 100\left(\frac{0.125}{100}\right) = \frac{12.5}{100} = \mathbf{12.5\%}$.

(b) $\frac{3}{40} = 0.075 = 100\left(\frac{0.075}{100}\right) = \frac{7.5}{100} = \mathbf{7.5\%}$.

(c) $6.27 = 100\left(\frac{6.27}{100}\right) = \frac{627}{100} = \mathbf{627\%}$.

(d) $0.0123 = 100\left(\frac{0.0123}{100}\right) = \frac{1.23}{100} = \mathbf{1.23\%}$.

(e) $\frac{3}{2} = 1.5 = 100\left(\frac{1.5}{100}\right) = \frac{150}{100} = \mathbf{150\%}$.

14. **(a)** $60\% = \frac{60}{10} = \mathbf{0.60}$.

(b) $\frac{2}{3}\% = \frac{\left(\frac{2}{3}\right)}{100} = \frac{2}{300} = \mathbf{0.00\overline{6}}$.

(c) $100\% = \frac{100}{100} = \mathbf{1}$.

15. $11\% \cdot \text{investment} = \$1020.80$ $\Rightarrow \text{investment} = \frac{\$1020.80}{0.11} = \mathbf{\$9280}$.

16. Percent defective $= 100\left[\frac{\left(\frac{5}{150}\right)}{100}\right] = \frac{\left(\frac{500}{150}\right)}{100} = \frac{3.\overline{3}}{100} = \mathbf{3.\overline{3}\%}$ or $\mathbf{3\frac{1}{3}\%}$.

17. Assume "nearest tenth" refers to the nearest tenth of a percent. Then percent correct $= 100\left(\frac{\frac{62}{70}}{100}\right) = \frac{\left(\frac{6200}{70}\right)}{100} \approx \frac{88.6}{100} = \mathbf{88.6\%}$.

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18. Let C be the cost four years ago. $60\% \cdot C = \$3450 \Rightarrow C = \frac{\$3450}{0.60} = \mathbf{\$5750}$.

19. A discount of $d\%$ means the customer pays $1 - \frac{d}{100}$ for the purchase. Discounts of 5%, 10%, and 20% mean the customer pays 0.95, 0.90, and 0.80, respectively, of cost. Their product, 0.648, is the same in any order, therefore there is **no difference**. The customer would pay 0.684 times the purchase price, for a discount of $1 - 0.684 = 0.316$, or 31.6%.

20. Let c be the cost of the bicycle. 100% of $c + 30\%$ of $c = \$104 \Rightarrow c + 0.3c = \$104 \Rightarrow 1.3c = \$104$. $c = \frac{\$104}{1.3} = \mathbf{\$80}$.

21. The price difference was $\$89.95 - \$62.00 = \$27.95$. The percentage difference was $100\left[\frac{\left(\frac{27.95}{89.95}\right)}{100}\right] \approx \mathbf{31.1\%}$.

22. Answers may vary. If the dress was originally priced at \$100, 60% off would result in a sale price of \$40. Then the 40% off coupon would result in a final price of $\$40 - (0.40 \cdot \$40) = \$24$. The reasoning could be applied to the actual list price of the dress.

23. If the item is \$100 then the markup is \$130. The sales price is then $\$130 - (.30)\$130 = \$91$ which is \$9 less than the whole sale price. The store would be losing 9%.

24. $I = Prt$, where $P = \$30,000$, $r = 12.5\% = 0.125$, and $t = 4$ years. $I = \$30,000 \cdot 0.125 \cdot 4 = \mathbf{\$15,000}$.

25. \$10,000 compounded quarterly at 14% annual interest for 3 years will, where $A = P(1+i)^n$, yield $A = \$10,000(1+\frac{0.14}{4})^{3\cdot 4} \approx \mathbf{\$15,110.69.}$

26. To reduce the crust from 25% to 20% the crust is decreased by 5% which is 25% of 20%. So it should be reduced by 25%.

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CHAPTER 8

Real Numbers and Algebraic Thinking

Assessment 8-1A: Real Numbers

1. Answers may vary. E.g., one such number could be 0.232233222333…, continuing the pattern of adding a 2 and a 3 to each succeeding group.

2. **(a)** $x^2 = 2^2 + (\sqrt{2})^2 = 4 + 2 = 6.$
$x = \sqrt{\mathbf{6}}.$

(b) $x^2 = (2)^2 + (2)^2 = 4 + 4 = 8.$
$x = \sqrt{8} = 2\sqrt{2}.$

(c) $5^2 = 2^2 + x^2 \Rightarrow x^2 = 5^2 - 2^2$
$= 25 - 4 = 21$
$x = \sqrt{21}.$

3. Line up the decimal points:

$0.9 = 0.90000000\ldots$
$0.\overline{9} = 0.99999999\ldots$
$0.\overline{98} = 0.98989898\ldots$
$0.9\overline{88} = 0.98888888\ldots$
$0.9\overline{98} = 0.99898989\ldots$
$0.\overline{898} = 0.89889889\ldots$
$\sqrt{0.98} = 0.98994949\ldots$

Ordering from greatest to least:

$0.\overline{9} > 0.9\overline{98} > \sqrt{0.98} > 0.\overline{98} > 0.9\overline{88} > 0.9 > 0.\overline{898}.$

4. **(a)** **Irrational**. There is no rational number s such that $s^2 = 51$.

(b) **Rational**. $8^2 = 64$.

(c) **Rational**. $18^2 = 324$.

(d) **Irrational**. There is no rational number s such that $s^2 = 325$.

(e) **Irrational**. The sum of a rational number and an irrational number is irrational.

(f) **Irrational**. The quotient of any non-zero rational number and any irrational number is irrational.

5. **(a)** $15 \cdot 15 = 225 \Rightarrow \sqrt{225} = \mathbf{15}$.

(b) $13 \cdot 13 = 169 \Rightarrow \sqrt{169} = \mathbf{13}$.

(c) $^-1 \cdot 9 \cdot 9 = {}^-81 \Rightarrow {}^-\sqrt{81} = {}^-9$.

(d) $25 \cdot 25 = 625 \Rightarrow \sqrt{625} = \mathbf{25}$.

(e) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \Rightarrow \sqrt{\frac{1}{4}} = \frac{1}{2}$.

(f) $0.01 \cdot 0.01 = 0.0001 \Rightarrow \sqrt{0.0001} = 0.01$.

6. **(a)** $2 < \sqrt{7} < 3$.
$(2.6)^2 = 6.76$ and $(2.7)^2 = 7.29$
$\Rightarrow 2.6 < \sqrt{7} < 2.7$.
$(2.64)^2 = 6.9696$ and $(2.65)^2 = 7.0225$
$\Rightarrow 2.64 < \sqrt{7} < 2.65$.
7 is closer to 7.0225 than to 6.9696 $\Rightarrow$
$\sqrt{7} \approx \mathbf{2.65}$.

(b) $0.1 < \sqrt{0.0120} < 0.2$.
$(0.10)^2 = 0.0100$ and $(0.11)^2 = 0.0121$
$\Rightarrow 0.10 < \sqrt{0.0120} < 0.11$.
0.0120 is closer to 0.0121 than to 0.0100
$\Rightarrow \sqrt{0.0120} \approx \mathbf{0.11}$.

7. Answers may vary.

(a) **False**. $0 + \sqrt{2}$ is irrational.

(b) **False**. $^-\sqrt{2} + \sqrt{2} = 0$, which is rational.

(c) **False**. $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational.

(d) **True**. $\sqrt{2} - \sqrt{2} = 0,$ which is rational.

8. Answers may vary.

(a) $\sqrt{2}, \sqrt{3}$, and $\sqrt{5}$ are three such irrational numbers.

(b) $0.\overline{54} = 0.54545454\ldots$
$< 0.546010010001...$
$< 0.547010010001...$
$< 0.548010010001...$
$< 0.\overline{55} = 0.55555555\ldots.$

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(c) $\frac{1}{3} = 0.\overline{3} = 0.333....$

$< 0.34010010001\ldots$

$< 0.35010010001...$

$< 0.36010010001... < 0.5.$

9. Consider 8(c). We form an irrational number between $0.\overline{3}$ and 0.5 by placing digits in lesser place-values so that the decimal does not terminate or repeat, e.g., 0.3434434443…. A similar approach can be taken for any two rational numbers. For example, 1.7171171117… is between 1.7 and 1.8.

10. **(a)** $Q \cup S = \boldsymbol{R}$. The set of real numbers contains any number which is either rational or irrational.

(b) $Q \cap S = \varnothing$. A rational number cannot be an irrational number.

(c) $Q \cap R = \boldsymbol{Q}$. $Q \subset R$.

(d) $S \cap W = \varnothing$. No whole number can be irrational.

(e) $W \cup R = \boldsymbol{R}$. $W \subset R$.

(f) $Q \cup R = \boldsymbol{R}$. $Q \subset R$.

11. In the tables of 11. and 12. below: N is the set of natural (or counting) numbers; I is the set of integers; Q is the set of rational numbers; R is the set of real numbers; and S is the set of irrational numbers. $N \subset I \subset Q \subset R; R = Q \cup S$.

		N	I	Q	S	R
(a)	6.7			√		√
(b)	5	√	√	√		√
(c)	$\sqrt{2}$				√	√
(d)	$^{-}5$		√	√		√
(e)	$3\frac{1}{7}$			√		√

12.

		$x =$	N	I	Q	S	R
(a)	$x^2 + 1 = 5$	$2, {}^{-}2$	√	√	√		√
(b)	$2x - 1 = 32$	$\frac{33}{2}$			√		√
(c)	$x^2 = 3$	$\sqrt{3}, {}^{-}\sqrt{3}$				√	√
(d)	$\sqrt{x} = {}^{-}1$	no solution					
(e)	$\frac{3}{4}x = 0.\overline{4}$	$\frac{16}{27}$			√		√

13. **(a)** $x = \mathbf{64}$. $\sqrt{64} = 8$.

(b) **No real values**. $\sqrt{x}$ is the principal square root of x.

(c) $x = {}^{-}\mathbf{64}$. $\sqrt{-(-64)} = \sqrt{64} = 8$.

(d) **No real values**. $\sqrt{-x}$ is the principal square root of x, if $x < 0$.

(e) All real numbers > 0. If $x = 0$ then $\sqrt{x} \not> 0$.

(f) **No real values**. $\sqrt{x}$ is the principal square root of x.

14. **(a)** $\sqrt{180} = \sqrt{36 \cdot 5} = \sqrt{36} \cdot \sqrt{5} = \mathbf{6\sqrt{5}}$.

(b) $\sqrt{363} = \sqrt{121 \cdot 3} = \sqrt{121} \cdot \sqrt{3} = \mathbf{11\sqrt{3}}$.

(c) $\sqrt{252} = \sqrt{36 \cdot 7} = \sqrt{36} \cdot \sqrt{7} = \mathbf{6\sqrt{7}}$.

15. **(a)** $\sqrt[3]{-54} = \sqrt[3]{-27 \cdot 2} = \sqrt[3]{-27} \cdot \sqrt[3]{2} = {}^{-}\mathbf{3}\sqrt[3]{\mathbf{2}}$.

(b) $\sqrt[5]{96} = \sqrt[5]{32 \cdot 3} = \mathbf{2}\sqrt[5]{\mathbf{3}}$.

(c) $\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = \mathbf{5}\sqrt[3]{\mathbf{2}}$.

(d) $\sqrt[5]{-243} = {}^{-}\mathbf{3}\sqrt[5]{\mathbf{1}} = {}^{-}\mathbf{3} \cdot \mathbf{1} = {}^{-}\mathbf{3}$.

16. **(a)** $a_1 = 5; a_4 = 10; n = 4$. Thus $10 = 5(r)^{4-1}$ $\Rightarrow 2 = r^3 \Rightarrow r = \sqrt[3]{2}$. The terms of the sequence are $5, \underline{\mathbf{5}\sqrt[3]{\mathbf{2}}}, \underline{\mathbf{5}\sqrt[3]{\mathbf{4}}}, 5\sqrt[3]{8} = 10$.

(b) $a_1 = 2; a_5 = 1; n = 5$. Thus $1 = 2(r)^{5-1}$ $\Rightarrow r = \sqrt[4]{\frac{1}{2}}$ or $r = {}^{-}\sqrt[4]{\frac{1}{2}}$. The terms of the

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sequence are then $2 = \sqrt[4]{16}, \underline{\sqrt[4]{8}}, \underline{\sqrt[4]{4}}, \underline{\sqrt[4]{2}},$
$\sqrt[4]{1} = 1$ or $2 = \sqrt[4]{16}, \underline{-\sqrt[4]{8}}, \underline{\sqrt[4]{4}}, -\sqrt[4]{2}, \sqrt[4]{1} = 1.$

(c) $a_2 = 5$ and $a_4 = 3$ Thus $5 = a_1(r)^{2-1}$ and $3 = a_1(r)^{4-1}$, so $5 = a_1 r$ and $3 = a_1 r^3$ Thus

$a_1 = \frac{5}{r}$ and $a_1 = \frac{3}{r^3} \Rightarrow \frac{5}{r} = \frac{3}{r^3} \Rightarrow$

$5r^3 = 3r \Rightarrow 5r^3 - 3r = 0 \Rightarrow r(5r^2 - 3) = 0$

$\Rightarrow r = 0$ and $5r^2 - 3 = 0 \Rightarrow 5r^2 = 3$

$\Rightarrow r^2 = \frac{3}{5} \Rightarrow r = \pm\sqrt{\frac{3}{5}} \Rightarrow r = \pm\frac{\sqrt{15}}{5}$

The terms of the sequence are

$\frac{5\sqrt{15}}{3}, 5, \sqrt{15}, 3$ or $-\frac{5\sqrt{15}}{3}, 5, -\sqrt{15}, 3$

(d) $a_2 = -2$ and $a_4 = -3$ Thus

$-2 = a_1(r)^{2-1}$ and $-3 = a_1(r)^{4-1}$, so $-2 = a_1 r$ and $-3 = a_1 r^3$ Thus

$a_1 = \frac{-2}{r}$ and $a_1 = \frac{-3}{r^3} \Rightarrow \frac{-2}{r} = \frac{-3}{r^3} \Rightarrow -2r^3 = -3r$

$\Rightarrow -2r^3 + 3r = 0 \Rightarrow -r(2r^2 - 3) = 0 \Rightarrow -r = 0$

and $2r^2 - 3 = 0 \Rightarrow 2r^2 = 3 \Rightarrow r^2 = \frac{3}{2}$

$\Rightarrow r = \pm\sqrt{\frac{3}{2}} \Rightarrow r = \pm\frac{\sqrt{6}}{2}$

The terms of the sequence when $r = \frac{\sqrt{6}}{2}$ are

$-\frac{2\sqrt{6}}{3}, -2, -\sqrt{6}, -3.$

The terms of the sequence when $r = -\frac{\sqrt{6}}{2}$ are

$\frac{2\sqrt{6}}{3}, -2, \sqrt{6}, -3.$

17. (a) $E(0) = 2^{10} \cdot 16^0 = \mathbf{2^{10}}$ **bacteria**.

(b) $E\left(\frac{1}{4}\right) = 2^{10} \cdot 16^{1/4} = 2^{10} \cdot 2$

$= \mathbf{2^{11}}$ **bacteria**.

(c) $E\left(\frac{1}{2}\right) = 2^{10} \cdot 16^{1/2} = 2^{10} \cdot 2^2$

$= \mathbf{2^{12}}$ **bacteria**.

18. (a) $(0.008)^{\frac{2}{3}} = (\frac{8}{1000})^{\frac{2}{3}} = (\frac{2^3}{10^3})^{\frac{2}{3}}$

$\frac{(2^3)^{\frac{2}{3}}}{(10^3)^{\frac{2}{3}}} = \frac{2^2}{10^2} = \frac{4}{100} = \frac{1}{25}$

(b) $(6.25)^{\frac{3}{2}} = (\frac{625}{100})^{\frac{3}{2}} = \left((\frac{25^2}{10^2})^{\left(\frac{1}{2}\right)}\right)^3$

$= \left(\frac{25}{10}\right)^3 = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

(c) $\left(\frac{25}{81}\right)^{-\frac{3}{2}} = \left(\frac{81}{25}\right)^{\frac{3}{2}} = (\frac{9^2}{5^2})^{\frac{3}{2}} = \frac{9^3}{5^3} = \frac{729}{125}$

(d) $(0.0000128)^{\frac{3}{7}} = (\frac{128}{10000000})^{\frac{3}{7}} = (\frac{2^7}{10^7})^{\frac{3}{7}}$

$= \frac{2^3}{10^3} = \frac{8}{1000} = \frac{1}{125}$

(e) $(-27)^{-\frac{4}{3}} = (-\frac{1}{27})^{\frac{4}{3}} = (-\frac{1^3}{3^3})^{\frac{4}{3}} = (-\frac{1}{3})^4 = \frac{1}{81}$

(f) $-(27)^{-\frac{4}{3}} = -(\frac{1}{27})^{\frac{4}{3}} = -(\frac{1^3}{3^3})^{\frac{4}{3}} = -(\frac{1}{3})^4 = -\frac{1}{81}$

19. (a) $3^x = 243 \Rightarrow 3^x = 3^5 \Rightarrow x = 5.$

(b) $9^{-x} = 27 \Rightarrow 3^{-2x} = 3^3 \Rightarrow -2x = 3 \Rightarrow x = -\frac{3}{2}$

$\left(\frac{9}{4}\right)^{3x} = \frac{32}{243} \Rightarrow \left(\frac{3^2}{2^2}\right)^{3x} = \frac{2^5}{3^5}$

(c) $\Rightarrow \left(\frac{3}{2}\right)^{6x} = \left(\frac{2}{3}\right)^5 \Rightarrow \left(\frac{3}{2}\right)^{6x} = \left(\frac{3}{2}\right)^{-5}$

$\Rightarrow 6x = -5 \Rightarrow x = -\frac{5}{6}$

(d) $\sqrt{-x} = 3\sqrt{2} \Rightarrow \sqrt{-x} = \sqrt{18} \Rightarrow$

$-x = 18 \Rightarrow x = -18$

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(e) $x^{-\frac{3}{4}} = 2 \Rightarrow (x^{-\frac{3}{4}})^{-\frac{4}{3}} = (2)^{-\frac{4}{3}} \Rightarrow x = (2)^{-\frac{4}{3}}$

(f) $(x-1)^2 = 2 \Rightarrow x-1 = \pm\sqrt{2} \Rightarrow x = 1 \pm \sqrt{2}$

20. Answers may vary.

(a) $\sqrt[5]{20} \Rightarrow x^5 - 20 = 0$

(b) $\sqrt[3]{-2} \Rightarrow x^3 + 2 = 0$

(c) $\sqrt[3]{10} - 1 \Rightarrow (x+1)^3 - 10 = 0$

(d) $\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow 3x^2 - 2 = 0$

21. Using "guess and check"

$4^3 = 64$

$4.6^3 = 97.3$

$4.7^3 = 103.8$

$5^3 = 125$

So, an integer approximation of $\sqrt[3]{103}$ is 5.

22. **(a)** $\frac{\sqrt{500}}{\sqrt{20}} = \frac{10\sqrt{5}}{2\sqrt{5}} = 5$ so it is rational.

(b) $8^{\frac{1}{3}} + 8^{-\frac{1}{3}} = (2^3)^{\frac{1}{3}} + (2^3)^{-\frac{1}{3}} = 2 + \frac{1}{2} = 2\frac{1}{2}$ so it is rational.

(c) $\frac{2}{\sqrt{2}} - \sqrt{2} = \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} = 0$ so it is rational.

(d) $\sqrt{1000} = 31.6227766...$ so it is irrational.

Assessment 8-1B

1. Answers may vary; one such could be 0.454455444555…, and continuing the pattern of adding a 4 and a 5 to each succeeding group.

2. **(a)** $x^2 = 3^2 + \left(\sqrt{3}\right)^2 = 9 + 3 = 12$

$\Rightarrow x = \sqrt{12} = \sqrt{4 \cdot 3} = \mathbf{2\sqrt{3}}$.

(b) $x^2 = (3)^2 + (3)^2 = 9 + 9 = 18$

$\Rightarrow x = \sqrt{18} = 3\sqrt{2}$

(c) $x^2 + 12^2 = 25^2 \Rightarrow x^2 = 25^2 - 12^2$

$x^2 = 481 \Rightarrow x = \sqrt{481}$

3. Line up the decimal points:

$0.8 = 0.80000\ldots$

$0.\overline{8} = 0.88888\ldots$

$0.\overline{89} = 0.89898\ldots$

$0.8\overline{89} = 0.88989\ldots$

$\sqrt{0.7744} = 0.88000\ldots$

Ordering from greatest to least:

$0.\overline{89} > 0.8\overline{89} > 0.\overline{8} > \sqrt{0.7744} > 0.8$.

4. **(a)** **Irrational**. There is no rational number n such that $n^2 = 78$.

(b) **Rational**. $9^2 = 81$.

(c) **Rational**. $7^3 = 343$.

(d) **Rational**. The sum of two rational numbers is rational, since the rationals are closed under addition.

(e) **Irrational**. The quotient of any non-zero rational number and any irrational number is irrational.

5. **(a)** $16 \cdot 16 = 256 \Rightarrow \sqrt{256} = \mathbf{16}$.

(b) $18 \cdot 18 = 324 \Rightarrow \sqrt{324} = \mathbf{18}$.

(c) **Impossible**. There is no real number n such that $n^4 = -1$ (or any other negative number).

(d) $32 \cdot 32 = 1024 \Rightarrow \sqrt{1024} = \mathbf{32}$.

6. **(a)** $4 < \sqrt{20.3} < 5$

$(4.5)^2 = 20.25$ and $(4.6)^2 = 21.16 \Rightarrow$

$4.5 < \sqrt{20.3} < 4.6$

$(4.50)^2 = 20.25$ and $(4.51)^2 = 20.3401 \Rightarrow$

$4.50 < \sqrt{20.3} < 4.51$

20.3 is closer to 20.3401 than to 20.25 $\Rightarrow$

$\sqrt{20.3} \approx \mathbf{4.51}$.

(b) $1 < \sqrt{1.64} < 2$

$(1.2)^2 = 1.44$ and $(1.3)^2 = 1.69 \Rightarrow$

$1.2 < \sqrt{1.64} < 1.3$

$(1.28)^2 = 1.6384$ and $(1.29)^2 = 1.6641 \Rightarrow$

$1.28 < \sqrt{1.64} < 1.29$

1.64 is closer to 1.6384 than to 1.6641 $\Rightarrow$

$\sqrt{1.64} \approx \mathbf{1.28}$.

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7. **(a)** **True**. The rationals are closed under addition.

(b) **False**. For example, $\sqrt{2} - \sqrt{2} = 0$, which is rational.

(c) **False**. If the rational number is 0, any product will be 0, which is rational.

8. Answers may vary.

(a) $\sqrt{9.1}, \sqrt{9.3}$, and $\sqrt{9.5}$ are three such numbers.

(b) $0.55555555\ldots = 0.\overline{55}$
$< 0.562010010001...$
$< 0.563010010001...$
$< 0.564010010001...$
$< 0.56565656\ldots. = 0.\overline{56}$

(c) 0.01
0.0101010010001…
0.0102010010001…
0.0103010010001…
0.011

9. Suppose $\frac{\sqrt{2}}{2}$ is a rational number which can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Then $\frac{\sqrt{2}}{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{2a}{b}$, which is a rational number. But $\sqrt{2}$ is an irrational number which cannot equal a rational number, so a contradiction exists and therefore $\frac{\sqrt{2}}{2}$ must be irrational.

10. **(a)** $Q \cap I = \boldsymbol{I}$. $I \subset Q$, since the set of rational numbers contains the set of integers.

(b) $S - Q = \boldsymbol{S}$. There are no irrational numbers which are also rational.

(c) $R \cup S = \boldsymbol{R}$. $S \subset R$, or irrational numbers are real.

(d) $\boldsymbol{R}$. Q, I, W, and S are all subsets of the set of real numbers.

(e) $\overline{S} = \boldsymbol{Q}$, where Q is the set of rational numbers. $Q \cup S = R; Q \cap S = \varnothing$.

11. In tables 11. and 12. below: N is the set of natural (or counting) numbers; I is the set of integers; Q is the set of rational numbers; R is the set of real numbers; and S is the set of irrational numbers. $N \subset I \subset Q \subset R; R = Q \cup S$.

		N	I	Q	S	R
(a)	$\sqrt{3}$				√	√
(b)	$4\frac{1}{2}$			√		√
(c)	$^{-}3\frac{1}{7}$			√		√

12.

		$x =$	N	I	Q	S	R
(a)	$x^2 + 2 = 4$	$\sqrt{2}, {}^{-}\sqrt{2}$				√	√
(b)	$1 - 2x = 32$	$\frac{^{-}31}{2}$			√		√
(c)	$x^3 = 4$	$\sqrt[3]{4}$				√	√
(d)	$\sqrt{x} = {}^{-}2$	no solution					
(e)	$0.\overline{7}x = 5$	$\frac{45}{7}$			√		√

13. **(a)** $\sqrt{x} = 5 \Rightarrow \sqrt{25} = 5 \Rightarrow x = 25$

(b) No real numbers. $\sqrt{x}$ is the principal square root of x.

(c) $\sqrt{^{-}x} = 5 \Rightarrow \left(\sqrt{^{-}x}\right)^2 = 5^2$
$\Rightarrow {}^{-}x = 25 \Rightarrow x = {}^{-}25$

(d) No real numbers. $\sqrt{x}$ is always positive, so its additive inverse would be negative.

(e) ${}^{-}\sqrt{x} = {}^{-}5 \Rightarrow \sqrt{x} = 5 \Rightarrow x = 25$

14. **(a)** $\sqrt{360} = \sqrt{36 \cdot 10} = \sqrt{36} \cdot \sqrt{10} = \boldsymbol{6\sqrt{10}}$.

(b) $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = \boldsymbol{2\sqrt{10}}$.

(c) $\sqrt{240} = \sqrt{16 \cdot 15} = \sqrt{16} \cdot \sqrt{15} = \boldsymbol{4\sqrt{15}}$.

15. **(a)** $\sqrt[3]{^{-}102} = \boldsymbol{^{-}1\sqrt[3]{102}}$. There is no number n such that $n^3 = 102$.

(b) $\sqrt[6]{64} = \sqrt[6]{2^6} = \boldsymbol{2}$.

(c) $\sqrt[3]{64} = \sqrt[3]{(2^2)^3} = 2^2 = \boldsymbol{4}$.

16. **(a)** $a_1 = 4, a_4 = 8$, and $n = 4$. Thus
$8 = 4(r)^{4-1} \Rightarrow r^3 = 2 \Rightarrow r = \sqrt[3]{2}$.
Then $a_2 = 4(\sqrt[3]{2})$ and $a_3 = 4(\sqrt[3]{2})^2 = 4\sqrt[3]{4}$.
The terms of the sequence are 4, $\underline{\boldsymbol{4\sqrt[3]{2},}}$ $\underline{\boldsymbol{4\sqrt[3]{4}}}$, 8.

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(b) $a_1 = 1, a_4 = 2$, and $n = 4$. Thus $2 = 1(r)^{4-1} \Rightarrow r^3 = 2 \Rightarrow r = \sqrt[3]{2}$.

Then $a_2 = 1(\sqrt[3]{2}) = \sqrt[3]{2}$ and $a_3 = (\sqrt[3]{2})^2 = \sqrt[3]{4}$. The terms of the sequence are $1, \underline{\sqrt[3]{2}}, \underline{\sqrt[3]{4}}, 2$.

(c) $a_3 = 2$ and $a_4 = \sqrt{2}$ Thus $2 = a_1(r)^{3-1}$ and $\sqrt{2} = a_1(r)^{4-1}$, so $2 = a_1r^2$ and $\sqrt{2} = a_1r^3$ Thus

$$a_1 = \frac{2}{r^2} \text{ and } a_1 = \frac{\sqrt{2}}{r^3} \Rightarrow \frac{2}{r^2} = \frac{\sqrt{2}}{r^3}$$
$$\Rightarrow 2r^3 = \sqrt{2}r^2 \Rightarrow 2r^3 - \sqrt{2}r^2 = 0$$
$$\Rightarrow r^2(2r - \sqrt{2}) = 0 \Rightarrow r^2 = 0$$
$$\text{and } 2r - \sqrt{2} = 0 \Rightarrow r = \frac{\sqrt{2}}{2}$$

The terms of the sequence are $4, 2\sqrt{2}, 2, \sqrt{2}$

(d) $a_2 = 1$ and $a_4 = 2$ Thus $1 = a_1(r)^{2-1}$ and $2 = a_1(r)^{4-1}$, so $1 = a_1r$ and $2 = a_1r^3$

$$a_1 = \frac{1}{r} \text{ and } a_1 = \frac{2}{r^3} \Rightarrow \frac{1}{r} = \frac{2}{r^3} \Rightarrow r^3 = 2r$$
$$\text{Thus } \Rightarrow r^3 - 2r = 0 \Rightarrow r(r^2 - 2) = 0 \Rightarrow r = 0$$
$$\text{and } r^2 - 2 = 0 \Rightarrow r^2 = 2 \Rightarrow r = \pm\sqrt{2}$$

The terms of the sequence are $-\frac{\sqrt{2}}{2}, 1, -\sqrt{2}, 2$ or $\frac{\sqrt{2}}{2}, 1, \sqrt{2}, 2$

17. Given the exponential function $E(t) = 8^t$, where $t \geq 0$ in this application.

(a) $E(0) = 8^0 = \mathbf{1}$.

(b) $E\left(\frac{1}{3}\right) = 8^{\frac{1}{3}} = \sqrt[3]{8} = \mathbf{2}$.

18. **(a)** $(0.008)^{-\frac{2}{3}} = (\frac{1000}{8})^{\frac{2}{3}} = (\frac{10^3}{2^3})^{\frac{2}{3}}$

$$\frac{(10^3)^{\frac{2}{3}}}{(2^3)^{\frac{2}{3}}} = \frac{10^2}{2^2} = \frac{100}{4} = 25$$

(b) $(0.04)^{\frac{3}{2}} = (\frac{4}{100})^{\frac{3}{2}} = (\frac{2^2}{10^2})^{\frac{3}{2}} = \frac{2^3}{10^3}$

$$= \frac{8}{1000} = \frac{1}{125}$$

(c) $\left(\frac{-8}{27}\right)^{-\frac{2}{3}} = \left(\frac{-27}{8}\right)^{\frac{2}{3}} = \left(\frac{-3^3}{2^3}\right)^{\frac{2}{3}} = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

(d) $(0.00000256)^{\frac{3}{8}} = (\frac{256}{100000000})^{\frac{3}{8}} = (\frac{2^8}{10^8})^{\frac{3}{8}}$

$$= \frac{2^3}{10^3} = \frac{8}{1000} = \frac{1}{125}$$

(e) $(-32)^{-\frac{4}{5}} = (-\frac{1}{32})^{\frac{4}{5}} = (-\frac{1^5}{2^5})^{\frac{4}{5}} = (-\frac{1}{2})^4 = \frac{1}{16}$

(f) $-(32)^{-\frac{4}{5}} = -\left(\frac{1}{32}\right)^{\frac{4}{5}} = -\left(\frac{1^5}{2^5}\right)^{\frac{4}{5}}$

$$= -\left(\frac{1}{2}\right)^4 = -\frac{1}{16}$$

19. **(a)** $3^x = -243 \Rightarrow 3^x = \left(-3\right)^5 \Rightarrow$ no solution.

(b) $81^{-x} = 27 \Rightarrow 3^{-4x} = 3^3 \Rightarrow -4x = 3 \Rightarrow x = -\frac{3}{4}$

$$\left(\frac{4}{9}\right)^{-3x} = \frac{32}{243} \Rightarrow \left(\frac{3^2}{2^2}\right)^{3x} = \frac{2^5}{3^5}$$

(c) $\Rightarrow \left(\frac{3}{2}\right)^{6x} = \left(\frac{2}{3}\right)^5 \Rightarrow \left(\frac{3}{2}\right)^{6x} = \left(\frac{3}{2}\right)^{-5}$

$$\Rightarrow 6x = -5 \Rightarrow x = -\frac{5}{6}$$

(d) $\sqrt[3]{-x} = 2\sqrt[3]{3} \Rightarrow \sqrt[3]{-x} = \sqrt[3]{24} \Rightarrow$

$$-x = 24 \Rightarrow x = -24$$

(e) $x^{-\frac{2}{5}} = 2 \Rightarrow (x^{-\frac{2}{5}})^{-\frac{5}{2}} = (2)^{-\frac{5}{2}} \Rightarrow x = (2)^{-\frac{5}{2}}$

(f) $(x+1)^2 = 2 \Rightarrow x + 1 = \pm\sqrt{2} \Rightarrow x = -1 \pm \sqrt{2}$

20. Answers may vary.

(a) $\sqrt[3]{5} \Rightarrow x^3 - 5 = 0$

(b) $\sqrt[5]{-2} \Rightarrow x^5 + 2 = 0$

(c) $\sqrt{2} - 1 \Rightarrow (x+1)^2 - 2 = 0$

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(d) $\dfrac{\sqrt{2}}{\sqrt[3]{2}} = \dfrac{2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{\frac{1}{2}-\frac{1}{3}} = 2^{\frac{3}{6}-\frac{2}{6}} = 2^{\frac{1}{6}} \Rightarrow x^6 - 2 = 0$

21. Using "guess and check"

$12^3 = 1728$

$12.6^3 = 2000.4$

$12.7^3 = 2048.4$

$13^3 = 2197$

So, an integer approximation of $\sqrt[3]{2001}$ is13.

22. **(a)** $\dfrac{\sqrt{320}}{\sqrt{20}} = \dfrac{8\sqrt{5}}{2\sqrt{5}} = 4$ is rational.

(b) $2\sqrt{50} - 8\sqrt{5} = 10\sqrt{2} - 8\sqrt{5}$ is irrational.

(c) $\dfrac{\sqrt{2}}{2} - \sqrt{2} = \dfrac{\sqrt{2}}{2} - \dfrac{2\sqrt{2}}{2} = \dfrac{\sqrt{2} - 2\sqrt{2}}{2}$ is irrational.

(d) $\sqrt{10^9} = 31622.7766...$ is irrational.

Assessment 8-2A: Variables

1. **(a)** Use the format for an arithmetic sequence: $a_n = a_1 + (n-1)d$, if $a_1 = 10$. Then $a_4 = 10 + (4-1)d = 10 + 3d$

(b) If n is the number, then twice the number is $2n$ and 15 less than that is $2n - 15$

(c) If n is the number, then its square is n^2 and 15 times that is $15n^2$

(d) If n is the number, its square is n^2 and twice the number is $2n$. Their difference is $\boldsymbol{n^2 - 2n}$.

2. **(a)** Let n be the number:

(i) Adding $\sqrt{3} \Rightarrow n + \sqrt{3}$

(ii) Multiplying the sum by $7 \Rightarrow 7(n + \sqrt{3})$

(iii) Subtracting $14 \Rightarrow 7(n + \sqrt{3}) - 14$

(iv) Dividing the difference by $7 \Rightarrow \frac{7(n+\sqrt{3})-14}{7}$

(v) Subtracting the original number $\Rightarrow \boldsymbol{\frac{7(n+\sqrt{3})-14}{7} - n}$.

(b) $\frac{7n+7\sqrt{3}-14}{7} - n = n + \sqrt{3} - 2 - n = \boldsymbol{\sqrt{3} - 2}$.

3. **(a)** There are 4, 6, 8, and 10 shaded tiles, respectively, in the four figures. Assume and arithmetic sequence with $a_1 = 4$ and $d = 2$. Thus $a_n = 4 + (n-1)2 = \boldsymbol{2n + 2}$ or $\boldsymbol{2(n + 1)}$.

(b) There are a total of $(n + 2)^2$ squares in each figure. Assume the pattern continues; then the number of white squares is $(n + 2)^2$ – the number of shaded squares, or $(n + 2)^2 - (2n + 2) = n^2 + 4n + 4 - 2n - 2 = \boldsymbol{n^2 + 2n + 2}$.

4. **(a)** Cost is $\$(50 + 60h)$

(b) Let n, d, and q be the number of nickels, dimes, and quarters, respectively. It is given that $n = 3d$ and $q = 3n = 9d$. Then the value of dimes + Value of nickels + value of quarters is $10d + 5(3d) + 25(9d) = 250d$ cents

(c) The sum of the three numbers is $x + (x+1) + (x + 2) = \boldsymbol{3x + 3}$.

(d) Make a table of the number of bacteria in terms of n minutes:

n minutes	Number of bacteria
1	$q \cdot 2$
2	$(q \cdot 2) \cdot 2 = q \cdot 2^2$
3	$(q \cdot 2^2) \cdot 2 = q \cdot 2^3$
⋮	⋮
n	$\boldsymbol{q \cdot 2^n}$

(e) The temperature after t hours is $\boldsymbol{(40 - 3t)°F}$.

(f) Pawel's salary is $\$s$ the first years; $\$(s + 5000)$ the second year; and $\$2(s + 5000) = \$(2s + 10,000)$ the third year, for a total of $\boldsymbol{\$(4s + 15,000)}$.

(g) The sum of the three numbers is $x + (x + 2) + (x + 4) = \boldsymbol{3x + 6}$.

(h) The sum of the three numbers is $(m-1) + m + (m + 1) = \boldsymbol{3m}$.

5. If the number of students is 45 times the number of professors, then $S = 45P$

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6. If there are five more girls than boys, then $\boldsymbol{g = b + 5}$.

7. Let m be the number of matchsticks Ryan uses. Then $m_1 = 13, m_2 = 19, m_3 = 25, \ldots$. Assume an arithmetic sequence with $m_1 = 13$ and $d = 6$. Thus $m_n = 13 + (n - 1) \cdot 6 = 6n + 7$ matchsticks.

8. **(a)** $P = \$8$ per hour $\times\ t$ hours, or $\boldsymbol{P = \$(8t)}$.

(b) Assume \$15 is paid as a flat fee for the first hour or fraction thereof. Afterwards $P = \$15 + \$10$ per hour $\times\ (t - 1)$ hours, or

$$P = \begin{cases} \$15 & \text{when } 0 < t < 1 \\ \$[15 + 10(t - 1)] & \text{when } t \geq 1. \end{cases}$$

9. Let r be total revenue. Then $r = 5x + 13(100) = \boldsymbol{\$(5x + 1300)}$.

10. **(a)** If the youngest receives $\$x$:
The eldest receives $\$3x$;
The middle receives $\frac{1}{2}(3x) = \$\frac{3}{2}x$.

(b) If the middle receives $\$y$:
The eldest receives $\$2y$;
The youngest receives $\frac{1}{3}(2y) = \$\frac{2}{3}y$.

(c) If the eldest receives $\$z$:
The middle receives $\$\frac{1}{2}z$;
The youngest receives $\$\frac{1}{3}z$.

11. **(a)** ***(i)*** The pattern appears to be subtracting 3 from each term to obtain its successor. At the 100th term 3 will be subtracted 99 times.
So the 100th term is $0 - 3(99) = {}^{-}297$.

(ii) The nth term can be found using the expression $\boldsymbol{0 - 3(n - 1) = {}^{-}3n + 3}$.

(b) ***(i)*** The pattern appears to be adding $2\sqrt{2}$ to each term to obtain its successor. At the 100th term $2\sqrt{2}$ will be added 99 times. The 100th term is $1 + 197\sqrt{2}$.

(ii) The nth term can be found using the expression $(1 - \sqrt{2}) + \boldsymbol{2}\sqrt{2}(n - 1)$
$1 - \sqrt{2} + 2\sqrt{2}\,n - 2\sqrt{2}$
$= 2\sqrt{\boldsymbol{2}}\boldsymbol{n} + \boldsymbol{1} - \boldsymbol{3}\sqrt{\boldsymbol{2}}$.

(c) ***(i)*** The pattern appears to be adding 2 to each term to obtain its successor. At the 100th term 2 will be added 99 times. The 100th term is
$\sqrt{3} + 0.5 + 2(99) = \sqrt{3} + \boldsymbol{198.5}$

(ii) $\boldsymbol{\sqrt{3} + 0.5 + 2(n - 1) = 2n + \sqrt{3} - 1.5}$

12. **(a)** ***(i)*** Multiply each term by $\sqrt{3}$ to obtain its successor. The 17th term is given by
$2 \cdot \sqrt{3}^{16} = 2 \cdot \left((\sqrt{3})^2\right)^8 = 2(3)^8 = 13{,}122$

(ii) The nth term is given by $\boldsymbol{2(\sqrt{3})^{n-1}}$.

(b) ***(i)*** Multiply each term by ${}^{-}\sqrt{2}$ to obtain its successor. The 17th term is given by

$$2 \cdot \left({}^{-}\sqrt{2}\right)^{16} = 2 \cdot \left(-2^{\frac{1}{2}}\right)^{16}$$

$$= 2 \cdot 256 = 512$$

(ii) The nth term is given by $2\left({}^{-}\sqrt{2}\right)^{n-1}$

(c) ***(i)*** Multiply each term by ${}^{-}\sqrt{5}$ to obtain its successor. The 17th term is given by

$${}^{-}\sqrt{5} \cdot \left({}^{-}\sqrt{5}\right)^{16} = {}^{-}\sqrt{5} \cdot \left(-5^{\frac{1}{2}}\right)^{16}$$

$$= {}^{-}\sqrt{5} \cdot 5^8$$

(ii) The nth is given by $({}^{-}\sqrt{5})({}^{-}\sqrt{5})^{\boldsymbol{n-1}} = \left({}^{-}\sqrt{5}\right)^n$

13. If the 100th term is $\sqrt{2}$ and the 200th term is $\sqrt{3}$ and the sequence is arithmetic, then

$$a_1 + d(100 - 1) = \sqrt{2} \Rightarrow a_1 = \sqrt{2} - 99d$$

$$a_1 + d(200 - 1) = \sqrt{3} \Rightarrow a_1 = \sqrt{3} - 199d$$

$$\Rightarrow \sqrt{2} - 99d = \sqrt{3} - 199d$$

$$\Rightarrow 100d = \sqrt{3} - \sqrt{2}$$

$$\Rightarrow d = \frac{\sqrt{3} - \sqrt{2}}{100}$$

and the first term is

$$a_1 + 99\left(\frac{\sqrt{3} - \sqrt{2}}{100}\right) = \sqrt{2}$$

$$a_1 = \sqrt{2} - 99\left(\frac{\sqrt{3} - \sqrt{2}}{100}\right).$$

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14. If the 11^{th} term is $^{-}128$ and the ratio is $^{-}\sqrt{2}$ then the first term is

$$^{-}128 = a_1(^{-}\sqrt{2})^{10} \Rightarrow {}^{-}128 = a_1\left(^{-}2^{\frac{1}{2}}\right)^{10}$$

$$^{-}128 = a_1 2^5 \Rightarrow a_1 = \frac{^{-}128}{32} = {}^{-}4$$

15. $10 - .20(10) = 8; 8 - .20(8) = 6.4;$
$6.4 - .20(6.4) = 5.12$
$5.12 - .20(5.12) = 4.096;$
$4.096 - .20(4.096) = 3.2768$ or 3.3 ft.

16. Starting with two hours and increasing by 20 minutes each day can be represented with an arithmetic sequence with $a_1 = 2$ and d=$\frac{1}{3}$ the n^{th} term is $2 + \frac{1}{3}(n-1)$. Set the n^{th} term equal to 12 hours and solve for n.

$$2 + \frac{1}{3}(n-1) = 12 \Rightarrow 2 + \frac{1}{3}n - \frac{1}{3} = 12$$
$$\Rightarrow 6 + n - 1 = 36 \Rightarrow 5 + n = 36 \Rightarrow n = 31$$

After 31 days Jake will be able to wear his contacts for 12 hours.

17. **(a)** The first 10 terms in simplest form:

$\sqrt{2}, \sqrt{3}, \sqrt{2}+\sqrt{3}, \sqrt{2}+2\sqrt{3},$
$2\sqrt{2}+3\sqrt{3}, 3\sqrt{2}+5\sqrt{3},$
$5\sqrt{2}+8\sqrt{3}, 8\sqrt{2}+13\sqrt{3},$
$13\sqrt{2}+21\sqrt{3}, 21\sqrt{2}+34\sqrt{3}$

(b) **(i)** The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, … $F_5 = 5$ and $F_6 = 8$ and $L_7 = 5\sqrt{2} + 8\sqrt{3}$ so

$L_7 = F_5\sqrt{2} + F_6\sqrt{3}$
$5\sqrt{2} + 8\sqrt{3} = 5\sqrt{2} + 8\sqrt{3}$
$L_8 = F_6\sqrt{2} + F_7\sqrt{3}$

(ii) $L_9 = F_7\sqrt{2} + F_8\sqrt{3}$
$L_{10} = F_8\sqrt{2} + F_9\sqrt{3}$

(iii) $L_n = F_{n-2}\sqrt{2} + F_{n-1}\sqrt{3}$

Assessment 8-2B

1. **(a)** If n is the number, then 10 more than the number is $\boldsymbol{n + 10}$.

(b) If n is the number, then 10 less than the number is $\boldsymbol{n - 10}$.

(c) If n is the number, then half times it is $\frac{1}{2}n$.

(d) If n is the number, then the sum of it and 10 is $\boldsymbol{n + 10}$.

(e) If n is the number, its cube is n^3 and the difference between its cube and its square is $n^3 - n^2$.

2. **(a)** Let n be the number. Then

(i) Adding $25 \Rightarrow n + 25$

(ii) Multiplying by $4 \Rightarrow 4(n + 25)$

(iii) Subtracting $60 \Rightarrow 4(n + 25) - 60$

(iv) Dividing by $4 \Rightarrow \frac{4(n+25)-60}{4}$

(v) Adding $5 \Rightarrow \frac{4(n+25)-60}{4} + 5$

(b) $\frac{4(n+25)-60}{4} + 5 = n + 25 - \frac{60}{4} + 5 = n + 15$

3. **(a)** There are 10,13, and 16 shaded tiles, respectively, in the three figures. Assume an arithmetic sequence with $a_1 = 10$ and $d = 3$. Thus $a_n = 10 + (n - 1)3 = \boldsymbol{3n + 7}$.

(b) There are a total of $(n + 3)^2$ squares in each figure. The number of white squares is then $(n + 3)^2$ − the number of shaded squares, or $(n + 3)^2 - (3n + 7) = n^2 + 6n + 9 - 3n - 7 = \boldsymbol{n^2 + 3n + 2}$.

4. **(a)** Cost is $\boldsymbol{\$(30 + xh)}$, where x is the cost per hour for h hours.

(b) Let n, d, and q be the number of nickels, dimes, and quarters, respectively. It is given that $n = 4d$ and $q = 3n = 12d$. Then the value of dimes + value of nickels + value of quarters is $10d + 5(4d) + 25(12d) = 330d$ cents.

(c) The sum of the three numbers is $x + (x - 1) + (x - 2) = \boldsymbol{3x - 3}$.

(d) Make a table of the number of bacteria in terms of n minutes:

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n minutes	Number of bacteria
1	$q \cdot 3^3$
2	$(q \cdot 3^3) \cdot 3^3 = q \cdot 3^6$
3	$(q \cdot 3^6) \cdot 3^3 = q \cdot 3^9$
⋮	⋮
n	$\boldsymbol{q \cdot 3^{3n}}$

(e) The temperature t hours ago was $\boldsymbol{(40 + 3t)^\circ F}$.

(f) Pawel's salary is $\$s$ the first year; $\$(s + 5000)$ the second year; and $\$(2s)$ the third year, for a total of $\boldsymbol{\$(4s + 5{,}000)}$.

(g) The sum of the three numbers is $x + (x - 2) + (x - 4) = \boldsymbol{3x - 6}$.

5. If there are 100 more men than women $\Rightarrow m = \boldsymbol{w + 100}$.

6. If there are 15 more chairs than tables $\Rightarrow \boldsymbol{c = t + 15}$.

7. Let m be the number of matchsticks Ryan uses. Then $m_1 = 3, m_2 = 5, m_3 = 7, \ldots$ Assume an arithmetic series with $a_1 = 3$ and $d = 2$.

(i) $a_n = 3 + 2 \cdot (n - 1) \Rightarrow 2n + 1$ matchsticks.

(ii) $a_{n-1} = 2(n - 1) + 1 = 2n - 1$ matchsticks.

8. **(a)** $P = \$d$ per hour $\times\ t$ hours, or $\boldsymbol{P = \$(dt)}$.

(b) Assume \$15 is paid as a flat fee for the first hour or fraction thereof. Afterwards $P = \$25 + \$k$ per hour $\times\ (t - 1)$ hours, or

$$P = \begin{cases} \$25 & \text{when } 0 < t < 1 \\ \$[25 + k(t - 1)] & \text{when } t \geq 1 \end{cases}$$

(c) $P = \$30 + \$10$ per hour $\times\ t$ hours, or $\boldsymbol{P = \$(30 + 10t)}$.

(d) $C = \$300 + \$4$ per day $\times\ n$ days, or $\boldsymbol{C = \$(300 + 4n)}$.

(e) $C = \$30 + \$0.35$ per mile $\times\ m$ miles, or $\boldsymbol{C = \$(30 + 0.35m)}$.

9. $dx + 2d \cdot 100 = dx + 200d$ dollars.

10. If Matt gives David 10 stickers, David would have $\boldsymbol{d + 10}$ stickers. Matt would then have $\boldsymbol{2d - 10}$ stickers.

11. **(a)** ***(i)*** The pattern appears to subtract 5 from each term to obtain its successor. The 100th term is given by $0 - 5(99) = {}^-495$

(ii) The nth term is given by $\boldsymbol{0 -}$ $5(n - 1) = 0 - 5n + 5 = {}^-5n + 5$.

(b) ***(i)*** The pattern appears to subtract $2\sqrt{2}$ from each term to obtain its successor. The 100th term is given by $1 + \sqrt{2} - 2\sqrt{2}(99) = 1 - 197\sqrt{2}$.

(ii) The nth term is given by $1 + \sqrt{2} - 2\sqrt{2}(n - 1) = 1 + \sqrt{2} - 2\sqrt{2}n + 2\sqrt{2} = 1 + 3\sqrt{2} - 2\sqrt{2}n$

(c) ***(i)*** The pattern appears to add $x + 2\sqrt{3}$ to each term to obtain its successor. The 100th term is given by $x + \sqrt{3} + (x + 2\sqrt{3})(99) = 199\sqrt{3} + 100x$.

(ii) The nth term is given by $x + \sqrt{3} + (x + 2\sqrt{3})(n - 1)$

12. **(a)** ***(i)*** The pattern is to multiply each term by $\sqrt{2}$ to obtain its successor. The 20th term is

$$\sqrt{\frac{3}{2}} \cdot \sqrt{2}^{19} = \frac{\sqrt{3}}{\sqrt{2}} \cdot 2^{\frac{19}{2}} = \frac{\sqrt{3} \cdot 2^{\frac{19}{2}}}{2^{\frac{1}{2}}}$$

$$= \sqrt{3} \cdot 2^{\frac{19}{2} - \frac{1}{2}} = \sqrt{3} \cdot 2^9 = 512\sqrt{3}$$

(ii) The nth term is given by $\sqrt{\frac{3}{2}}(\sqrt{2})^{n-1}$.

(b) ***(i)*** The pattern is to multiply each term by $\sqrt{2}$ to obtain its successor. The 20th term is given by $\sqrt{2}(\sqrt{2})^{19} = (\sqrt{2})^{20} = 2^{10}$.

(ii) The nth term is given by $\sqrt{2}(\sqrt{2})^{n-1} = \left(\sqrt{2}\right)^n$

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(c) ***(i)*** The pattern is to multiply each term by $^{-}\sqrt{3}$ to obtain its successor. The 20th term is $(^{-}\sqrt{3})(^{-}\sqrt{3})^{19} = (^{-}\sqrt{3})^{20} = 3^{10}$.

(ii) The *n*th term is given by $^{-}\sqrt{3} \cdot \left(^{-}\sqrt{3}\right)^{n-1}$ $(^{-}\sqrt{3})^{n}$.

(d) ***(i)*** The pattern is to multiply each term by $^{-}\sqrt{3}$ to obtain its successor. The 20th term is $3(^{-}\sqrt{3})^{19} = 3^{1} \cdot {}^{-}3^{\frac{19}{2}} = (^{-}1) \cdot 3^{1+\frac{19}{2}} = (^{-}1) \cdot 3^{\frac{21}{2}} = (^{-}1) \cdot 3^{\frac{1}{2}+\frac{20}{1}} = {}^{-}\sqrt{3} \cdot 3^{10}$.

(ii) The *n*th term is given by $3 \cdot \left(^{-}\sqrt{3}\right)^{n-1}$

13. If the 10th term is 25 and the 20th term is 100 and the sequence is geometric, then

$25 = a_1(r)^9$ and $100 = a_1(r)^{19}$

$a_1 = \dfrac{25}{r^9}$ and $a_1 = \dfrac{100}{r^{19}}$

$\Rightarrow \dfrac{25}{r^9} = \dfrac{100}{r^{19}} \Rightarrow 25r^{19} = 100r^9$

$25r^{19} - 100r^9 = 0 \Rightarrow 25r^9(r^{10} - 4) = 0$

$25r^9 = 0$ and $r^{10} - 4 = 0$

$r = 0$ and $r^{10} = 4 \Rightarrow r = \sqrt[10]{4}$

The first term is

$$a_1 = \frac{25}{r^9} \Rightarrow a_1 = \frac{25}{\left(\sqrt[10]{4}\right)^9}$$

$$\Rightarrow a_1 = \frac{25}{4^{\frac{9}{10}}} = \frac{25}{4^{\frac{5}{10}+\frac{4}{10}}} = \frac{25}{4^{\frac{1}{2}} \cdot 4^{\frac{2}{5}}}$$

$$= \frac{25}{\sqrt{4} \cdot (2^2)^{\frac{2}{5}}} = \frac{25}{2\sqrt[5]{2^4}}$$

14. **(a)** The common difference in the arithmetic sequence is 3.

(b) The common ratio in the geometric sequence is $3^{\frac{3}{2}}$.

15. $20 - .10(20) = 18$;
$18 - .10(18) = 16.2$;
$16.2 - .10(16.2) = 14.58$;
$14.58 - .10(14.58) = 13.122$ or 13.122 m.

16. Create a table:

0.01	100
0.11	100.01
0.21	100.02
0.31	100.03
0.41	100.04
0.51	100.05
…	…
0.01 + (0.1)(*n* - 1)	100 + (0.01)(*n* - 1)

When is 0.01 + (0.1)(*n* - 1) > 100 + (0.01)(*n* - 1)?

$0.01 + (0.1)(n-1) > 100 + (0.01)(n-1)$

$0.01 + 0.1n - 0.1 > 100 + 0.01n - 0.01$

$0.1n - 0.09 > 0.01n + 99.99$

$0.09n > 100.08$

$n > 1112$

At 1112 terms they are equal so at the 1113th term the number in the first sequence be greater than the number in the second sequence.

17. (a) 1, 2, 3, 5, 8, 13, 21

(b) 1 + 2 + 3 = 6

(c) 1 + 2 + 3 +5 = 11

(d) 1 + 2 + 3 + 5 + 8= 19

(e) 1 + 2 + 3 +5 + 8 + 13= 32

(f) 1 + 2 + 3 +5 + 8 + 13 + 21 = 53

(g) The sum of the first *n* terms is 2 less than two terms later in the sequence.

(h) If the *n*th term is F_n then $F_1 + F_2 + F_3 + \ldots + F_n = F_{n+2} - 2$.

Mathematical Connections 8-2: Review Problems

11. Answers may vary. For example, $\sqrt{2} \approx 1.414213\ldots$, so rational numbers between 1.41 and $\sqrt{2}$ include **1.411** and **1.412**. Irrationals between 1.41 and $\sqrt{2}$ include **1.411011101111…** and **1.412022022022222…**. Alternatively,

irrational numbers can be obtained by adding an

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irrational number to 1.41, e.g., $1.41 + \frac{1}{1000 \cdot \sqrt{2}}$ is an irrational number between 1.41 and $\sqrt{2}$.

12. (a) $\sqrt[3]{728} = 2\sqrt[3]{91}$ is irrational.

(b) $\frac{2}{\sqrt{2}} = \sqrt{2}$ is irrational.

(c) $\sqrt[3]{2^9 \cdot 3^{12}} = 2^3 \cdot 3^4 = 8 \cdot 81 = 648$ is rational.

(d) $\sqrt[3]{2} \cdot \sqrt[3]{4} = \sqrt[3]{8} = 2$ is rational.

(e) $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} + \sqrt{2}\right) = \sqrt{9} + \sqrt{6} - \sqrt{6} - \sqrt{4}$
$= 3 - 2 = 1$
is rational

(f) $\left(\sqrt{3} - \sqrt{2}\right)^2 = \left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right)$
$= \sqrt{9} - \sqrt{6} - \sqrt{6} + \sqrt{4} = 5 - 2\sqrt{6}$
is irrational.

13. **(a)** $\left(\frac{^-8}{27}\right)^{\frac{^-2}{3}} = \left(\frac{^-27}{8}\right)^{\frac{2}{3}} = \left(\frac{^-3^3}{2^3}\right)^{\frac{2}{3}} = \left(\frac{^-3}{2}\right)^2 = \frac{9}{4}$

(b) $\left(\frac{^-9}{4}\right)^{\frac{1}{2}} = \sqrt{\frac{^-9}{4}}$ is not real.

(c) $^-\left(\frac{9}{4}\right)^{\frac{3}{2}} = {}^-\left(\frac{3^2}{2^2}\right)^{\frac{3}{2}} = {}^-\frac{3^3}{2^3} = {}^-\frac{27}{8}$

14. **(a)** $\sqrt{^-x} = 5 \Rightarrow {}^-x = 25 \Rightarrow x = {}^-25$

(b) $\sqrt{x^2} = {}^-x \Rightarrow$ true for all $x \le 0$.

(c)

$\left(^-x\right)^{\frac{3}{2}} = 125 \Rightarrow {}^-x = \left(5^3\right)^{\frac{2}{3}} \Rightarrow {}^-x = 25 \Rightarrow x = {}^-25$

$n < \sqrt[3]{50} < n + 1$

15. $n < 3.684031499... < n + 1$
$3 < 3.684031499... < 4$
If n is an integer, then n is 3.

Assessment 8-3A: Equations

1. The balance scales imply addition, so:
$\Delta + \square = 12.$
$O + O + \Delta + \square = 18;$
$2O + 12 = 18;$ so
$\mathbf{O = 3}.$
$2O + \square = 10;$
$6 + \square = 10;$ so
$\square = \mathbf{4}.$
$\Delta + \square = 12;$
$\Delta + 4 = 12;$ so
$\Delta = 8.$

2. **(a)** $x + \sqrt{3} = 2\sqrt{3} - x \Rightarrow 2x = \sqrt{3}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

(b) $\frac{3}{2}x - 3 = x + \sqrt{2} \Rightarrow \frac{3}{2}x - x = 3 + \sqrt{2}$

$\Rightarrow \frac{1}{2}x = 3 + \sqrt{2} \Rightarrow x = 6 + 2\sqrt{2}$

(c) $5\left(2x + \sqrt{2}\right) + 7\left(2x + \sqrt{2}\right) = 12 \Rightarrow$

$12(2x + \sqrt{2}) = 12 \Rightarrow 2x + \sqrt{2} = 1$

$\Rightarrow x = \frac{1 - \sqrt{2}}{2}$

(d) $3(\sqrt{x} - 3) = 5(\sqrt{x} - 3)$

$3(\sqrt{x} - 3) - 5(\sqrt{x} - 3) = 0$

$^-2(\sqrt{x} - 3) = 0$

$\sqrt{x} - 3 = 0 \Rightarrow \sqrt{x} = 3 \Rightarrow x = 9$

(e) $\left(\sqrt{x} - 5\right)^2 = 9$

$\sqrt{x} - 5 = 3$ and $\sqrt{x} - 5 = {}^-3$

$x = 64$ and $x = 4$

3. Let m be the number of matchsticks Ryan uses.
Then $m_1 = 4, m_2 = 7, m_3 = 10, ..., m_n = 67.$
Assume an arithmetic series with $m_1 = 4, d = 3,$ and $m_n = 67.$

Thus $67 = 4 + (n - 1) \cdot 3 \Rightarrow 67 = 4 + 3n - 3 \Rightarrow 66 = 3n \Rightarrow n =$ **22 squares**.

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4. If s is the number of student tickets sold, then $812 - s$ is the number of nonstudent tickets. Adding their values gives the total amount taken in:

$\$2 \cdot s + \$3 \cdot (812 - s) = \$1912$

$2s + 2436 - 3s = 1912$

$\Rightarrow s =$ **524 student tickets sold**.

5. Let e be the amount the eldest receives,
m be the amount the middle sibling receives,
y be the amount the youngest receives.

Then $e = 3y$ and $m = y + 14{,}000$

So $e + m + y = 486{,}000$

$3y + (y + 14{,}000) + y = 486{,}000$

$5y + 14{,}000 = 486{,}000$

$5y = 472{,}000$

Thus $\boldsymbol{y = \$94{,}400}$

$\boldsymbol{m = \$108{,}400}$

$\boldsymbol{e = \$283{,}200}$

6. Let e be the length of the equal length pieces.

Then $2e + (e - 3) = 120$ inches

$3e - 3 = 120$

$3e = 123 \Rightarrow e = 41$

So the equal length pieces are **41 inches** and the short piece is **38 inches** (disregarding the width of the saw cuts).

7. Let d be the number of dimes; then $67 - d$ is the number of nickels. $0.10d$ is the amount of money in dimes and $0.05(67 - d)$ is the amount of money in nickels.

Thus $0.10d + 0.05(67 - d) = 4.20$, or

$10d + 5(67 - d) = 420$

$10d + 335 - 5d = 420$

$5d = 85 \Rightarrow d =$ **17 dimes**

$67 - d =$ **50 nickels**.

8. Let m be Miriam's age now; then $m - 10$ is Ricardo's age now $m - 2$ was Miriam's age two years ago. Thus $m - 2 = 3(m - 10) \Rightarrow m - 2 = 3m - 30 \Rightarrow 28 = 2m$, or **Miriam is 14**

Ricardo is 4.

9. Let g be the number of graduate students; $20g$ is the number of undergraduates. Then

$g + 20g = 21{,}000$

$21g = 21{,}000 \Rightarrow 1000$ graduate students.

10. Let the perpendicular sides to the river be of length a; then the parallel side is of length b = 2a. Thus $a + 2a + a = 1200, 4a = 1200 \Rightarrow a = 300$.

The perpendicular sides are then **300 yards**; the parallel side is **600 yards**.

11. In the given sequence $d = 3$, thus

$n + (n + 3) + (n + 6) = 903$

$3n + 9 = 903$

$3n = 894$

$n = 298$

The three terms are 298, 301, and 304.

12. Let a be the shorter side and b be the longer side.

The shorter side is $\frac{1}{3}$ of the length of the longer side; $a = \frac{1}{3}b$. The perimeter is 100 ft or

$2a + 2b = 100 \Rightarrow 2\left(\frac{1}{3}b\right) + 2b = 100$

$\Rightarrow \frac{2}{3}b + \frac{6}{3}b = 100 \Rightarrow \frac{8}{3}b = 100$

$b = \frac{300}{8} = \frac{75}{2}$ and $a = \frac{1}{3} \cdot \frac{75}{2} = \frac{25}{2}$

Using Pythagorean Theorem, the length of the diagonal is

$a^2 + b^2 = diagonal^2 \Rightarrow$

$\left(\frac{25}{2}\right)^2 + \left(\frac{75}{2}\right)^2 = diagonal^2$

$\frac{625}{4} + \frac{5625}{4} = diagonal^2 \Rightarrow \frac{6250}{4} = diagonal^2$

$\frac{3125}{2} = diagonal^2 \Rightarrow$

$diagonal = \sqrt{\frac{3125}{2}} = \sqrt{1562.5} = 39.53\ ft$

Assessment 8-3B

1. If $\square = 2\Delta$, then $\Delta < \square$.

If $\Delta = 2O$, then $O < \Delta$.

So $O < \Delta < \square$.

(a) $\square$ weighs the most.

(b) **O** weighs the least.

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2. **(a)** $x - \sqrt{2} = 3\sqrt{2} - x \Rightarrow 2x = 3\sqrt{2} + \sqrt{2}$

$$x = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

(b) $\frac{-3}{2}x - 2 = x - \sqrt{2} \Rightarrow -3x - 4$

$= 2x - 2\sqrt{2} \Rightarrow -5x = 4 - 2\sqrt{2}$

$\Rightarrow x = \dfrac{2\sqrt{2} - 4}{5}$

(c) $5(2x - \sqrt{3}) - 7(2x - \sqrt{3}) = 2$

$-2(2x - \sqrt{3}) = 2 \Rightarrow 2x - \sqrt{3} = -1$

$x = \dfrac{\sqrt{3} - 1}{2}$

(d) $2(x^2 - 3) = (x^2 - 3)$

$2(x^2 - 3) - (x^2 - 3) = 0$

$x^2 - 3 = 0$

$x = \pm\sqrt{3}$

(e) $(\sqrt{-x} - 5)^2 = 9 \Rightarrow \sqrt{-x} - 5 = \pm 3$

$\sqrt{-x} = 5 \pm 3 \Rightarrow \sqrt{-x} = 8$ or $\sqrt{-x} = 2$

$-x = 64$ or $-x = 4 \Rightarrow x = -64$ or $x = -4$

3. Let m be the number of matchsticks Ryan uses. Then $m_1 = 4, m_2 = 7, m_3 = 10, \ldots$. Assume an arithmetic sequence with $d = 3$. Let m_n be the number of matchsticks in the last figure; then $m_n + (m_n - 3) = 599 \Rightarrow m_n = 301$ matchsticks. Thus the last two figures used **298** and **301** matchsticks, respectively.

4. Step 1: Write a mathematical model. To do this, let x represent the number of tickets sold to students. Then, $723 - x$ represents the number of tickets sold to non-students. Thus,

$$\begin{aligned} 3x + 5(723 - x) &= 2815 \\ 3x + 3615 - 5x &= 2815 \\ -2x + 3615 - 3615 &= 2815 - 3615 \\ -2x &= -800 \\ x &= 400 \end{aligned}$$

The number of non-student tickets sold is $723 - 400 = \mathbf{323}$.

5. Let e be the amount left to the eldest, m be the amount left to each of two middle siblings, and y be the amount left to the youngest.

It is given that $e = 3y$ and each $m = y + 16{,}000$.

Thus if $e + m + m + y = 2{,}000{,}000$ then

$3y + y + 16{,}000 + y + 16{,}000 + y = 2{,}000{,}000 \Rightarrow$

$6y + 32{,}000 = 2{,}000{,}000 \Rightarrow y = 328{,}000$.

The youngest will receive \$328,000, the middle children each will receive \$344,000, and the eldest will receive \$984,000.

6. Let a be the Alex's age now. Her age ten years from now will be $a + 10$. Then $a + 10 = 3a \Rightarrow a = 5$. Alex is **5 years old**.

7. Let d be the number of Dave's stickers; Matt then has $2d$ stickers.

So $d + 2d = 2(120) \Rightarrow d = 80$ and $2d = 160$.

If **Matt gives Dave 40 stickers**, each will then have 120.

8. **(a)** Let m be Miriam's age now and r be Ricardo's age now, so $r = m - 4$. Miriam's age 10 years ago was $m - 10$ and Ricardo's age 10 years ago was $(m - 4) - 10 = m - 14$.

Thus $m - 10 = 3(m - 14) \Rightarrow m - 10 = 3m - 42 \Rightarrow \boldsymbol{m = 16}$ and $\boldsymbol{r = 12}$.

(b) Ten years ago Miriam was 6 and Ricardo was 2; Miriam was then three times as old as Ricardo.

9. Let s be the number of students; then the number of professors is $\frac{1}{13}s$.

Thus $s + \frac{1}{13}s = 28{,}000 \Rightarrow 1\frac{1}{13} \cdot s = 28{,}000$

$\Rightarrow s = \mathbf{26{,}000\ students}$.

10. Let a be the length of the parallel side, so the length of each of the perpendicular sides is $2a$.

Then $2a + a + 2a = 800 \Rightarrow a = 160$. The pasture's dimensions are thus **320 by 160 yards**.

11. In a geometric sequence, $a_n = a_1 r^{n-1}$ and the first two terms are thus a_1 and $a_1 r$. Then $a_1 + a_1 r = 100a_1 \Rightarrow a_1(1 + r) = 100a_1 \Rightarrow 1 + r = 100$

$\Rightarrow \boldsymbol{r = 99}$.

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12. Let a be the shorter side and b be the longer side. Then $a = \frac{1}{3}b$. Using Pythagorean Theorem:

$$a^2 + b^2 = diagonal^2$$
$$\left(\frac{1}{3}b\right)^2 + b^2 = 9^2$$
$$\frac{1}{9}b^2 + b^2 = 81$$
$$\frac{10}{9}b^2 = 81 \Rightarrow b^2 = \frac{729}{10}$$
$$b = \sqrt{\frac{729}{10}} = \frac{27}{\sqrt{10}} = \frac{27\sqrt{10}}{10} \text{ and } a = \frac{9}{\sqrt{10}} = \frac{9\sqrt{10}}{10}$$

Mathematical Connections 8-3: Review Problems

13. $\boldsymbol{x = 3y}$.

14. If the middle even number is $2n$, then the next two are $2n + 2$ and $2n + 4$; the previous two are $2n - 4$ and $2n - 2$. Their sum is $2n - 4 + 2n - 2 + 2n + 2n + 2 + 2n + 4 = 10n$**.**

15. Let $\boldsymbol{x}$ be the number of Jack's CD's. Then the number of Julie's is $\boldsymbol{2x}$; the number of Tyto's is $3(2x) = \boldsymbol{6x}$.

16. **(a)** $P = 30 + (30 + d) + ((30 + d) + d) = 90 + 3d$

(b) $\$(d + 2d + 4d + 8d) = \boldsymbol{\$15d}$.

17. **(a)** If the middle term is x and the difference is d, then the terms are $x - d, x, x + d$. Their sum is $x - d + x + x + d = 3x$. Thus, the sum of any three consecutive terms in an arithmetic sequence is three times the middle term.

(b) If the middle term is x and the ratio is r then the terms are $\frac{x}{r}, x, xr$. Their product is $\frac{x}{r} \cdot x \cdot xr = x^3$. Thus, the product of any three consecutive terms in a geometric sequence is the middle term to the third power.

18. The sixth term is given by $\sqrt{5}(0.5)^5 = \sqrt{5} \cdot \left(\frac{1}{2}\right)^5$

$= \sqrt{5}\left(\frac{1}{32}\right) = \boldsymbol{\frac{\sqrt{5}}{32}}$.

19. We can write a mathematical model. Let r be the ratio. Then, the fourth term can be obtained by the expression $12 \cdot r^3$. Thus, $12 \cdot r^3 = \sqrt{5} \Rightarrow$ $\boldsymbol{r = \frac{\sqrt[6]{5}}{\sqrt[3]{12}}}$.

Assessment 8-4A: Functions

1. Where x is the first element in each ordered pair and $f(x)$ is the second:

(a) **Multiply** the input number by $^-2$; i.e., $f(x) = {}^-2x$.

(b) **Add 6** to the input number; i.e., $f(x) = x + 6$.

(c) **Square the input; i.e.** $f(x) = x^2$

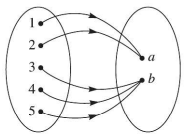
2. **(a)** **Not a function**. The element 1 is paired with both a and d.

(b) **A function**. Each element in the first set is paired with a unique element from the second.

(c) **A function**. Each distinct element in the first set is paired with an element from the second.

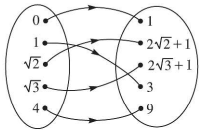
(d) **A function**. Each distinct element in the first set is paired with an element from the second.

3. **(a)** Answers may vary; e.g.:



(b) Each of the five elements in the domain have two choices for a pairing $\Rightarrow 2^5 = 32$ possible functions.

4. **(a)**



(b) $\{(0, 1), (1, 3), (\sqrt{2}, 2\sqrt{2} + 1), (\sqrt{3}, 2\sqrt{3} + 1), (4, 9)\}$.

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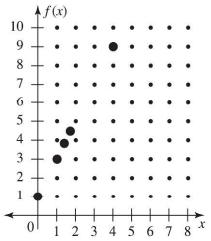
(c)

x	$f(x)$
0	1
1	3
$\sqrt{2}$	$2\sqrt{2}+1$
$\sqrt{3}$	$2\sqrt{3}+1$
4	9

(d)



5. (a) Function from $\mathbb{R}$ to {2}, a subset of R.

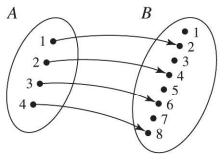
$f(x) = 2$ is called a constant function and its only output is 2.

(b) Function from $\mathbb{R}$ to $\{x \mid x \geq 0\}$, a subset of R. $f(x) = \sqrt{x}$ has output of nonnegative numbers only because $\sqrt{x}$ is the principal square root.

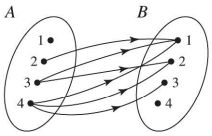
(c) Not a function because (4, ⁻2) and (4, 2) both satisfy the relation, but inputs can have only one associated output.

(d) Function from $\mathbb{R}$ to $\mathbb{R}$. $x = y^3 \Rightarrow y = \sqrt[3]{x}$ has output of all real numbers only because $\sqrt[3]{x}$ is the cubed root of all real numbers.

6. (a) (*i*)



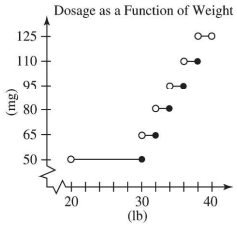
(*ii*)



(b) (*i*) A function from A to B because each element in A corresponds to one and only one element in B. The range is {2, 4, 6, 8}.

(*ii*) Not a function. Elements 3 and 4 in A correspond to more than one element in B; element 1 in A does not correspond to any element in B.

7. This is an example of **step function**; note in the following graph that a child weighing exactly 30, 32, 34,… pounds uses the lower dosage at the break point:

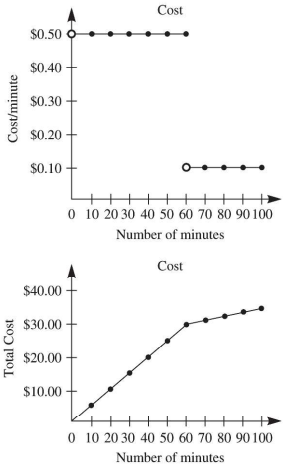


8. (a) $L(n)$ could be an arithmetic sequence with $a_3 = 8$ and $d = 3$. Then $8 = a_1 + (3-1)\cdot 3 \Rightarrow a_1 = 2$.

$L(n) = 2 + (n-1)\cdot 3 = \mathbf{3n - 1}$.

(b) $L(n)$ could be obtained by squaring n and adding 1, or $\boldsymbol{L(n) = n^2 + 1}$.

9. (a)



(b) It must be assumed that the company charges only for the **exact fraction of minutes used**.

(c) The two segments represent the **two different rates** per minute; the steeper one comes from the $50¢$ per minute charge for the first 60 minutes.

(d) If $c(t)$ in dollars is cost as a function of time t in minutes, then

$$c(t) = \begin{cases} \mathbf{0.50t} & \textbf{if } t \leq 60 \\ \mathbf{30 + 0.10(t - 60)} & \textbf{if } t > 60 \end{cases}$$

Where \$30 is the cost of the first 60 minutes.

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10. Let n be the elements of the domain:

(a) Assume an arithmetic sequence with $a_1 = 3$ and $d = 5$. Thus $\boldsymbol{f(n) = 3 + (n-1)\cdot 5 = 5n - 2}$.

(b) The output is obtained if 3 is raised to the *nth* power, so $\boldsymbol{f(n) = 3^n}$.

(c) A constant function, so $f(n) = 3$.

(d) Assume a geometric sequence with $a_1 = 3$ and $r = 3^{\frac{1}{3}}$. Thus

$$f(n) = 3\left(3^{\frac{1}{3}}\right)^{n-1} = 3^1 3^{\frac{1}{3}n - \frac{1}{3}} = 3^{\frac{1}{3}n + \frac{2}{3}} = 3^{\frac{n+2}{3}}$$

11. **(a)** $(g \circ f)(6) = g[f(6)] = f(6) - 5 = 7(6) - 5 = 37$

(b) $(g \circ f)(10) = g[f(10)] = f(10) - 5 = 7(10) - 5 = 65.$

(c) $(g \circ f)(\sqrt{10}) = g[f(\sqrt{10})] = f(\sqrt{10}) - 5 = 7(\sqrt{10}) - 5$.

(d) $(g \circ f)(0) = g[f(0)] = f(0) - 5 = \boldsymbol{^-5}$.

(e) $(g \circ f)(\frac{5}{8}) = g[f(\frac{5}{8})] = f(\frac{5}{8}) - 5 = 7(\frac{5}{8}) - 5 = \frac{35}{8} - \frac{40}{8} = {}^-\frac{5}{8}.$

(f) $(g \circ f)(n) = g[f(n)] = f(7n) - 5 = 7n - 5$.

12. $7n - 5 = 2n \Rightarrow 5n = 5 \Rightarrow n = 1.$

13. **(a)** **(i)** $(f \circ g)(6) = f[g(6)] = f[6 - 5] = f(1) = 7(1) = 7.$

(ii) $(\text{f} \circ g)(10) = f[\text{g}(10)] = f(10 - 5) = f(5) = 7(5) = 35.$

(iii) $(\text{f} \circ g)(\sqrt{10}) = f[\text{g}(\sqrt{10})] = f(\sqrt{10} - 5) = 7(\sqrt{10} - 5) = 7\sqrt{10} - 35$.

(iv) $(f \circ g)(0) = f[\text{g}(0)] = f(0 - 5) = f(^-5) = (^-5)(7) = {}^-35.$

(v) $(f \circ g)(\frac{5}{8}) = f[g(\frac{5}{8})] = f(\frac{5}{8} - 5) = f(\frac{5}{8} - \frac{40}{8}) = f(\frac{^-35}{8}) = \frac{^-35}{8} \cdot 7 = {}^-\frac{245}{8}.$

(vi) $(f \circ g)(n) = f[g(n)] = f(x - 5) = 7(x - 5) = 7x - 35$.

$(f \circ g)(x) = x$

(b) $7x - 35 = x \Rightarrow 6x = 35$

$x = \frac{35}{6}.$

14. **(a)** $f(f(0)) = f(^-0 + b) = {}^-(^-0 + b) + b = 0.$

(b) $f(f(2) = f(^-2 + b) = {}^-(^-2 + b) + b = 2$**.**

(c) $f(f(x)) = f(^-x + b) = {}^-(^-x + b) + b = x.$ This works for all real numbers.

15. **(a)** ***(i)*** **Yes**. $4n - 3 = 1 \Rightarrow t(1) = 1.$

(ii) **Yes**. $4n - 3 = 385 \Rightarrow t(97) = 385.$

(iii) **Yes**. $4n - 3 = 389 \Rightarrow t(98) = 389.$

(iv) **No**. If $4n - 3 = 392, n = \frac{395}{4}$ which is not a natural number.

(b) ***(i)*** **No**. 0 is not a natural number.

(ii) **Yes**. $n^2 = 25 \Rightarrow t(5) = 25.$

(iii) **Yes**. $n^2 = 625 \Rightarrow t(25) = 625.$

(iv) **No**. If $n^2 = 1000, n = \sqrt{1000}$ which is not a natural number.

(v) **No**. If $n^2 = 90, n = \sqrt{90}$, which is not a natural number.

(c) ***(i)*** **Yes**. $n(n - 1) = 0 \Rightarrow t(1) = 0.$

(ii) **Yes**. $n(n - 1) = 2 \Rightarrow t(2) = 2.$

(iii) **Yes**. $n(n - 1) = 5 \Rightarrow t(5) = 20.$

(iv) **No**. If $n(n - 1) = 999 \Rightarrow t(32.11) \approx 999$, but 32.11 is not a natural number.

16. **(a)** $f(x) = x^2 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}.$

(b) $f(x) = x^3 \Rightarrow x^3 = 2 \Rightarrow x = \sqrt[3]{2}.$

17. **(a)** ***(i)*** $(1,7) \Rightarrow 2 \cdot 1 + 2 \cdot 7 = \mathbf{16}.$

(ii) $(2,6) \Rightarrow 2 \cdot 2 + 2 \cdot 6 = \mathbf{16}.$

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(iii) $(6,2) \Rightarrow 2 \cdot 6 + 2 \cdot 2 = \mathbf{16}$.

(iv) $(\sqrt{5},\sqrt{5}) \Rightarrow 2 \cdot \sqrt{5} + 2 \cdot \sqrt{5} = \mathbf{4\sqrt{5}}$.

(b) If output (or perimeter) $= 20$ then
$2\ell + 2w = 20 \Rightarrow 2(\ell + w) = 20$
$\Rightarrow \ell + w = 10$.
The possibilities are **any ordered pair** $\mathbf{(l, 10 - l)}$ **with** $\mathbf{0 < l < 10}$.

(c) Domain:
$\{(\ell, w) \mid \ell$ and w are any positive real numbers$\}$.
The range is any positive real number.

18. (a) There were 600 cars on the road at 6:30; there were (as nearly as can be determined from the graph) 650 cars at 7:00 $\Rightarrow 650 - 600 =$ **50 cars** increase.

(b) The graph rises most steeply **between 6:00 and 6:30**; i.e., that is the period in which the increase in number of cars was greatest (by 200 cars).

(c) The graph is flat between 8:00 and 8:30; i.e., there was **no increase** in the number of cars.

(d) Only one period of the graph shows a drop: 700 cars at **8:30,** decreasing to 600 at **9:00**, or $700 - 600 =$ **100 cars** decrease.

(e) Segments are used because the data are continuous rather than discrete; e.g., there is a unique number of cars at 5:47 A.M. The assumption of a linear increase or decrease in traffic between hours may, however, be invalid and could lead to erroneous conclusions.

19. (a) ***(i)*** $H(2) = 128(2) - 16(2)^2 =$ **192 feet**.

(ii) $H(6) = 128(6) - 16(6)^2 =$ **192 feet**.

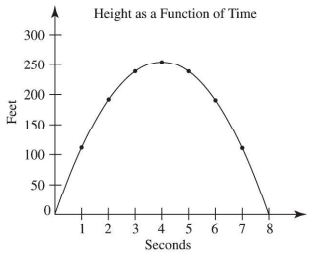
(iii) $H(3) = 128(3) - 16(3)^2 =$ **240 feet**.

(iv) $H(5) = 128(5) - 16(5)^2 =$ **240 feet**.

Some of the heights correspond to the height of the ball as it rises; some to its height as it falls.

(b) Plot:

t	$H(t) = 128t - 16t^2$
0	0
1	112
2	192
3	240
4	256
5	240
6	192
7	112
8	0



The ball will reach its highest point of **256 feet** at $t =$ **4 seconds**.

(c) $H(t) = 0$ at $t = 8$; i.e., the ball will hit the ground in **8 seconds**. Note that $H(t) = 0$ also at $t = 0$.

(d) Domain: $\mathbf{\{t | 0 \le t \le 8\}}$ **seconds.**

(e) Range: $\mathbf{\{H(t) | 0 \le H(t) \le 256\}}$ **feet**.

20. 7, 17, 31,… The number of vertical matchsticks in each shape are square numbers: 4, 9, 16, … = $(n+1)^2$. The horizontal match sticks are $3 \cdot 1, 4 \cdot 2, 5 \cdot 3, \ldots : (n+2)n$ So $S(n) = (n+1)^2 + (n+2)n$.

21. 1, 4, 16. …It is a geometric sequence with $a_1 = 1$ and $r = 4$. Then $S(n) = 4^{n-1}$.

22. (a) **A function**. $x + y = 2 \Rightarrow y = 2 - x$; for any input x, y is unique.

(b) **Not a function**. $x - y < 2 \Rightarrow y > x - 2$; for any input x, y may be any value greater than $x - 2$.

(c) **A function**. y is unique for any input x.

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(d) **A function**. $xy = 2 \Rightarrow y = \frac{2}{x}$; for any input x (except $x = 0$, for which y is undefined), y is unique.

(e) **A function**. y is unique for any input x.

(f) $|y| = x$ is not a function from x to y, $x = 1 \Rightarrow y = \pm 1$ which is not unique.

(g) $x^2 + y^2 = 1$ is not a function from x to y, $x = 0 \Rightarrow y = \pm 1$ which is not unique.

(h) **A function**. y is unique for any input x.

23. **(a)** **A function**. There is only one value of y for each value of x.

(b) **Not a function**. When $x = 1$ there are four values of y.

(c) **A function**. There is only one value of y for each value of x.

24. **(a)** B and H must be boys since there are no "is the sister of arrows" from either. The remainder; A, C, D, G, I, J, E, and F; must be girls.

(b) $\{(A, B), (A, C), (A, D), (C, A), (C, B), (C, D), (D, A), (D, B), (D, C), (E, H), (F, G), (G, F), (I, J), (J, I)\}$.

(c) **No**. A "is the sister of" three different people $\Rightarrow$ the relation is not a function.

25. **(a)** **Yes**. The set of all people has exactly one mother (assuming biological mothers only) each–more or less than one is not possible.

(b) **No**. Some elements of the set of all boys do not have a brother or may have more than one.

26. **(a)** **Not an equivalence relation:**

(i) Not reflexive. A person cannot be a parent to him/herself.

(ii) Not symmetric. John can be a parent to Jane, but Jane cannot be a parent to John.

(iii) Not transitive. If John is the parent of James and James is the parent of Joseph, John is not the parent of Joseph.

(b) **An equivalence relation** (reflexive, symmetric, and transitive):

(i) Reflexive. Juan is the same age as Juan.

(ii) Symmetric. If Juan is the same age as Juanita, then Juanita is the same age as Juan.

(iii) Transitive. If Juan is the same age as Jose and Jose is the same age as Victor, then Juan is the same age as Victor.

(c) **An equivalence relation:**

(i) Reflexive. Jo Ann has the same last name as herself.

(ii) Symmetric. If Jo Ann has the same last name as Cheryl, then Cheryl has the same last name as Jo Ann.

(iii) Transitive. If Jo Ann has the same last name as Cheryl and Cheryl has the same last name as Penelope, then Jo Ann has the same last name as Penelope.

(d) **An equivalence relation:**

(i) Reflexive. Vicky is the same height as herself.

(ii) Symmetric. If Barbara is the same height as Margarita, then Margarita is the same height as Barbara.

(iii) Transitive. If Willy is the same height as Billy and Billy is the same height as Don, then Willy is the same height as Don.

(e) **Not an equivalence relation:**

(i) Not reflexive. Cindy cannot be married to herself.

(ii) Symmetric. If Arnold is married to Pam, then Pam is married to Arnold.

(iii) Not transitive. If John is married to Clara and Clara is married to John, then John is not married to John.

(f) **Not an equivalence relation:**

(i) Reflexive. Peter lives within 10 miles of himself.

(ii) Symmetric. If Jon lives within 10 miles of Evangeline, then Evangeline lives within 10 miles of Jon.

(iii) Not transitive. If Fred lives within 10 miles of Jim and Jim lives within 10 miles of Herb, then Fred does not necessarily live within 10 miles of Herb.

(g) **Not an equivalence relation:**

(i) Not reflexive. Juan cannot be older than himself.

(ii) Not symmetric. If Jose is older than Mireya then Mireya cannot be older then Jose.

(iii) Transitive. If Jean is older than Mike and Mike is older than Cybil, then Jean is older than Cybil.

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Assessment 8-4B

1. (a) **Multiply ⁻3** to the input number; i.e., $f(x) = (^{-}3)x$.

(b) **Add 5** the input number ; i.e., $f(x) = x + 5$.

(c) **Square** the input number; i.e., $f(x) = x^2$.

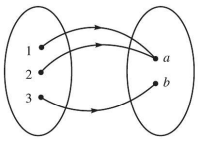
2. (a) **Not a function**. The element 2 from {1, 2, 3} is not paired with any element from the set $\{a, b, c, d\}$.

(b) **Not a function**. 1 is paired to more than one element from $\{a, b, c, d\}$, while 2 and 3 are not paired at all.

(c) **Function.**

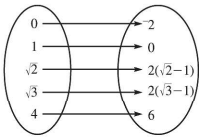
(d) **Not a function.** (2, b), and (2, c) both satisfy the relation, but inputs can have only one associated output.

3. (a) Answers may vary; for example:



(b) Each of the three elements in the domain have two choices for a pairing $\Rightarrow$ $\mathbf{2^3 = 8}$ possible functions.

4. (a)



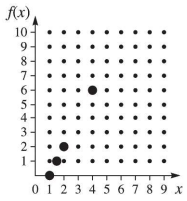
(b)

$\{(0, {}^{-}2), (1, 0), (\sqrt{2}, 2(\sqrt{2} - 1)), (\sqrt{3}, 2(\sqrt{3} - 1)), (4, 6)\}$.

(c)

x	$f(x)$
0	$^{-}2$
1	0
$\sqrt{2}$	$2(\sqrt{2} - 1)$
$\sqrt{3}$	$2(\sqrt{3} - 1)$
4	6

(d)



5. (a) **A function**. The function is {(0, 0), (1, 0), (2, 0), (3, 0), (x, 3)} where x is any real number other than 0,1,2 or 3.

(b) **Not a function**. Some elements of the domain correspond with more than one element of the range; e.g., (3, 0) and (3, 1), (4, 0) and (4, 1),….

(c) **A function**. {(0, 0), (1, 1), (2, 2),…, (9, 9)}.

(d) **Not a function** because (2, 2) and (2, ⁻2) both satisfy the relation, but inputs can only have one associated output in a function.

6. (a) If each element in A is doubled and then increased by 1, it corresponds to a unique element in B. Let the elements in A be x and the elements in B be $f(x)$; the rule is $f(x) = 2x + 1$.

(b) If each first element of the ordered pairs in A is squared and then added to the second element of the ordered pair, they correspond to a unique element in B. Let the elements in A be (x, y) and the elements in B be $f((x, y))$; the rule is $f((x, y)) = x^2 + y$.

7. (a) $T = 70°$ F. Then $C = 70 - 40 = 30$ chirps per 15 seconds $=$ **2 chirps** each second.

(b) 40 chirps per minute $=$ 40 chirps per 60 seconds $=$ 10 chirps per 15 seconds. $10 = T - 40 \Rightarrow T = 10 + 40 =$ **50°F**.

8. These solutions are possible if the patterns continue:

(a) $\boldsymbol{L(n) = n(n + 1)}$; e.g., if $n = 6$ then $n + 1 = 7$ and $6 \cdot 7 = 42$. Note that n and $L(n)$ do not have to be in increasing or decreasing order.

(b) $L(n)$ is obtained by taking the nth power of 2, or $\boldsymbol{L(n) = 2^n}$; e,g., if $n = 5$, $L(n) = 2^5 = 32$.

9. (a) The "Cost per Minute" graph shows 45¢ for the **cost of the sixth minute**; the "Total Cost for Calls" graph shows the **cost for a six-minute call** to be $2.70.

(b) It must be assumed that the cost per minute is always the same regardless of the length of

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the call, and that the company charges $45¢$ per minute for each fraction of a minute.

(c) ***(i)*** $c = 45¢$ per minute in the first graph.

(ii) At $45¢$ per minute, $c = 45t$ in the second graph, where c is cost in cents and t is time in minutes.

10. Let n be the elements of the domain; if the patterns continue:

(a) The output is obtained if n is doubled, or, alternatively, this is an arithmetic sequence with $a_1 = 2$ and $d = 2$. Thus $f(n) = 2 + (n - 1) \cdot 2 = \mathbf{2n}$.

(b) This is a geometric sequence with first tern 3 and common ratio 3. Thus $f(n) = \mathbf{3^{n-1}}$.

(c) This is a geometric sequence with first tern $\sqrt{3}$ and common ratio 1.
Thus $f(n) = \sqrt{3}(1)^{n-1} = \sqrt{3}$.

(d) This is a geometric sequence with first tern 3^{-1} and common ratio $3^{\frac{1}{2}}$. Thus
$f(n) = 3^{-1}(3^{\frac{1}{2}})^{n-1} = 3^{-1}3^{\frac{1}{2}n-\frac{1}{2}}$
$= 3^{\frac{n-3}{2}}$

11. **(a)** $(g \circ f)(5) = g[f(5)] = g(5 - 1) = 7 \cdot 4 = \mathbf{28}$.

(b) $(g \circ f)(\sqrt{3}) = g[f(\sqrt{3})] = g(\sqrt{3} - 1) = 7 \cdot (\sqrt{3} - 1) = 7\sqrt{3} - 7$.

(c) $(g \circ f)(10) = g[f(10)] = g(10 - 1) = 7 \cdot 9 = \mathbf{63}$.

(d) $(g \circ f)(a) = g[f(a)] = g(a - 1) = \mathbf{7(a - 1)}$.

(e) $(g \circ f)(\frac{7}{6}) = g[f(\frac{7}{6})] = g(\frac{7}{6} - 1) = g(\frac{1}{6}) = 7 \cdot \frac{1}{6} = \frac{7}{6}$.

12. $7x - 7 = 2x \Rightarrow 5x = 7 \Rightarrow x = \frac{7}{5}$

13. **(a)** **(i)** $(f \circ g)(5) = f[g(5)] = f(35) = 35 - 1 = 34$.

(ii) $(f \circ g)(\sqrt{3}) = f[g(\sqrt{3})] = f(7\sqrt{3}) = 7\sqrt{3} - 1$.

(iii) $(f \circ g)(10) = f[g(10)] = f(70) = 70 - 1 = 69$.

(iv) $(f \circ g)(a) = f[g(a)] = g(7a) = 7a - 1$.

(v) $(f \circ g)(\frac{7}{6}) = f[g(\frac{7}{6})] = g(\frac{49}{6}) = \frac{49}{6} - 1 = \frac{43}{6}$.

(b) $7x - 1 = x \Rightarrow 6x = 1 \Rightarrow x = \frac{1}{6}$

14. **(a)** $f(f(0)) = f(^-(0 + b)) = ^-(^-(0 + b) + b) = 0$

(b) $f(f(\sqrt{2})) = f(^-(\sqrt{2} + b)) = ^-(^-(\sqrt{2} + b) + b) = \sqrt{2}$

(c) $f(f(x)) = f(^-(x + b)) = ^-(^-(x + b) + b) = x$
So for all real numbers.

15. **(a)** $\frac{2x-1}{2x+1} = 2 \Rightarrow 2x - 1 = 2(2x + 1)$
$2x - 1 = 4x + 2 \Rightarrow ^-2x = 3 \Rightarrow x = ^-\frac{3}{2}$

(b) $\frac{2x-1}{2x+1} = 0 \Rightarrow 2x - 1 = 0(2x + 1)$
$2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

(c) $\frac{2x-1}{2x+1} = ^-\frac{1}{2} \Rightarrow 2x - 1 = ^-\frac{1}{2}(2x + 1)$
$2x - 1 = ^-x - \frac{1}{2} \Rightarrow 3x = \frac{1}{2} \Rightarrow x = \frac{1}{6}$

(d) $\frac{2x-1}{2x+1} = 1 \Rightarrow 2x - 1 = 1(2x + 1)$
$2x - 1 = 2x + 1 \Rightarrow 0x = 2 \Rightarrow 0 = 2$
This is not possible.

16. **(a)** ***(i)*** **Yes**. $n^2 = 1 \Rightarrow t(1) = 1$.

(ii) **Yes**. $n^2 = 4 \Rightarrow t(2) = 4$.

(iii) **Yes**. $n^2 = 9 \Rightarrow t(3) = 9$.

(iv) **No**. If $n^2 = 10$, then $n = \sqrt{10}$ which is not a natural number.

(v) **Yes**. $n^2 = 900 \Rightarrow t(30) = 900$.

(b) ***(i)*** **Yes**. $n(n + 1) = 2 \Rightarrow t(1) = 2$.

(ii) **Yes**. $n(n + 1) = 12 \Rightarrow t(3) = 12$.

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(iii) **Yes**. $n(n+1) = 2550 \Rightarrow t(50) = 2550$.

(iv) **No**. $n(n+1) = 2600 \Rightarrow t(50.49) \approx 2600$, but 50.49 is not a natural number.

17. (a) ***(i)*** $(1, 4) \Rightarrow 2 \cdot 1 + 2 \cdot 4 = \mathbf{10}$.

(ii) $(2, 1) \Rightarrow 2 \cdot 2 + 2 \cdot 1 = \mathbf{6}$

(iii) $(1, 2) \Rightarrow 2 \cdot 1 + 2 \cdot 2 = \mathbf{6}$.

(iv) $(\sqrt{3}, \sqrt{3}) \Rightarrow 2 \cdot \sqrt{3} + 2 \cdot \sqrt{3} = \mathbf{4\sqrt{3}}$.

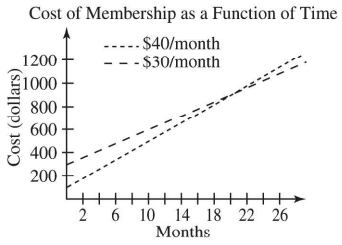
(v) $(x, y) \Rightarrow \mathbf{2 \cdot x + 2 \cdot y}$.

(b) If output (or perimeter) = 20 then $2\ell + 2w = 20 \Rightarrow 2(\ell + w) = 20 \Rightarrow \ell + w = 10$. The possibilities are **{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)}**.

(c) **No**. Outputs are single numbers, not ordered pairs.

18. (a) $C(x) =$ \$100 + \$40 per month $(x) \Rightarrow$ $\boldsymbol{C(x) = \$(100 + 40x)}$.

(b) and (c)



(d) The graphs cross at **20 months** where $300 + 30x = 100 + 40x$; after that the second plan is less expensive.

19. (a) ***(i)*** $H(2) = 128(2) - 16(2)^2 = \mathbf{192\ feet}$.

(ii) $H(6) = 128(6) - 16(6)^2 = \mathbf{192\ feet}$.

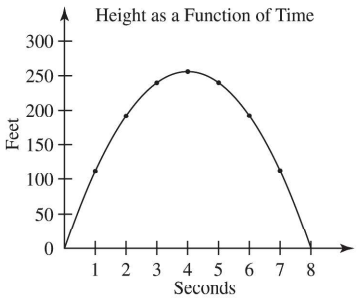
(iii) $H(3) = 128(3) - 16(3)^2 = \mathbf{240\ feet}$.

(iv) $H(5) = 128(5) - 16(5)^2 = \mathbf{240\ feet}$.

Some of the heights correspond to the height of the ball as it rises; some to its height as it falls. The graph is a parabola with vertex (4, 256), so **256** is the highest point.

(b) Plot:

t	$H(t) = 128t - 16t^2$
0	0
1	**112**
2	192
3	240
4	256
5	240
6	192
7	**112**
8	0



The ball will reach its highest point of **256 feet** at $\boldsymbol{t = 4}$ **seconds**.

(c) $H(t) = 0$ at $t = 8$; i.e., the ball will hit the ground in **8 seconds**. Note that $H(t) = 0$ also at $t = 0$.

(d) Domain: $\boldsymbol{\{t \mid 0 \le t \le 8\}}$ **seconds**.

(e) Range: $\boldsymbol{\{H(t) \mid 0 \le H(t) \le 256\}}$ **feet**.

20. (a) Assuming the matchstick figure patterns continue, expand each to the fourth figure and count: Each figure adds one row and one column to the preceding figure, so the fourth figure would have five rows and five columns. The two matchsticks at the lower left, though, are removed, leaving **58** matchsticks.

(b) If we include the missing 2 matchsticks from the bottom left then there are $2 \cdot 3 = (n+1)(n+2)$ vertical matchsticks and $2 \cdot 3 = (n+1)(n+2)$ horizontal matchsticks in the first shape. There are $3 \cdot 4 = (n+1)(n+2)$ vertical matchsticks and $3 \cdot 4 = (n+1)(n+2)$ horizontal matchsticks in the second shape. There are $4 \cdot 5 = (n+1)(n+2)$ vertical matchsticks and $4 \cdot 5 = (n+1)(n+2)$ horizontal matchsticks in the third shape Each shape has $(n+1)(n+2) + (n+1)(n+2) = 2(n+1)(n+2)$

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matchsticks. If the 2 are subtracted from the bottom left, there are

$S(n) = 2(n + 1)(n + 2) - 2$ matchsticks.

21. **(a)** 2,5,9,… $S(n) = \frac{n(n+1)}{2} + n$.

(b) $S(n) = n + 2 + 2n = \mathbf{3n + 2}$.

22. **(a)** **A function**. $x - y = 2 \Rightarrow y = x - 2$; for any input x, y is unique.

(b) **Not a function**. $x + y < 20 \Rightarrow y < 20 - x$; for any input x, y may be any value less than $20 - x$. Thus, it is easy to find two outputs y for each input x.

(c) **A function**. For any input x, y is unique.

(d) **A function**. For any input x, y is unique.

(e) **A function**. For any input x, y is unique.

(f) $|y| = |x|$ is not a function; if $x = \sqrt{2}$ then $y = \pm 1$ so there is no unique output.

(g) $x^2 - y^2 = 1$ is not a function; if $x = 1$ then $y = \pm 1$ so there is no unique output.

(h) **A function**. For any input x, y is unique.

23. **(a)** **A function**. For any value x, there is a unique value of y.

(b) **A function**. For any value x, there is a unique value of y.

(c) **A function**. For any value x, there is a unique value of y.

(d) **A function**. For any value x, there is a unique value of y.

(e) **Not A function**. For example, for x=1, there is two distinct y values.

24. **(a)** ***(i)*** **Symmetric**. If $x = a$ and $y = b$ satisfies the relation, then $x = b$ and $y = a$ also satisfies it (by the commutative property).

(ii) **Not symmetric**. $a - b \neq b - a$.

(iii) **Symmetric**. If $x = a$ and $y = b$ satisfies the relation, then $x = b$ and $y = a$ also satisfies it.

(iv) **Symmetric**. If $y = x$, then $x = y$.

(v) **Not symmetric**. $(2, 4) \neq (4, 2)$.

(b) **All** are functions of y in terms of x. In each, for any input x there is a unique output y.

25. **(a)** B and F must be boys, since others are "the sister of" them but they are not "the sister of" in return. E and H are not paired, so their gender is indeterminate. The remainder; A, C, D, I, J, and G, must be girls.

(b) $\{(A, B), (A, C), (A, D), (C, A), (C, B), (C, D), (D, A), (D, B), (D, C), (G, F), (I, J), (J, I)\}$.

(c) **No**. A "is the sister of" three different people $\Rightarrow$ the relation is not a function.

26. **(a)** **A function**. Each element of the domain is paired with one and only one element of the range.

(b) A relation but **not a function**. New York from the domain is paired with two elements of the range.

(c) A relation but **not a function**. "mother" from the domain could be paired with more than one element of the range.

(d) **A function**. Each element of the domain is paired with one and only one element of the range.

(e) A relation but **not a function**. The element 1 from the domain could be paired with any odd number from the range to produce an even number.

27. In each of the following, use the nonempty set $\{1, 2, 3\}$ as an example

(a) **An equivalence relation**:

(i) Reflexive. Assume that the nonempty set is called A. The relation "is equal to" is reflexive, because for any subset of A, call it B, $B = B$ by the definition of set equality.

(ii) Symmetric. Assume that B and C are subsets of A, and that $B = C$. This means that every element in B is in C and every element in C is in B. Thus, $C = B$ is also true, making the relation "is equal to" symmetric on the set of subsets of A.

(iii) Transitive. Assume that B, C, and D are subsets of A and that $B = C$ and $C = D$. Then because of set equality, every element of B is an element of C and every element of C is an element of D, making B a subset of D. Further, every element of D is an element of C and every element of B, so D is a subset of B. Thus $B = D$, proving that "is equal to" is transitive.

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(b) Not an equivalence relation:

(i) Not reflexive. $\{1\} \not\subset \{1\}$.

(ii) Not symmetric. If $\{1\} \subset \{1, 2\}$ then $\{1, 2\} \not\subset \{1\}$.

(iii) Transitive. If $\{1\} \subset \{1, 2\}$ and $\{1, 2\} \subset \{1, 2, 3\}$ then $\{1\} \subset \{1, 2, 3\}$.

(c) Not an equivalence relation:

(i) Not reflexive. $\{1\}$ is not $\neq \{1\}$.

(ii) Symmetric. If $\{1\} \neq \{2\}$ then $\{2\} \neq \{1\}$.

(iii) Not transitive. If $\{1\} \neq \{2\}$ and $\{2\} \neq \{1\}$ then $\{1\}$ is not $\neq \{1\}$.

(d) An equivalence relation:

(i) Reflexive. $n(\{1, 2\}) = n(\{2, 1\})$.

(ii) Symmetric. If $n(\{1, 2\}) = n(\{1, 3\})$ then $n(\{1, 3\}) = n(\{1, 2\})$.

(iii) Transitive. If $n(\{1, 2\}) = n(\{1, 3\})$ and $n(\{1, 3\}) = n(\{2, 3\})$ then $n(\{1, 2\}) = n(\{2, 3\})$.

Mathematical Connections 8-4: Review Problems

22. **(a)**
$$2\sqrt{2} - \sqrt{2}x = x$$
$$2\sqrt{2} = x + \sqrt{2}x$$
$$2\sqrt{2} = x(1 + \sqrt{2})$$
$$x = \frac{2\sqrt{2}}{1+\sqrt{2}}$$

(b)
$$\frac{3}{4}x = \frac{\sqrt{2}}{2}x + 3 \Rightarrow \frac{3}{4}x - \frac{\sqrt{2}}{2}x = 3$$
$$x\left(\frac{3}{4} - \frac{\sqrt{2}}{2}\right) = 3 \Rightarrow x\left(\frac{3}{4} - \frac{2\sqrt{2}}{4}\right) = 3$$
$$x\left(\frac{3-2\sqrt{2}}{4}\right) = 3 \Rightarrow x = \frac{12}{3-2\sqrt{2}}$$

(c)
$$(x-1)^2 = 2 \Rightarrow x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

(d)
$$\sqrt{2}(2x-3) = \sqrt{3}(2x-3)$$
$$\sqrt{2}(2x-3) - \sqrt{3}(2x-3) = 0$$
$$(2x-3)(\sqrt{2} - \sqrt{3}) = 0$$
$$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$$

(e) $\sqrt{(x-1)^2} = x - 1 \Rightarrow$ True for all $x \geq 1$.

(f) $\sqrt{(x-1)^2} = 1 - x \Rightarrow$ True for all $x \leq 1$.

23. Since distance = rate × time, the fast car will travel $70t$ miles in the same time as the slow car travels $60t$ miles. Then $70t - 40 = 60t \Rightarrow 10t = 40 \Rightarrow$ ***t* = 4 hours**.

24. **(a)** Answers may vary. $\sqrt{3} \approx 1.7320508$, so **1.8** and **1.85** are two rational numbers between $\sqrt{3}$ and 2.

(b) Answers may vary. $\frac{11}{13} \approx 0.84615$ and $\frac{12}{13} \approx 0.92307$ Irrational numbers between $\frac{11}{13}$ and $\frac{12}{13}$ include **0.901001000…** and **0.9020020002…** .

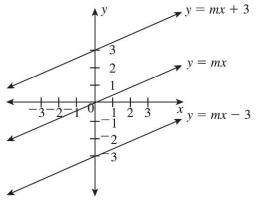
25. Answers may vary. For example, $\sqrt{2} + {}^{-}\sqrt{2} = 0$, a rational number.

Assessment 8-5A: Equations in a Cartesian Coordinate System

1. The graph of $y = mx + 3$ contains the point $(0, 3)$ and is parallel to the line $y = mx$. Similarly, the graph of $y = mx - 3$ contains the point $(0, {}^{-}3)$ and is parallel to $y = mx$ (see below).

(a) Parallel line; *y*-intercept = 3.

(b) Parallel line; *y*-intercept = ${}^{-}3$.



(c) The graph of $y = mx$ with a slope three times *m* and the same *y*-intercept.

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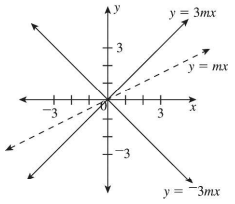
(d) The graph of $y = mx$ with a slope negative three times m and the same y-intercept.



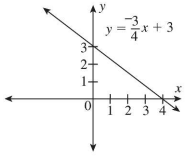
2. Each of the following is in the form $y = mx + b$, where m represents the slope of the line and b represents the y-coordinate of the y-intercept. The x-intercept in each case can be found by setting y equal to 0 and solving for x.

(a) Given $y = \frac{-3}{4}x + 3$:

(i) $m = \frac{-3}{4}$ and $b = 3$.

(ii) If $y = 0 = \frac{-3}{4}x + 3 \Rightarrow x = 4$; the x-intercept is at $(4, 0)$.

(iii) Draw a line through $(0, 3)$ and $(4, 0)$.

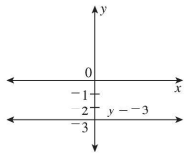


(b) Given $y = {}^{-}3$:

(i) $m = 0$ and $b = {}^{-}3$.

(ii) There is no x-intercept; i.e., $y = {}^{-}3$ is a horizontal line.

(iii) Draw a line through $(0, {}^{-}3)$ parallel to the x-axis.

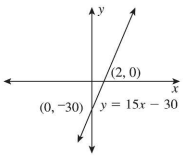


(c) Given $y = 15x - 30$:

(i) $m = 15$ and $b = {}^{-}30$.

(ii) If $y = 0 = 15x - 30 \Rightarrow x = 2$.

(iii) Draw a line through $(0, {}^{-}30)$ and $(2, 0)$.



3. When the equations in Problem 2 are in the form $y = mx + b$, b is the y-intercept and is located at $(0, b)$. The x-intercept may be found by setting y equal to 0 and solving for x. If x is some value a, the x-intercept is located at $(a, 0)$.

(a) $y = \frac{-3}{4}x + 3$:

(i) $b = 3$. y-intercept is at **(0, 3)**.

(ii) If $y = 0 = \frac{-3}{4}x + 3 \Rightarrow x = 4$. x-intercept is at **(4, 0)**.

(b) $y = {}^{-}3$:

(i) $b = {}^{-}30$. y-intercept is at **(0, ⁻3)**.

(ii) There is **no *x*-intercept**; i.e., $y = {}^{-}3$ is a horizontal line parallel to the x-axis.

(c) $y = 15x - 30$.

(i) $b = {}^{-}30$. y-intercept is at **(0, ⁻30).**

(ii) If $y = 0 = 15x - 30 \Rightarrow x = 2$. x-intercept is at **(2, 0)**.

4. **(a)** For ease in calculating, use the points $(0, 32)$ and $(100, 212)$. Slope $(m) = \frac{212-32}{100-0} = \frac{9}{5} \Rightarrow$ $F = \frac{9}{5}C + b$.

The F-intercept is at $(0, 32)$. $b = 32$; thus $\boldsymbol{F = \frac{9}{5}C + 32}$.

(b) $F = \frac{9}{5}C + 32 \Rightarrow \frac{9}{5}C = F - 32 \Rightarrow$ $C = \boldsymbol{\frac{5}{9}(F - 32)}$.

(c) Consider the relationship $F = \frac{9}{5}C + 32$. Substitute C for F:

$$\begin{aligned} C &= \tfrac{9}{5}C + 32 \\ 5c &= 5\left(\tfrac{9}{5}C + 32\right) \\ 5c &= 9c + 160 \\ -4c &= 160 \\ c &= -40. \end{aligned}$$

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The ordered-pair $\mathbf{(-40, -40)}$ represents where degrees Celsius and degrees Fahrenheit are the same.

5. When the equations are in slope-intercept form, $y = mx + b$, m is the slope of the line and b is the y-intercept, located at $(0, b)$.

(a) ***(i)*** $3y - x = 0 \Rightarrow 3y = x \Rightarrow \boldsymbol{y = \frac{1}{3}x}$.

(ii) $m = \frac{1}{3}$; y-intercept is at $(0, 0)$.

(b) ***(i)*** $x + y = 3 \Rightarrow \boldsymbol{y = {}^{-}x + 3}$.

(ii) $m = {}^{-}1$; y-intercept is at $(0, 3)$.

(c) ***(i)*** $x = 3y \Rightarrow \boldsymbol{y = \frac{1}{3}x}$.

(ii) $m = \frac{1}{3}$; y-intercept is at $(0, 0)$.

6. **(a)** $m = \frac{{}^{-}2-3}{1-{}^{-}4} = \frac{{}^{-}5}{5} = {}^{-}1$, thus $y = {}^{-}x + b$.

Substituting x and y from $({}^{-}4, 3)$: $(3) = {}^{-}({}^{-}4) + b$, thus $b = {}^{-}1$.

So $\boldsymbol{y = {}^{-}x - 1}$.

(b) $m = \frac{1-0}{2-0} = \frac{1}{2}$, thus $y = \frac{1}{2}x + b$. The line goes through the origin, thus $b = 0$.

So $\boldsymbol{y = \frac{1}{2}x}$.

(c) $m = \dfrac{0 - \frac{{}^{-}1}{2}}{\frac{1}{2} - 0} = \dfrac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = 1$, thus $y = x + b$.

Substituting x and y from $\left(\frac{1}{2}, 0\right)$: $(0) = \left(\frac{1}{2}\right) + b$, thus $b = \frac{{}^{-}1}{2}$.

So $\boldsymbol{y = x - \frac{1}{2}}$.

(d) $m = \frac{{}^{-}3 - {}^{-}3}{{}^{-}1 - 0} = 0$, thus $y = 0x + b$.

${}^{-}3 = 0x + b \Rightarrow b = {}^{-}3$

So $y = {}^{-}3$.

7. Answers may vary.

(a) Both points include the coordinate $y = 2$, thus the equation of the line is $y = 2$. Other points could be $({}^{-}3, 2), (5, 2), \ldots$.

(b) Both points include the coordinate $x = 0$, thus the line is the y-axis. Other points could be $(0, 1), (0, {}^{-}6), \ldots$.

8. **(a)** $({}^{-}2, 0), ({}^{-}2, 1)$, and (x, y) are collinear, thus the value of x at each point is ${}^{-}2$.

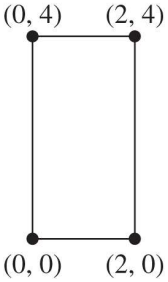
(i) $\boldsymbol{x = {}^{-}2}$ is the equation of the line.

(ii) y may assume **any real value**.

(b) ***(i)*** The fourth quadrant implies that $\boldsymbol{x > 0}$ and $\boldsymbol{y < 0}$.

(ii) x and y may assume **any real value** greater than zero or less than zero, respectively.

9. The rectangle is shown below:



It has dimensions of $2 - 0 = 2$ and $4 - 0 = 4$.

(i) Area $= 2 \cdot 4 = \mathbf{8}$ **square units**.

(ii) Perimeter $= 2 \cdot 2 + 2 \cdot 4 = \mathbf{12}$ **units**.

10. **(a)** $\boldsymbol{x = 3}$, a vertical line through $(3, 0)$.

(b) $\boldsymbol{y = 5}$, a horizontal line through $({}^{-}4, 5)$.

11. In each case, slope $(m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

(a) $m = \frac{0-3}{{}^{-}5-4} = \frac{{}^{-}3}{-9} = \boldsymbol{\frac{1}{3}}$.

(b) $m = \frac{2-2}{1-\sqrt{5}} = \frac{0}{1-\sqrt{5}} = \mathbf{0}$.

(c) $m = \frac{b-a}{b-a} = \mathbf{1}$ (if $b \neq a$).

(d) $m = \frac{{}^{-}2-2}{\sqrt{5}-\sqrt{5}} = \frac{{}^{-}4}{0}$ does not exist.

12. In each case, since m is known, substitute x and y values into the equation $y = mx + b$ and solve for b.

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(a) Use the point $(4,3)$. $(3) = \frac{1}{3}(4) + b \Rightarrow$ $b = \frac{5}{3}$, thus $\boldsymbol{y = \frac{1}{3}x + \frac{5}{3}}$.

(b) $\boldsymbol{y = 2}$ (a horizontal line).

(c) Use the point $(a, a) \Rightarrow (a) = 1(a) + b \Rightarrow$ $b = 0$, thus $\boldsymbol{y = x}$ (if $b \neq a$).

13. Answers may vary, depending on estimates from the fitted line; e.g.:

(a) From the fitted line, estimate point coordinates of $(50,10)$ and $(80,40)$. $m = \frac{40-10}{80-50} = \frac{30}{30} = 1$.

Use the point $(50,10)$ and substitute $T = 50$ and $C = 10$ into $C = 1T + b$ (i.e., an equation of the form $y = mx + b$). $(10) = 1(50) + b \Rightarrow b = -40$. The equation is then $\boldsymbol{C = T - 40}$.

(b) **If** $T = 90°$, then $C = (90) - 40 = \mathbf{50}$ **chirps** per 15-second interval.

(c) $N = 4C$; i.e., there are four 15-second intervals in one minute. $N = 4(T - 40)$, or $\boldsymbol{N = 4T - 160}$.

14. **(a)** $y = 2x - 20$ is line $\overleftrightarrow{BC}$ because the x-intercept is (10,0) and the y-intercept is (0, -20). Then $y = 4 - 2x$ is line $\overleftrightarrow{AB}$.

(b) Point D has the same x-coordinate as point C which is 10. Point D lies on $\overleftrightarrow{AB}$, so $y = 4 - 2(10) = -16$. Point D has coordinates $(10, -16)$.

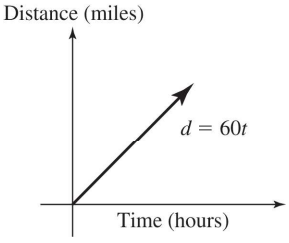
15. **(a)** The lines have the same x-intercept, -3.

(b) The lines have the same x-intercept, 1.

(c) The lines have the same slope, -2.

(d) The lines have the same slope, -1.

16. **(a)** Graphs may vary.



(b) Use the points $(0,0)$ and $(1,60)$. Then $m = \frac{60-0}{1-0} = \mathbf{60}$.

17. **(a)** If $y = 3x - 1$ and $y = x + 3$, then $3x - 1 = x + 3 \Rightarrow 2x = 4 \Rightarrow x = 2$.

Substituting $x = 2$ into $y = x + 3 \Rightarrow y = 5$; or a unique solution of **(2, 5)**.

(b) Multiply $3x + 4y = -17$ by $2 \Rightarrow$ $6x + 8y = -34$. Multiply $2x + 3y = -13$ by $-3 \Rightarrow -6x + -9y = 39$.

Add the equations:

$$\begin{array}{rcrcr} 6x & + & 8y & = & -34 \\ -6x & + & -9y & = & 39 \\ \hline & & -y & = & 5 \end{array}$$

or $y = -5$.

Substitute $y = -5$ into $3x + 4y = -17$.

$3x + 4(-5) = -17 \Rightarrow x = 1$; or a unique solution of **(1, −5)**.

(c) Multiply $-\frac{2}{3}x + y = \frac{1}{3}$ by $3 \Rightarrow -2x + 3y = 1$

$$-2x + 3y = 1$$

and add the equations: $\underline{2x - 3y = -1}$. There $0 = 0$

are infinitely many solutions of the form $\left(x, \frac{2}{3}x + \frac{1}{3}\right)$.

(d) Substitute $y = 1 - x$ in $y = x - 1$:

$1 - x = x - 1 \Rightarrow 2 = 2x \Rightarrow x = 1$. Substitute 1 for x: $y = 1 - 1 \Rightarrow y = 0$. There is a unique solution (1,0).

18. Let g be the number of gallons of gasoline and let k be the number of gallons of kerosene on the truck.

Then: $g + k = 5000 \Rightarrow g = 5000 - k$.

$\$0.13g + \$0.12k = \$640 \Rightarrow 13g + 12k = 64000$.

Substitute $g = 5000 - k$ into $13g + 12k = 64000 \Rightarrow 13(5000 - k) + 12k = 64000 \Rightarrow$ $k = \mathbf{1000}$ **gallons of kerosene**.

Substitute $k = 1000$ into $g = 5000 - k \Rightarrow$ $g = \mathbf{4000}$ **gallons of gasoline**.

19. Let d be the number of dimes and let q be the number of quarters. Then:

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$d + q = 27 \Rightarrow d = 27 - q.$

$\$0.10d + \$0.25q = \$5.25 \Rightarrow 10d + 25q = 525.$

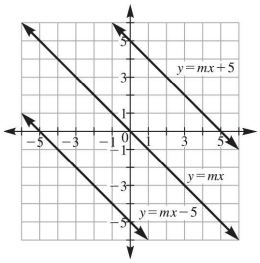
Substitute $d = 27 - q$ into $10d + 25q = 525 \Rightarrow$ $10(27 - q) + 25q = 525$. $q =$ **17 quarters**.

Substitute $q = 17$ into $d = 27 - q$. $d =$ **10 dimes**.

Assessment 8-5B

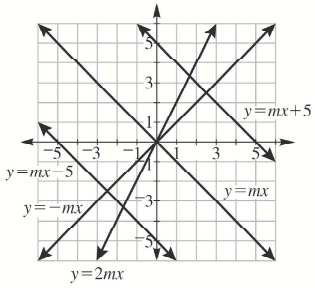
1. (a) The graph of $y = mx + 5$ contains the point $(0, 5)$ and is parallel to the line $y = mx$.

(b) Similarly, the graph of $y = mx - 5$ contains the point $(0, -5)$ and is parallel to the line $y = mx$.



(c) The graph of $y = {}^{-}mx$ has the same intercept as $y = mx$, but a negative slope.

(d) The graph of $y = 2mx$ has the same intercept as the graph of $y = mx$, but a slope twice as steep.



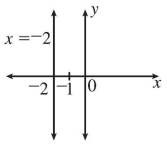
2. Each of the following [except part (a)] is in the form $y = mx + b$, where m represents the slope of the line and b represents the y-coordinate of the y-intercept. The x-intercept in each case can be found by setting y equal to 0 and solving for x.

(a) Given $x = {}^{-}2$:

(*i*) m is undefined, thus there is no y-intercept.

(*ii*) The x-intercept is at $x = {}^{-}2$.

(*iii*) Draw a line through $({}^{-}2, 0)$ parallel to the y-axis.

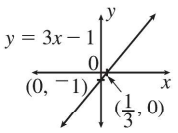


(b) Given $y = 3x - 1$:

(*i*) $m = 3$ and $b = {}^{-}1$.

(*ii*) If $y = 0 = 3x - 1 \Rightarrow x = \frac{1}{3}$.

(*iii*) Draw a line through $(0, {}^{-}1)$ and $\left(\frac{1}{3}, 0\right)$.

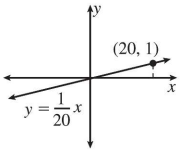


(c) Given $y = \frac{1}{20}x$:

(*i*) $m = \frac{1}{20}$ and $b = 0$.

(*ii*) To find another point, aribitrarily assign a value to x and solve for y; e.g., if $x = 20 \Rightarrow y = 1$.

(*iii*) Draw a line through $(0, 0)$ and $(20, 1)$.



3. When the equations in problem lare in the form $y = mx + b$, b is the y-intercept and is located at $(0, b)$. The x-intercept may be found by setting y equal to 0 and solving for x. If x is some value a, the x-intercept is located at $(a, 0)$.

(a) $x = {}^{-}2$:

(*i*) There is **no *y*-intercept**; i.e., $x = {}^{-}2$ is a vertical line.

(*ii*) The x-intercept is at $x = {}^{-}2$, or **$({}^{-}2, 0)$**.

(b) $y = 3x - 1$:

(*i*) $b = {}^{-}1$. y-intercept is at **$(0, {}^{-}1)$**.

(*ii*) if $y = 0 = 3x - 1 \Rightarrow x = \frac{1}{3}$.

x-intercept is at $\left(\frac{1}{3}, 0\right)$.

(c) $y = \frac{1}{20}x$:

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(i) $b = 0$. y-intercept is at **(0, 0)**.

(ii) If $y = 0 = \frac{1}{20}x \Rightarrow x = 0$. x-intercept is at **(0, 0)**.

4. Think of slope as a ratio: $\frac{39.37 \text{ in}}{1 \text{ m}}$. If x is a measure in meters, then $39.37 \text{ in/m} \cdot x = 39.37 \text{ in}$. If y is the measure in inches, then the equation is $\boldsymbol{y = 39.37x}$.

5. When the equations are in slope-intercept form, $y = mx + b$, m is the slope of the line and b is the y-intercept, located at $(0, b)$.

(a) ***(i)*** $\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 12\left(\frac{x}{3} + \frac{y}{4}\right) = 12(1) \Rightarrow$

$4x + 3y = 12 \Rightarrow 3y = {}^{-}4x + 12.$

$\boldsymbol{y = \frac{-4}{3}x + 4}$.

(ii) $m = \frac{-4}{3}$; y-intercept is at $(0, 4)$.

(b) ***(i)*** $3x - 4y + 7 = 0 \Rightarrow {}^{-}4y =$

${}^{-}3x - 7 \Rightarrow 4y = 3x + 7$. $\boldsymbol{y = \frac{3}{4}x + \frac{7}{4}}$.

(ii) $m = \frac{3}{4}$; y-intercept is at $\left(0, \frac{7}{4}\right)$.

(c) ***(i)*** $x - y = 4(x - y) \Rightarrow x - y =$

$4x - 4y \Rightarrow 0 = 3x - 3y$. $\boldsymbol{y = x}$.

(ii) $m = 1$; y-intercept is at $(0, 0)$.

6. **(a)** Both points include the coordinate $y = 1$. The equation of the line is $\boldsymbol{y = 1}$.

(b) Both points include the coordinate $x = 2$. The equation of the line is $\boldsymbol{x = 2}$.

(c) Both points include the coordinate $y = 0$. The equation of the line is $\boldsymbol{y = 0}$.

(d) Both points include the coordinate $y = {}^{-}\sqrt{3}$. The equation of the line is $y = {}^{-}\sqrt{3}$.

7. Answers may vary.

(a) Both points include the coordinate $x = {}^{-}1$, thus the equation of the line is $x = {}^{-}1$. Other points could be $({}^{-}1, 7), ({}^{-}1, {}^{-}5), \ldots$.

(b) Both points have the same x and y value; i.e., the equation of the line is $y = x$. All other points are of the form (a, a); other points could be $(3, 3), ({}^{-}1, {}^{-}1), \ldots$.

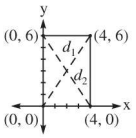
8. **(a)** $({}^{-}2, 1), (0, 1)$, and (x, y) are collinear, thus the value of y at each point is 1.

(i) $\boldsymbol{y = 1}$ is the equation of the line.

(ii) x may assume **any real value**.

(b) $(x, y), (0, 0)$, and $({}^{-}1, {}^{-}1)$ are collinear, and in $(0, 0)$ and $({}^{-}1, {}^{-}1)$ the values of x and y are the same. Since the points are collinear, $x = y$.

9. **(a)** Sketches may vary.



(b) Coordinates of the other vertices are (0, 0), (0, 6), and (4, 0).

(c) There are two diagonals, one with positive slope and one with negative slope.

(i) Positive slope: $m = \frac{6-0}{4-0} = \frac{3}{2}$. The y-intercept is at (0, 0). Thus $\boldsymbol{y = \frac{3}{2}x}$.

(ii) Negative slope: $m = \frac{0-6}{4-0} = \frac{-3}{2}$. The y-intercept is at (0, 6). Thus $\boldsymbol{y = \frac{-3}{2}x + 6}$.

(d) The point of intersection is at the intersection of the two diagonals, or where $\frac{3}{2}x =$

$\frac{-3}{2}x + 6 \Rightarrow 3x = 6 \Rightarrow x = 2$. If $x = 2$, $y = \frac{3}{2}(2) = 3$. The coordinates of the intersection are then **(2, 3)**.

10. **(a)** $\boldsymbol{y = {}^{-}2}$, a horizontal line through $(0, {}^{-}2)$.

(b) $\boldsymbol{x = {}^{-}4}$, a vertical line through $({}^{-}4, 5)$.

11. **(a)** $m = \frac{2-1}{5-{}^{-}4} = \boldsymbol{\frac{1}{9}}$.

(b) $m = \frac{198-81}{{}^{-}3-{}^{-}3} = \frac{117}{0} \Rightarrow$ **undefined** (or no slope).

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(c) $m = \frac{10-12}{1-1.0001} = \frac{^-2}{^-0.0001} = \mathbf{20{,}000}$.

(d) $m = \frac{\sqrt{5}-\sqrt{5}}{^-2-2} = 0$.

12. (a) Use the point $(5,2)$. $(2) = \frac{1}{9}(5) + b \Rightarrow$ $b = \frac{13}{9}$, thus $\boldsymbol{y = \frac{1}{9}x + \frac{13}{9}}$.

(b) $\boldsymbol{x = ^-3}$ (a vertical line).

(c) Use the point $(1,10) \Rightarrow (10) = 20{,}000(1) + b \Rightarrow$ $b = ^-19{,}990$, thus $\boldsymbol{y = 20{,}000x - 19{,}990}$.

(d) $y = \sqrt{5}$.

13. (a) In eight months, $\$2180 - \$2100 = \$80$ simple interest was earned, or $\$10$ per month. (10 months) $\cdot$ ($\$10$ per month) $= \$100$ interest earned in the first ten months. The original balance was $\$2100 - \$100 = \mathbf{\$2000}$.

(b) Simple interest (I) $=$ principal (p). annual rate (r) $\cdot$ time (t) in years.
$r = \frac{1}{pt} = \frac{\$100}{\$2000 \cdot \frac{10}{12}} = 0.06$, or **6%**.

14. $y = x - 4$ is line $\overleftrightarrow{BC}$; $y = 2 - 2x$ is line $\overleftrightarrow{AB}$; and $y = 5$ is line $\overleftrightarrow{CD}$. Point A is the x-intercept of line $\overleftrightarrow{AB}$, $0 = 2 - 2x \Rightarrow x = 1$ so (1,0). Point B is the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$, $2 - 2x = x - 4 \Rightarrow 6 = 3x \Rightarrow x = 2$ so $(2,^-2)$. Point C is the intersection of $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$, $x - 4 = 5 \Rightarrow x = 9$ so (9,5).

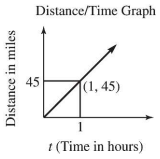
15. (a) The lines have the same x-intercept, 3.

(b) The lines have the same x-intercept, 3.

(c) The lines have the same slope, 1.

(d) All non-vertical lines through the point (1,1).

16. (a)



(b) Think of slope as a ratio, 45 miles per 1 hour. So slope or $\frac{45}{1} = \mathbf{45}$.

17. (a) Rewrite each equation into the form $y = mx + b$. $2x - 6y = 7 \Rightarrow y = \frac{1}{3}x - \frac{7}{6}$ and $3x - 9y = 10 \Rightarrow y = \frac{1}{3}x - \frac{10}{9}$.

Since both equations have slope $\frac{1}{3}$ and different y-intercepts, they are parallel. Since they never intersect at a single point there is **no solution**.

This can be illustrated by solving the system. Set $\frac{1}{3}x - \frac{7}{6} = \frac{1}{3}x - \frac{10}{9} \Rightarrow 18\left(\frac{1}{3}x - \frac{7}{6}\right) = 18\left(\frac{1}{3}x - \frac{10}{9}\right) \Rightarrow 6x - 21 = 6x - 20 \Rightarrow 0 = 1$.

Since $0 \neq 1$, it can be seen that there is no solution.

(b) Rewrite each equation into the form $y = mx + b$. $4x - 6y = 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{6}$ and $6x - 9y = 1.5 \Rightarrow y = \frac{2}{3}x - \frac{1}{6}$.

Since both equations represent the same line, $y = \frac{2}{3}x - \frac{1}{6}$, there are an infinite number of solutions of the form $\left(x, \frac{2}{3}x - \frac{1}{6}\right)$.

In attempting to solve these types of systems, a solution such as $0 = 0$ will always be obtained. An answer of this type means an infinite number of solutions is possible.

(c) Substitute $y = 2 - x$ in $y = x - 2$:

$2 - x = x - 2 \Rightarrow 4 = 2x \Rightarrow x = 2$. Substitute 2 for x: $y = 2 - 2 \Rightarrow y = 0$. There is a unique solution (2,0).

(d) Multiply $^-\frac{3}{2}x + y = \frac{1}{2}$ by $2 \Rightarrow$ $^-3x + 2y = 1$

$3x - 2y = ^-1$

and add the equations: $\underline{^-3x + 2y = \quad 1}$. There

$0 = 0$

are infinitely many solutions of the form $\left(x, \frac{3}{2}x + \frac{1}{2}\right)$.

18. The equation of the segment connecting $(5,0)$ and $(6,8)$ is $\boldsymbol{y = 8x - 40}$ (determined by finding m; then substituting x and y from one of the points into the equation $y = mx + b$ to find b).

The equation of the segment connecting $(10,0)$ and $(3,4)$ is $\boldsymbol{y = \frac{-4}{7}x + \frac{40}{7}}$.

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The equation of the segment connecting $(0,0)$ and $(8,4)$ is $\boldsymbol{y = \frac{1}{2}x}$.

Equating these three values of y (i.e., $8x - 40 = \frac{-4}{7}x + \frac{40}{7} = \frac{1}{2}x$) yields $x = \frac{16}{3}$.

Substituting $x = \frac{16}{3}$ into any of the three equations yields $y = \frac{8}{3}$. The coordinates of the common intersection are $\left(\frac{16}{3}, \frac{8}{3}\right)$.

Mathematical Connections 8-5: Review Problems

15. **(a)** $\boldsymbol{\sqrt[3]{x} = x^2 - 3}$.

(b) $\boldsymbol{x^2 + y^2 = 36}$.

(c) $\boldsymbol{\sqrt{0.\overline{9}} = 1}$.

16. $\sqrt{6} \approx \mathbf{2.45}$.

17. **(a)** $x\sqrt{2} - 3y = 0.\overline{4} \Rightarrow x\sqrt{2} = 0.\overline{4} + 3y \Rightarrow x = \frac{0.\overline{4}+3y}{\sqrt{2}}$.

(b) $x^2 - 81 = 0 \Rightarrow (x-9)(x+9) = 0$
$\Rightarrow x - 9 = 0$ or $x + 9 = 0$
$\Rightarrow \boldsymbol{x = 9}$ or $\boldsymbol{x = -9}$.

(c) $3x < -\sqrt{7} \Rightarrow x < \frac{-\sqrt{7}}{3}$.

18. **(a)** $f(3) = 3\sqrt{7} - \sqrt{7} = \mathbf{2\sqrt{7}}$.

(b) $f(\sqrt{7}) = \sqrt{7} \cdot \sqrt{7} - \sqrt{7} = \mathbf{7 - \sqrt{7}}$.

(c) $f(-4) = -4\sqrt{7} - \sqrt{7} = -5\sqrt{7}$.

19. $12 = 3x - \sqrt{2} \Rightarrow 12 + \sqrt{2} = 3x$
$\Rightarrow x = \frac{\mathbf{12+\sqrt{2}}}{\mathbf{3}}$.

20. **(a)** For $f(x) = \sqrt{x+1}$ to have real-valued range values, $x + 1$ must not be negative. So $x + 1 \geq 0 \Rightarrow \boldsymbol{x \geq -1}$. Write as $\{x \mid x \in \mathbb{R}$ and $x \geq -1\}$.

(b) For $f(x) = \sqrt{-x}$ to have real-valued range values, $-x$ must not be negative. So $-x \geq 0$ $\Rightarrow \boldsymbol{x \leq 0}$. Write as $\boldsymbol{\{x \mid x \in R \text{ and } x \leq 0\}}$.

Chapter 8 Review

1. **(a)** **Irrational**. The pattern never repeats in blocks of the same length.

(b) **Irrational**. Any non-zero rational number divided by any irrational number is irrational.

(c) **Rational**. The ratio of two integers.

(d) **Rational**. The pattern repeats in blocks of 0011.

(e) **Irrational**. The pattern does not repeat.

2. **(a)** $\sqrt{484} = 22$

(b) $\sqrt{288} = \sqrt{144 \cdot 2} = \mathbf{12\sqrt{2}}$.

(c) $\sqrt{180} = \sqrt{36 \cdot 5} = \mathbf{6\sqrt{5}}$.

(d) $\sqrt[3]{162} = \sqrt[3]{27 \cdot 6} = \mathbf{3\sqrt[3]{6}}$.

3. **(a)** **No**. $-\sqrt{2} + \sqrt{2} = 0$ is rational.

(b) **No**. $\sqrt{2} - \sqrt{2} = 0$ is rational.

(c) **No**. $\sqrt{2} \cdot \sqrt{2} = 2$ is rational.

(d) **No**. $\frac{\sqrt{2}}{\sqrt{2}} = 1$ is rational.

4. $4 < \sqrt{23} < 5$.

$(4.7)^2 = 22.09$ and $(4.8)^2 = 23.04 \Rightarrow$
$4.7 < \sqrt{23} < 4.8$;

$(4.79)^2 = 22.9441$ and $(4.80)^2 = 23.04 \Rightarrow$
$4.79 < \sqrt{23} < 4.80$;

$(4.795)^2 = 22.992025$ and $(4.796)^2 = 23.001616 \Rightarrow 4.795 < \sqrt{23} < 4.796$;

23 is closer to 23.001616 than to 22.992025, thus $\sqrt{23} \approx \mathbf{4.796}$.

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5. $1 < \sqrt[3]{2} < 2$

$1.2^3 = 1.728$ and $1.3^3 = 2.197$.

$1 \cdot 2 < \sqrt[3]{2} < 1.3$.

$1.25^3 = 1.953125$ and $1.26^3 = 2.000376$.

Thus, $1.25 < \sqrt[3]{2} < 1.26$.

2 is closer to 2.000376 that it is to 1.953125,

so $\sqrt[3]{2} \approx \mathbf{1.26}$.

6. **(a)** Since $a_1 = 5$ and $a_3 = 10, 10 = 5(r)^2$. This implies that $r^2 = 2$ and $r = \sqrt{2}$ or $r = {}^-\sqrt{2}$. So, $a_2 = \mathbf{5\sqrt{2}}$ **or** $\mathbf{{}^-5\sqrt{2}}$.

(b) Since $a_1 = 1$ and $a_5 = \frac{1}{4}, \frac{1}{4} = 1(r)^4$. This implies that $r^4 = \frac{1}{4}$ and $r = \frac{1}{\sqrt[4]{4}}$ or $\frac{-1}{\sqrt[4]{4}}$.

Thus, two possible solutions are: $\mathbf{1, \frac{1}{\sqrt[4]{4}}, \frac{1}{\sqrt[4]{16}}} = \mathbf{\frac{1}{2}, \frac{1}{2\sqrt[4]{4}}}, \frac{1}{4}$ and $\mathbf{1, \frac{-1}{\sqrt[4]{4}}, \frac{1}{2}, \frac{-1}{2\sqrt[4]{4}}}$,

7. $\sqrt{7}({}^-1)^{10} = \sqrt{7}$. The ratio $r = {}^-1$.

8. The difference is $a_2 - a_1 = \sqrt{2} - 1$. Thus, $\boldsymbol{a_n = 1 + (n-1)(\sqrt{2} - 1)}$.

9. $\boldsymbol{S = 13P}$.

10. There are 103 times as many girls as boys.

11. $\boldsymbol{f = 3y}$.

12. Let $S = n_1 + n_2 + n_3 + \cdots$. Then $S_{new} = (10n_1 - 10) + (10n_2 - 10) + (10n_3 - 10) + \cdots = 10[(n_1 - 10) + (n_2 - 10) + (n_3 - 10) + \cdots] = \mathbf{10S - 10n}$.

13. Let n be the whole number. Then $12\left(\frac{n}{13}\right) - 20 + 89 = 93 \Rightarrow 12\left(\frac{n}{13}\right) = 24 \Rightarrow \frac{n}{13} = 2 \Rightarrow \boldsymbol{n = 26}$.

14. **(a)** Let n be the number. Then:

(i) $n + 17$

(ii) $2(n + 17) = 2n + 34$

(iii) $(2n + 34) - 4 = 2n + 30$

(iv) $2(2n + 30) = 4n + 60$

(v) $(4n + 60) + 20 = 4n + 80$

(vi) $\frac{4n+80}{4} = n + 20$

(vii) $n + 20 - 20 = \boldsymbol{n}$.

(b) Answers may vary; for example, the next two lines could be to add 3, divide by 4, and subtract 17. Then $\frac{[4(n+18)-7]+3}{4} - 17 = n$.

15. **(a)** $4(7x - 21) = 14(7x - 21) \Rightarrow 28x - 84 = 98x - 294 \Rightarrow 210 = 70x \Rightarrow \boldsymbol{x = 3}$.

(b) $3(\sqrt{x} - 1) = \sqrt{9x} + 5 \Rightarrow 3\sqrt{x} - 3 = 3\sqrt{x} + 5 \Rightarrow 3\sqrt{x} - 3\sqrt{x} = 8 \Rightarrow 0 \neq 8$ so there is no solution.

(c) $4x - 2 = 3x + \sqrt{10} \Rightarrow \boldsymbol{x = 2 + \sqrt{10}}$.

(d) $2(x + \sqrt{3}) = 3(x - \sqrt{3}) \Rightarrow 2x + 2\sqrt{3} = 3x - 3\sqrt{3} \Rightarrow 5\sqrt{3} = x$**.**

16. Let m be the number of Mikes's cards, j be the number of Jordan's cards, and p be the number of Paige's cards.

It is given that $j = 2p$ and $m = 3j = 3(2p) = 6p$.

Then If $m + j + p = 999 \Rightarrow p + 2p + 6p = 999 \Rightarrow 9p = 999 \Rightarrow p = 111$.

So Paige has **111 cards**

Jordan has **222 cards**

Mike has **666 cards.**

17. Let s be the number of science book overdue days and c be the number of children's book overdue days. It is given that $c = s - 14$.

Then $8(0.20)(s - 14) + 2(0.20)s = 11.60 \Rightarrow 1.6s - 22.4 + 0.4s = 11.60 \Rightarrow 2s = 34 \Rightarrow s = 17$.

Each science book was overdue by **17 days**

Each children's book was overdue by $17 - 14 =$ **3 days**.

18. Let j be the number of papers delivered by Jacobo, d be the number of papers delivered by Dahlia, and r be the number of papers delivered by Rashid.

It is given that $d = r + 100$ and $j = 2d = 2(r + 100)$.

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Then if $j + d + r = 500 \Rightarrow r + (r + 100) + 2(r + 100) = 500 \Rightarrow 4r + 300 = 500 \Rightarrow r = 50$.

So Rashid delivered **50 papers**

Dahlia delivered $50 + 100 =$ **150 papers**

Jacobo delivered $2(50 + 100) =$ **300 papers**.

19. **(a)** **A function**. Each component of the domain corresponds to a unique component of the range.

(b) **Not a function**. a and b both correspond to two components of the range.

(c) **A function**. a and b correspond to unique components of the range.

(d) **A function**. Each component of the domain corresponds to a value of 2 in the range.

20. **(a)** $\{0, 1, 2, 3\} + 3 \Rightarrow$ Range is $\{x + 3 | x \in \{0, 1, 2, 3\}\} =$ **{3, 4, 5, 6}**.

(b) Range is $\{3x - 1 | x \in \mathbb{R}\} = \mathbb{R}$

(c) The domain is $\{x^2 | x \in \mathbb{R}\}$ so the range is $\{x \mid x \geq 0\}$.

(d) $\{0, 1, 2\}^2 + 3\{0, 1, 2\} + 5 \Rightarrow$ Range is $\{x^2 + 3x + 5 \mid x \in \{0, 1, 2\}\} =$ **{5, 9, 15}**.

21. **(a)** **Not a function**. A student may have more than one major.

(b) **A function**. The range is the subset of the natural numbers that includes the number of pages in each book in the library.

(c) **A function**. The range is $\{x | x \geq 6$ and x is even$\}$.

(d) **A function**. The range is $\{0, 1\}$.

(e) **A function**. The range is the set of all natural numbers N.

22. **(a)** $\boldsymbol{C(x) = \$[200 + 55(x - 1)]}$ where x is the number of months.

(b) Plot:

x	$C(x)$
1	200
2	255
3	310
4	365
5	420
6	475
7	530
8	585
9	640
10	695
11	750
12	805

(c) The cost will exceed \$600 beginning with the **ninth month**.

(d) If $C(x) = 200 + 55(x - 1) = 6000$ then $55x - 55 = 5800 \Rightarrow x = 106.5$.

Or the cost will exceed \$6000 in the **107th month**.

23. **(a)** $4x - 5 = 15 \Rightarrow 4x = 20 \Rightarrow x = 5$.

(b) $x^2 - 1 = 2 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$.

(c) $\sqrt{^-x} = 3 \Rightarrow {}^-x = 9 \Rightarrow x = {}^-9$.

(d) $\sqrt{x} = {}^-3$ has no solutions. All square roots have positive answers.

(e) $\dfrac{x+1}{x+2} = {}^-1 \Rightarrow x + 1 = ({}^-1)(x + 2)$

$x + 1 = {}^-x - 2 \Rightarrow 2x = {}^-3 \Rightarrow x = {}^-\dfrac{3}{2}$

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(f) $\dfrac{x+1}{x+2} = 1 \Rightarrow x+1 = x+2 \Rightarrow 0x = 1 \Rightarrow 0 \neq 1$ no solution.

24. (a) **A function**. Each value on the x-axis corresponds to a unique value on the y-axis.

(b) **Not a function**. 4 and 5 on the x-axis each correspond to two values on the y-axis.

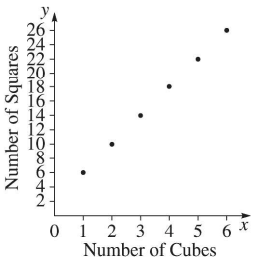
(c) **Not a function**. The x-value 5 corresponds to two values on the y-axis.

25. Assume Jilly starts with unpainted blocks each time:

(a)

Number of cubes	Number of squares to paint
1	6
2	10
3	14
4	18
5	22
6	26

(b)



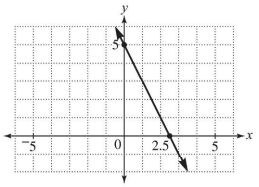
(c) This is a function which is an arithmetic sequence with $a_1 = 6$, $d = 4$, and $a_6 = 26 \Rightarrow a_n = 6 + (n-1)4 \Rightarrow 4n + 2$. The function is $y = \mathbf{4x + 2}$ for $x = 1, 2, ..., 6$.

(d) **No**. The graph does represent a function, but not a straight line because values of x cannot assume anything but natural numbers.

26. Use a spreadsheet approach, where values of x are in column A and corresponding values of the functions are in column B.

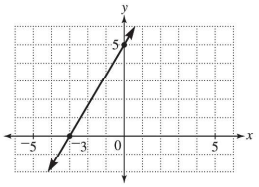
(a) $y = -2x + 5$:

A	B
−1	7
0	5
1	3
2	1
⋮	⋮



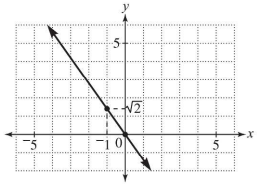
(b) $-\dfrac{x}{3} + \dfrac{y}{5} = 1 \Rightarrow -5x + 3y = 15 \Rightarrow y = \dfrac{5}{3}x + 5$ (values of x are multiples of 3):

A	B
−6	−5
−3	0
0	5
3	10
6	15
⋮	⋮



(c) $y = -\sqrt{2}x$ (values of x in column A are selected to yield whole-number values of y in column B):

A	B
$-\sqrt{2} \approx -1.4$	2
0	0
$\sqrt{2} \approx 1.4$	−2
$\sqrt{8} \approx 2.8$	−4
⋮	⋮



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27. **(a)** $4x+3y-12=0$ is the graph of $\overleftrightarrow{CD}$.

(b) $x-y-1=0 \Rightarrow y=x-1$

$4x+3y-12=0 \Rightarrow y=4-\frac{4}{3}x$

$x-1=4-\frac{4}{3}x \Rightarrow 3x-3=12-4x$

$7x=15 \Rightarrow x=\frac{15}{7}$

$y=\frac{15}{7}-1 \Rightarrow y=\frac{8}{7}$

The coordinates of point E are $\left(0,\frac{8}{7}\right)$.

(c) $x-y-1=0 \Rightarrow 0-y-1=0 \Rightarrow y=^{-}1$, the coordinates of point B are $(0,^{-}1)$. Point F has the same y-coordinate. Using the equation for $\overleftrightarrow{CD}$ substitute $^{-}1$ for y and solve for x.

$4x+3(^{-}1)-12=0 \Rightarrow 4x-15=0 \Rightarrow x=\frac{15}{4}$. The distance of $\overline{BF}$ is $\frac{15}{4}-0=\frac{15}{4}$.

28. **(a)** $^{-}3x+12=23 \Rightarrow ^{-}3x=11 \Rightarrow x=^{-}\frac{11}{3}$

(b)

$\sqrt{2}x-5=\sqrt{8}x+1$

$\sqrt{2}x-5=2\sqrt{2}x+1$

$\sqrt{2}x-2\sqrt{2}x=6$

$^{-}\sqrt{2}x=6 \Rightarrow x=^{-}\frac{6}{\sqrt{2}}=^{-}3\sqrt{2}$

(c) $x^3=^{-}2 \Rightarrow x=\sqrt[3]{^{-}2}$

(d) $4x^2-33=3 \Rightarrow 4x^2=36 \Rightarrow x^2=9 \Rightarrow \sqrt{x^2}=\sqrt{9} \Rightarrow |x|=3 \Rightarrow \boldsymbol{x=\pm 3}$.

(e) $(x-1)^3=2 \Rightarrow x-1=\sqrt[3]{2} \Rightarrow x=1+\sqrt[3]{2}$.

(f) $(2x+1)^4=5 \Rightarrow 2x+1=\pm\sqrt[4]{5} \Rightarrow x=\frac{^{-}1\pm\sqrt[4]{5}}{2}$.

(g) Set each factor equal to zero and solve.

$x - 1 = 0 \Rightarrow x = 1$

So

or $2x + 1 = 0 \Rightarrow x = \frac{1}{2}$.

29. Yes. The new ratio is the cube root of the original ratio. If x and y are successive terms in the original sequence with ratio r, so that $r=\frac{y}{x}$, then $\frac{\sqrt[3]{y}}{\sqrt[3]{x}}\sqrt[3]{\frac{y}{x}}=\sqrt[3]{r}$ which is constant.

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